Title: Toulmin Analysis: A Tool for Analyzing Teaching and Predicting Student Performance in Proof-Based Classes

Abstract: This paper provides a method for analyzing undergraduate teaching of proof-based courses based on Toulmin's model of argumentation. The paper then describes how that analysis can be used as a predictor of subsequent student proof-writing performance and shows that the predictions are reasonable approximations of students' subsequent proof-writing. The method of analysis was developed via research in a lecture-based abstract algebra class, it has application, to any lecture-based, proof-intensive course. This method provides one possible way to directly link classroom teaching activities to subsequent student performance that would force instructors to assume more responsibility for their students' demonstrated end-of-course performance.

Keywords: proof, Toulmin analysis, abstract algebra, classroom research

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Introduction and research questions

It is suggested in numerous studies (Dreyfus, 1999; Dubinsky, et. al, 1994; Leron, Hazzan, & Zazkis, 1995; Weber, 2001) that students are not learning at the level that faculty desire. What needs to change? Advisory reports from national research associations have called upon faculty to move away from the lecture format and adopt other teaching methods (National Science Foundation (*NSF*), 1992; Mathematical Sciences Education Board (*MSEB*), 1991). However, instructors often believe that students bear the responsibility of learning (Wu, 1999). New insight is needed to help these instructors understand how their actions in the classroom affect their students' ability to master the material. This study will develop new tools needed to analyze lecture-based teaching and will directly connect instruction to student learning in proof-based courses. In particular the study:

1) Uses Toulmin's (1969) model of argumentation to analyze the teaching of proof-based undergraduate courses taught via lecture.
2) Proposes that students in the class will adopt the argumentation methods as modeled by their instructor.
3) Analyzes student work to determine whether the Toulmin-analysis of teaching does predict students' proof-writing behavior.

1 Literature

Despite the lack of use of non-traditional curricula and pedagogies, the *NSF* and professional organizations such as the Mathematical Associan of America (*MAA*) continue to fund the development of new curricula and professional development in the hope of changing collegiate mathematics teaching and improving student learning. Each of these curriculum projects has promised to disseminate their work, typically via professional development activities at mathematics conferences. This curriculum development and implementation model almost exactly mirrors the process used at the K-12 level in which "many efforts over the past decade or so have been aimed at providing well-designed curricula for school mathematics… Each has spawned an industry of workshops and conferences focused on helping teachers prepare to use the materials in their classrooms" (Sowder, 2007, p. 177). It has been shown that these efforts "have not been particularly successful in educational projects" (Richardson & Placier, 2001, p. 906); that is, they have not affected meaningful change in teaching practices.

New studies of pedagogical methods suggest that it is essential that teachers reflect on their beliefs and practices in order to affect meaningful change (Richardson & Placier, 2001). Yet, at the undergraduate level faculty generally hold the belief that "the professor gives an outline of what and how much students should learn, and students do the work on their own outside" of the class meetings (Wu, 1999, p. 267). This contract implies that as long as the instructor delivers a clear lecture and communicates appropriately, the students are responsible for their own failure to fully comprehend the instructor's intent or apprehend the deeper structure of the material as well as adopt the mathematical behaviors, such as proof-writing strategies, that the instructor models.

I posit that students are appropriating some of the modeled behavior, but not always the aspects that faculty believe are most important. This argument is supported by new research on transfer-in-pieces (Wagner, 2006). This research claims that experts and novices perceive the same aspects of an instructor's presentation as having differing levels of importance, and as a result attend to different aspects. Wagner (2006) then demonstrated that students would transfer aspects of a structure from one problem to another, even when then they seem mathematically inappropriate from an expert's perspective.

In a proof-based course, such as abstract algebra, the instructors are modeling proof-writing strategies and the types of arguments that they expect from their students. Thus, we should be able to observe which aspects of proof-writing and types of arguments are appropriated by the students. After analyzing the classroom teaching via Toulmin analysis (1969), this paper analyzes student work and then provides a preliminary means of linking the analyses of classroom instruction with student learning.

