Making the familiar strange: An Analysis of Language in Postsecondary Calculus Textbooks then and now

Contributed Research Report
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Abstract*:* Three calculus textbooks covering a span of about 40 years were examined to determine whether and how the language used has changed given the reform movement and the impetus to make mathematics accessible to all. Placed in a discourse analytic framework using Halliday's (1978) theory of functional components –ideational, interpersonal and textual, and using the exposition of the concept of a function as a unit of comparison, the study showed that language is an integral indicator of the author's view of mathematics and an important factor for textbook adoption in the pursuit of student success.

Keywords*:* discourse analysis, calculus textbooks, language of mathematical discourse

Introduction

In the late 1980s, the Calculus Consortium at Harvard (CCH) was funded by the National Science Foundation to redesign the Calculus curriculum with a view to making Calculus more applied, relevant, and accessible. The intent was to re/think and re/present the content so as to focus on real-world applications, to emphasize concepts and graphical representations, and to take advantage of the increasingly sophisticated technology. Calculus is now presented in a manner radically different from the traditional approach of abstraction, formal notation and symbolism, and algebraic conventions.

The goal of this research is to see whether and how calculus textbooks designed for the postsecondary level in 'regular' Calculus courses have changed over the years with respect to the language used in the exposition and by inference, the view of mathematics manifested. One concept, that of a function and in particular its definition, is chosen and used to trace the dimensions of the language over the years and the consequent shifts in the view and presentation of mathematics in calculus textbooks. The research questions are: Has the language of calculus textbooks changed over time and if so, in what ways? Has the language changed from one that is exclusive (mathematics as an elite subject with an elite community) to one that is inclusive and accessible to all? From the language, how are the authors' views of mathematics characterized and how have they changed over time?

The three textbooks I have chosen are *Calculus* by Spivak (1967), *The Calculus of a Single Variable with Analytic Geometry*, 5th edition by Leithold (1986), and *Single Variable Calculus: Early Transcendentals*, 5th edition by Stewart (2003). Textbooks may be studied subjectively to describe the interaction between the student and the written material or to describe teachers' use of textbooks and the subsequent effect on the teacher (Remillard et al, 2009). However, following Herbel-Eisenmann (2007), I seek to examine the 'voice' of calculus textbooks over the years as *objectively given structure* (emphasis in the original, p.396). This examination will be placed in a discourse analytic framework which attends to the aspects of text relating to language, voice, agency and identity.

Analytic Framework

Language has been increasingly seen as an important issue relating to mathematics teaching and learning. Rowland (2000) emphasizes two principles in studying language: the linguistic principle ('language as means of accessing thought') and the deictic principle

(language as a means of communication and a 'code to express and point to concepts, meanings and attitudes') (p. 2). In his *Language as a Social Semiotic*, Halliday (1978) identifies three functional components or functions of language– the ideational, the interpersonal, and the textual –from which meaning is apprehended. The ideational functional component of the text answers the questions: What is the view of mathematics as presented in the text? How is the subject of mathematics envisioned in the mind of the author of the text and in what style is it rendered? The interpersonal functional component describes the social and personal roles and relationships among the authors and readers. Evidence of this function is discerned by considering the use of personal pronouns (first, I/we/us/our, and second person, you), imperatives, and modality. The textual functional component describes the content matter or the mathematics presented in the text, the theme and modes of reasoning, the arguments and their forms, and the narratives of mathematical activity.

Each of the textbooks will be examined as to the "voice" that emerges, the extent of agency, and the construction of the identity of the reader by the text.

Method

The data consists of the 10 – 14 pages from the each of the three Calculus textbooks that cover the exposition of the concept of a function. Exposition includes the preliminary introductory commentary and the definition (or definitions) of a function. I mined the relevant pages carefully with respect to the linguistic markers for the three functions as articulated by Halliday.

Findings and Discussion

Table 1 gives the results of the comparison of the textbooks across markers for the functional components of language with respect to the concept of a function.

Table 1. Comparison across markers for the functional components.

