Authority in the Negotiation of Sociomathematical Norms

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In 1996, Yackel and Cobb introduced the study of sociomathematical norms in an attempt to understand how students' mathematical autonomy might be fostered by their mathematical beliefs and values and to make sense of the complexity of mathematical activity in the classroom. They defined sociomathematical norms to be "normative aspects of mathematics discussion specific to students' mathematical activity (p. 461). In other words sociomathematical norms can be seen as the reoccurring mathematical aspects of discourse that focus on mathematical thinking rather than thinking about mathematics. For example, the social norm that you should justify your answer does not, by itself, insure that your justifications will be accurate, rigorous, or convincing. However, a sociomathematical norm that defines what constitutes a convincing argument can be introduced to set an expectation in the classroom that encourages strong mathematical activity in the form of justification (Yackel & Cobb, 1996; Kazemi & Stipek, 2001).

We propose that four components must be identified for an expectation to qualify as a sociomathematical norm. *The four components of a sociomathematical norm are: 1. a mathematical expectation is set forth, 2. a mathematical interpretation of the expectation occurs, 3. the expectation is agreed upon, and 4. the expectation is validated as legitimate.*

Researchers have documented sociomathematical norms introduced by a research team (McClain & Cobb, 2001), teachers (Yackel, Rasmussen, King 2000), or students (Hershkowitz & Schwarz, 1999). Once introduced, sociomathematical norms are negotiated and re-negotiated by various participants in the class. The negotiation process often allows the expectation, embedded within the sociomathematical norm, to become clear and understandable as it is interpreted by both teacher and students so that it is genuinely agreed upon. However, recently Levenson, Tirosh, and Tsamir, (2009) found that teacher endorsed norms, enacted norms, and student perceived norms may all be different within the same classroom. In this case, the expectation in teacher endorsed norms was not genuinely agreed upon.

In previous work, we suggested that authority in the classroom hinges on three major concepts: authority relation, legitimacy, and change (Gerson & Bateman, submitted June, 2010). The authority relation is a relationship between two or more people, with at least one person acting as the bearer of authority and at least one person acting as the receiver of authority. The bearer of authority makes a claim; the receiver of authority recognizes the claim as legitimate and is influenced to change his or her behavior. In traditional classroom settings, authority is usually hierarchal with the teacher acting as bearer of authority and the student acting as receiver of authority (Herbel-Eisenmann, Wagner, & Cortes, 2008). In inquiry-based classroom settings the bearer and receiver of authority are more fluid roles taken on by both teacher and student at different times (Hamm & Perry, 2002; Wilson & Lloyd, 2000). We define legitimacy of authority as "the knowledge, skills, position, or experiences that influence a person or group within an authority relationship (Gerson & Bateman, submitted June 2010)."

In addition we defined four general types of authority in the mathematics classroom by the knowledge, skills, position, or experiences that legitimize the authority: Hierarchal, Expertise, Mathematical, and Performative (Gerson & Bateman, submitted 2010). Briefly, Hierarchal authority is the authority a person holds because of their position in the class (e.g. as an instructor, or presenter). Expertise Authority is legitimized by the perceived expertise of the bearer either by proving their mathematics expertise or by having ownership in the creation of a solution. Mathematical authority is legitimized by mathematical argument and justification. Performative authority is legitimized by the ability to engage the class. These authorities can all be held by both instructors and students.

Method

Our research is set in a teaching experiment in a university honors calculus class by Hope Gerson and Janet Walter. Students worked on tasks designed or selected to elicit conceptually important calculus content without prior instruction. The corpus of data from this study is taken from two, 2-hour class periods in Calculus II taught in the fall of 2007. The data were chosen for two reasons. First they occurred early in the semester as sociomathematical norms were still being actively negotiated. Second, a compelling episode occurred at the beginning of the second day, where Michael, a student in the class, introduced a new way of thinking about what constitutes a mathematical difference and how mathematical difference should be explored. We recognized this episode as pertaining to the negotiation of sociomathematical norms and wanted to further understand the dynamics in play, in particular, under what authority are sociomathematical norms introduced and negotiated?

We analyzed four hours of videotape gathered on January 29 and 31, 2007. Members of the research team transcribed and independently verified videodata. Together, the authors used open coding on one-half hour of videodata to identify key ideas, such as authority, agency, social norms, and sociomathematical norms. Later, the authors independently coded surrounding episodes. We, then, came back together to build consensus about which codes were important, how to define them and how to recognize them in the data. This helped us refine our definition of sociomathematical norms and to more accurately recognize them in the data. We then used axial coding to look for patterns in the data.

Analysis and Discussion

In the following excerpt, after two groups presented their solution to the same task, Michael introduced a new expectation (lines 1 and 4) for what constitutes a mathematical difference. Heber and Tyler interpreted the expectation and began to negotiate with Michael the meaning of the new expectation.