2 Data and Methodology for analysis

I took detailed field notes while observing 18 class meetings. I also collected demographic information on all students in the class and work from 6 participants of 12 students on a content assessment that required them to write a ring-theory proof.

I transcribed all text on the board in addition to classroom dialogue. I reviewed all classroom video recordings and made a log of all episodes that included proof-writing or presentation. Criteria for proof-production or presentation was rather straightforward. An incident was logged as such when any member of the class community was writing or showing a formal mathematical proof that drew on symbolic notation and logical reasoning. To pursue the question of whether students had appropriated the proof-writing behavior that Dr. Tripp had modeled required analysis of the proofs she wrote during class and the student's work outside of class. I analyzed all such arguments using Toulmin's (1969) model into one of the following: data, warrant, backing, qualifier or conclusion. I also noted when an aspect of argumentation was written or spoken aloud. In this paper I present two sets of arguments, the first set was chosen because, collectively, they were the only observed instance of Dr. Tripp demonstrating all of the properties for a sub-ring proof. The second set of examples was chosen because they mirrored the sub-ring property proofs in a different structure. I employed a Toulmin analysis on the student work similar to that used to analyze instruction.

3 Results

3.1 Dr. Tripp's presentation

In the proof to be discussed in this paper we see the following patterns in Dr. Tripp's written presentation of mathematical property-verification proofs (excluding spoken comments):

	Data	Warrant	Backing	Qualifier	Conclusion
Written	13	5	0	0	13

A similar pattern was recapitulated across all of Dr. Tripp's proof-writing. When Dr. Tripp models proof writing, she always writes the data and conclusion but she was never observed to write a backing or a qualifier. Moreover, she only infrequently wrote warrants but, when her spoken comments were included in the analysis she included warrants in more than half of all arguments. Because the students were only submitting written work, the table above does not take into account the fact that Dr. Tripp always spoke warrants aloud, nor does it reflect any of her statements of backing.

When we consider this mixed pattern of writing out versus only speaking the warrant for a particular piece of data as modeling the written arguments that she expected of her students, we should expect some mixed results. In terms of writing the data and conclusions, Dr. Tripp has always modeled writing those and, as a result, we should see that students always or almost always write the data and conclusion. But, her mixed writing of warrants may not provide her students a consistent model for their own work. Finally, we should expect that the students never, or almost never, write backing or a qualifier in their proofs.

3.2 Students' work

When all the student data on property-verification arguments is aggregated, without reference to the validity of the student's claim, we see the summary of written argument elements table below.

	Data	Warrant	Backing	Qualifier	Conclusion
Stated	32	14	2	1	30
Implied	1	18	7	4	3

The first observation is that the students almost universally wrote both the data and conclusion of their arguments. They were much more varied in writing out warrants or, at least, implying warrants in their workof the 33 written arguments, only 14 included a written warrant. Finally, the students almost never wrote out a backing or a quantifier, writing only two backings and one quantifier out of 33 arguments. In short, this is exactly what we would have expected from the students given Dr. Tripp's given Dr. Tripp's presentations of written proof.

4 Significance and directions for further research

This paper makes two significant contributions. First, I showed one way to draw upon Toulmin's (1969) model of argumentation to analyze proof-based courses including traditional taught abstract algebra courses. Moreover, Toulmin's model helped explain the relationship between the instructor's written proof and the classroom dialogue that she led. When Dr. Tripp's presentation is taken as modeling the type of mathematical behavior that she wants her students to demonstrate we would infer that she wants her students to always be able to articulate the data, warrant and conclusion of an argument.

Second, I analyzed student work by again drawing upon Toulmin's (1969) model for argumentation. When taken as a whole, the students' collective proof-writing that they also wrote a level of detail similar to that modeled by their instructor and used argumentation elements at a similar rate as Dr. Tripp. That is, the students almost perfectly demonstrated that they had appropriated Dr. Tripp's modeled proof-writing in terms of the level of detail that they included in their written work.