	Spivak (1967)	Leithold (1986)	Stewart (2003)
Pronouns - 1st person	we/us/our 32 instances	we/us 5 instances	we/us 24 instances
Pronouns – 2nd person	you 9 instances	None	you 3 instances
Imperatives Inclusive	let's 1 instance	call, compare, let, note, observe, recall 6 instances	consider, determine, let, notice, remember 7 instances
Imperatives Exclusive	None	find, read 4 instances	draw, find, sketch, use 6 instances
Modal verbs	May 2 instances	None	None
Questions	2	None	1
Conditionals	If 6 instances if … then 10 instances	Given 3 instances given that 2 instances	If 3 instances if … then 4 instances

Beginning with the interpersonal component, the most striking occurrence is that of 32 instances of first-person pronouns in Spivak as compared with five in Leithold and 23 in Stewart. In Spivak, there were 29 uses of *we*, two of *us* and one of *our*. From the opening paragraph in his liberal use of *we* and *us*, Spivak sets the tone of including the reader in his deliberations. Spivak clearly views the reader as someone who is part of the community of people doing or studying mathematics. Another possible reading is that the use of *we, us*, and *our* suggests a more general form indicative of the register of mathematicians. In comparison, the five occurrences of *we* in Leithold read clinically as in 'we see that' or 'we observe that'. The use of personal pronouns indicates the presence or absence of humans in the activity and the implied distance and degree of formal relationship between the author and the reader (Morgan, 1996). Leithold deploys his words in a detached 'scientific' manner, the very opposite of the kind of writing that Burton and Morgan (2000) exhort mathematicians to adopt.

The frequency of imperatives in a text indicates the degree to which the author wishes to draw the reader's attention to a point in the text (note that, observe that), to encourage the reader to reflect (consider, compare, recall, remember), or to give a simple command (find, sketch, use). Both Leithold and Stewart use a similar number of imperatives that indicate the usual textbook framing (consider, notice, observe, recall) and that signal the ability of the author (determine, evaluate, find, sketch, use) to tell the reader what to do. It is note-worthy that Spivak does not use any of these imperatives but still manages by his use of personal pronouns to convey a sense of introducing the reader to and including the reader in the activity that mathematicians undertake.

Modality, as a feature of language, enables authors and speakers to express their feelings, values, attitudes, and judgments about the propositions in their texts. Demonstrations of modality include modal auxiliary verbs such as 'may' and 'can', adverbs relating to the uncertain state of knowledge such as 'possibly' and 'maybe', the use of moods and tenses, and the use of hedges (Rowland, 2000, p. 65). For these three textbooks there was little or no evidence of modality. There were two instances of 'may' in Spivak ('You may feel that we have also reached…' and 'Two consolations may be offered', p. 45). These have nothing to do with the mathematics involved but indicate concern for and offer solace to the reader. Leithold and Stewart offer no suggestion that that there is any uncertainty related to mathematical activity and by their lack of use of modality, indicate a view of mathematics that strongly holds to an absolute, ideal perspective.

For the textual component, all three authors use the mode of discourse characterized by exposition (evident of the raison d'être of the textbook) in laying out a clear and concrete treatment of the subject matter. Questions as evidence of a conversational or dialogic style of exposition were barely used; there were two questions in Spivak, none in Leithold and one in Stewart.

The ideational functional component in each of the three textbooks is very nearly identical in that the authors' content and meaning are similar. Each author is interested in communicating the content of the concept of a function and introducing the objects and relations that are under consideration when discussing the concept of a function. Each encodes in the text his individual vision of mathematics. The view of mathematics evinced in all three is fixed, absolute, and formal.

As seen from these linguistic markers, the tenor of the language in evoking the relationship between the author and the reader in the three textbooks is markedly different. Spivak and Leithold are diametrically opposite in the use of the first and second person pronouns and imperatives in engaging and addressing the reader with Stewart striking a moderate note in this regard. In summary, the three textbooks are similar in their theme and message but differ

considerably in the interpersonal component with Stewart capturing a moderate position between what may be considered the extremes of linguistic markers by Spivak and Leithold.

Implication

The language of mathematics is often seen as foreign with its own lexicon, grammar, and modes of argument. More than being able to negotiate the language, students of mathematics must become fluent in it. Bakhtin declares that '[e]ach text presupposes a generally understood (that is, conventional within a given collective) system of signs, a language (if only the language of art)' (1953/1986, p. 105). Hence the mathematics textbook has a conventional system of signs which is part of a language that is to be understood if one wishes to be a member of the community involved in mathematical activity.

The differences in language in a textbook account for much of the reader's regard for the textbook. In this paper I have teased out the subconscious linguistic markings in the text and have shown that there is more to the text than meets the eye; that what we have taken as familiar is indeed strange: a nebulous complex of beliefs and ideas about mathematics which we adopt and perpetuate without realizing the implications and consequences. This analysis suggests that it behooves us as teachers to re/examine our practices in making textbook choices for the betterment of ourselves and our students and to be aware of the functions and forms of language that subtly maintain hegemonic practices in the teaching and learning of mathematics.

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