1 Michael:	Now, we it looks like we've got two different equations? *[expectation]*
2 Heber:	Yeah. *[interpretation]*
3 Tyler:	But they're the same. *[interpretation]*
4 Michael:	Um, they're not the same equation, they both model something different, and I'm 90 percent sure that I know what that is, the difference. *[negotiation of meaning]*

In order for this new expectation that two equations are different if they "model something different" to become normative, Michael's expectation needed to be interpreted, agreed upon, and legitimized.

For the next ten minutes, Michael and the class continued to negotiate the meaning of Michael's expectation that they explore what the two different equations model. For example in the next excerpt, Michael re-states the expectation embedded in a mathematical argument in line 7, and Robert interprets that to mean solving part a, and expresses that he understands Michael's expectation.

5 Michael:	if you just get the volume function and just start evaluating the volume function *[mathematical argument]*
6 Derrick:	[inaudible] you could get a general equation *[interpretation]*
7 Michael:	Right, but I'm saying take an indefinite in, er a definite integral of their equation. What would that model? What, if you plug in the value of one, into their indefinite integral, what does that represent? *[mathematical argument and restatement of the expectation]*
8 Robert:	'Cause, 'cause do you want me to solve part a? 'cause it [inaudible] [moves thumb and index finger together] I see what you're asking. *[interpretation]*

Michael's initial statement in line 4, that "they both model something different" did not supply enough mathematical information for Robert and the rest of the class to build a mathematical interpretation of the expectation, nor to judge whether it was legitimate. Michael's argument both made explicit how he wanted to compare the two equations, and offered legitimization of the expectation through mathematical authority. Although Michael held granted authority to present his ideas to the class, and expertise authority for his own solution, it was Michael's mathematical authority legitimized through his mathematical argument that allowed others to interpret and agree on the expectation.

We found similar results with the other sociomathematical norms that were introduced by students. The fact that mathematical authority played so large a role in building a mathematical interpretation, agreeing on the interpretation , and validating the expectation lead us to believe that mathematical authority is important in the negotiation of sociomathematical norms. While any authority could validate an expectation, we found that mathematical authority, when activated, played a role in every component of the sociomathematical norm.

When instructors introduce sociomathematical norms, as in the study by Levenson, Tirosh, and Tsamir (2009), we suspect that their hierarchal authority and expertise authority legitimize the expectation before its meaning is fully interpreted. Therefore students are likely to accept the norm before they agree on its mathematical meaning. But when students introduce the expectation, it opens the path for mathematical authority to legitimize the claim. We suggest creating a classroom environment where students rather than the teacher are encouraged to initiate and negotiate sociomathematical norms will lead to better agreement on the expectations among the members of the class. We also believe that if teachers introduce a sociomathematical norm, they should be aware of the potentially obstructive role their hierarchal authority and expertise authority may play in the negotiation of that norm.

References

Clark, P. G., Moore, K. C., and Carlson M. P. (2008) Documenting the emergence of "speaking with meaning" as a sociomathematical norm in professional learning community discourse. The Journal of Mathematical Behavior 27(4) 297-310

Gerson, H. and Bateman, E. (submitted, June 2010) Authority in an Agency-centered, Inquiry-based University Calculus Classroom.

Hamm, J. V., and Perry, M. (2002). Learning mathematics in first-grade classrooms: on whose authority? *Journal of Educational Psychology, 94*(1), 126-137.

Herbel-Eisenmann, B., Wagner, D. Cortes, V. (2008) Encoding Authority: Persvasive Lexical Bundles in Mathematics Classrooms. Proceedings of the Joint Meeting of the International Group for the Psychology of Mathematics Education and the International Group for the Psychology of Mathematics Education North American Chapter (Merida, Mexico, July 17-21, 2008)

Hershkowitz, R., and Schwarz, B. (1999). The emergent perspective in rich learning environments: Some roles of tools and activities in the construction of sociomathematical norms. Educational Studies in Mathematics, 38, 149-166.

Kazemi, E. & Stipek, D. (2001). Promoting conceptual understanding in four upper elementary mathematics classrooms. The Elementary School Journal, 102 (1). 59-80.

Levinson, E., Tirosh, D., and Tsamir, P. (2009) Students' perceived sociomathematical norms: the missing paradigm. The Journal of Mathematical Behavior 28, 171-187.

Levinson, E., Tirosh, D., and Tsamir, P. (2006) mathematically and practically-based explanations: individual preferences and sociomathematical norms. International Journal of Science and Mathematical Educations 4, 319-344

Lopez, L. M. & Allal, L. (2007). Sociomathematical norms and the regulation of problem solving in classroom microcultures. International Journal of Educational Research, 46 252-265.

McClain, K. & Cobb, P. (2001). An approach for supporting teachers' learning in social context. In F. Lin, and T. Cooney (Eds.) Making sense of Mathematics Teacher Education (pp. 207-232). Dordrecht, the Netherlands: Kluwer.

Yackel, E. and Cobb, P. (1996) Sociomathematical norms, argumentation, and autonomy in mathematics. Journal for Research in Mathematics Education, 22, 390-408.

Yackel, E., Rasmussen, C, King, K. (2000) Social and sociomathematical norms in an advanced undergraduate mathematics course. Journal of Mathematical Behavior, 19, 275-287.