This research immediately suggests two future directions for research; both directed towards better understanding the development of students' mathematical proficiency. First, this use of Toulmin's framework to analyze teaching was helpful in making sense of some aspects of Dr. Tripp's writing and her classroom dialogue, but cannot explain all aspects of her modeling of proof-writing. We need significant research that studies teaching of proof (Harel & Sowder, 2007; Harel & Fuller, 2009) and, in particular, lecture-based teaching of proof. Moreover, we need new theoretical lenses to make sense of what lecture-based teachers are doing in classes and that also provide a means to explain student mathematical proficiency.

Furthermore, it is also worth pursuing the idea that teachers consciously model appropriate mathematical behavior for their students as a means of making sense of lecture-based undergraduate courses. For example, we should be exploring how instructors model for their students other fundamental mathematical skills such as exploring definitions and examples, organizing and linking knowledge, and abstraction and generalization from examples to name but a few. In general, significantly more research is needed to explain traditional lecture-based instruction of proof-based content courses and abstract algebra courses in particular.

Lastly, in this study I reported seemingly inconsistent behavior on the part of the instructor which was linked to inconsistent behavior on the part of the students. We might suggest that classroom professors make explicit statements, like, "listen to the types of questions I ask you and ask myself, these are the kinds of things *you* should be asking while you write proofs." Then, "to decide when you should write down the answers to these questions you should

…" This type of meta-level dialogue could lead the students to a better understanding of when warrants and backing need to be explicitly stated in a proof as well as decreasing their writing of invalid or incorrect proofs.

References:

Dreyfus, T. (1999). Why Johnny can't prove. *Educational Studies in Mathematics, 40*(1), 85-109.

Dubinsky, E., Dautermann, J., Leron, U., & Zazkis, R. (1994). On learning fundamental concepts of group theory. *Educational Studies in Mathematics, 27*(3), 267-305.

Harel, G. & Fuller, E. (2009). Contributions toward perspectives on learning and teaching proof. In D. Stylianou, M. Blanton, and E. Knuth (Eds.), *Teaching and learning proof across the grades; A K-16 perspective.* New York, NY: Routledge.

Harel, G. & Sowder, L. (2007). Towards a comprehensive perspective on proof. In F. Lester (ed.) *Second handbook of research on mathematical teaching and learning*. NCTM: Washington, DC.

Leron, U., Hazzan, O., & Zazkis, R. (1995). Learning group isomorphism: A crossroads of many concepts. *Educational studies in mathematics, 29*(2), 153-174.

Mathematical Sciences Education Board (MSEB). (1991). *Moving beyond myths: Revitalizing undergraduate mathematics*. Washington DC: National Academies Press.

National Science Foundation. (1992). *America's academic future: A report of the Presidential Young Investigator Colloquium on U.S. engineering, mathematics, and science education for the year 1010 and beyond*. Washington DC: author.

Richarson, V. & Placier, P., (2001). Teacher Change. In Richardson, V. (Ed). *Handbook of Research on Teaching*, (pp. 905-947). Washington DC: American Educational Research Association.

Sowder, J. (2007) The mathematical education and development of teachers. (Pp. 157-224.) In Lester, F. (Ed) Second Handbook of research on mathematics teaching and learning. Reston, VA: NCTM.

Toulmin, S. (1969). The uses of arguments. Cambridge: Cambridge University Press.

Wagner, J. F. (2006). Transfer in pieces. *Cognition and Instruction, 24*(1), 1-71.

Weber, K., (2001). Student difficulties in constructing proofs: The need for strategic knowledge. *Educational Studies in Mathematics, 48*(1), 101-119.

Wu, H. (1999). The joy of lecturing—With a critique of the romantic tradition of education writing. In S. G. Krantz (Ed.), *How to teach mathematics* (pp. 261-271). Providence, RI: American Mathematical Society.