University Students' Understanding of Function is Still a Problem!

Zahra Gooya & Mehdi Javadi
Shahid Beheshti University
Iran
gooya@sbu.ac.ir, Zahra_gooya@yahoo.com

Abstract

A research study was designed using the conceptual model consisting two cells of concept images and concept definition developed by Vinner (1983) and has been used by many researchers since then, to investigate students' understanding of different concepts of calculus. A related literature review made us believe that students' understanding of function as one of the pillar of calculus is still problematic. 53 first year university students participated in this study that its purpose was to shed more light into the students' understanding of function in terms of their concept images and concept definitions. The study showed that the most common concept images of function among the students were having a rule, and using a machine as a metaphor for a function. The study also indicated that a concept image of having a rule for each function acted as an obstacle for students to understand the concept definition of function.

Key words: Conceptual Model, Concept Image, Concept Definition, Function, Calculus..

Introduction

Long time ago (1988), the first author did a research about students' understanding of calculus focusing on two fundamental concepts of calculus that are function and derivative. The study carried out at the University of British Columbia in Canada in which, the drop out rate of calculus by then was about %50 (1988). For that study, she used constructivism (Kilpatrick, 1987) as a general theoretical framework and used task based interviews (Jones, 1985) and adopted the idea of teacher as researcher and model builder (Cobb & Steffe, 1983) during the interviews. Finally, Gooya (1988) used a conceptual model consisting of concept image and concept definition developed by Vinner (1983). The finding of that study revealed that the nature of university students' conceptual understanding of function was as follows:

- *A number of students held proper concept images of function which should lead to the development of an appropriate concept definition.*
- *Few of the students, understood function as a relation between two variables (without having the restriction that for each x there is only one y.)*
- *For some students, a function was only considered to be an algebraic function.*

(p.104).

In 2008, both authors felt that university students' understanding of function is still a problem! The second author was high school teacher interested to find out that why despite teaching calculus to students majoring mathematics and physics or natural sciences at secondary school, they still have difficulty understanding it and the first author's experience was that this difficulty still exists at the university level as well. And this was the beginning of our explicating journey to find out more about this extremely important issue in the teaching and learning of calculus. So we did start our journey in the following way.

We conducted a study that its main purpose was to investigate the first year university students' understanding of function. 53 first year university students completing Calculus 1 and Foundation of Mathematics- from three universities in Tehran- participated in this study that its aim was to shed more light into university students' understanding of function in terms of their concept images and concept definitions. The research was exploratory in nature and the data were collected through 8 carefully designed questions. Those who participated in this study were volunteered students majoring mathematics at their universities and all studied calculus at high school. For this purpose, we took Harel's (2004) advice and developed aforementioned conceptual model (Vinner, 1983) to explore students' common concept images and concept definitions regarding the concept of function. Further, Bingolbali and Monaghan (2007) have also mentioned that this conceptual model is still has its own place among mathematics education researchers and the first paper that introduced this model is among the classics of mathematics education research. The data for this study were collected through a set of eight carefully designed questions based on the research findings in this field. For instance, the first three questions asked the students to explain that "whether the showing graphs represent a function or not and why", and the fourth question asked "is there exist a function that all its values are equal?" The next three questions gave us a chance to explore the students' concept images of the function. The purpose of the last question was to investigate the students' personal concept definitions of function.

After the analysis of the data, the students' concept images and their personal concept definitions were categorized according to the above conceptual model. The result was similar in various ways with what was found by Gooya in 1988 and that became our concern about teaching and learning of calculus. In particular, the findings showed that one of the major concept images among the students was that having a rule for function, is a relation between two things, and it is recognized by the test of perpendicular line. However, the students did not consider "arbitrariness" as one of the important characteristics of function – meaning that the value of a function at any given point is independent of the value at other points, and the domain and range can be arbitrary sets. Indeed, the idea of having a rule for function was in contradiction with their images about the arbitrariness of the correspondence of function. This concept image acted as an obstacle for the formation of the formal concept definition of function. In addition, "univalence" (meaning that for each element in the domain, there is a unique element in the range) as another key element in the formal concept definition of function, had not a solid formation in the students' concept images. Therefore, the students' concept images did not provide a proper foundation for them to understand the two essential features of the concept of function namely; the arbitrariness and the univalence.

However, there are many potential opportunities in high school and university calculus textbooks to develop and enrich students' concept images but they were mainly used as examples for different features and characteristics of the concept of function. Thus to conclude, the researchers speculate that teaching plays a major role to cause students' difficulties with understanding *function* and they suggest further research to attest this speculation.

References

Bingolbali, E. & Monaghan, J. (2007). "Concept image revisited". *Educational Studies in Mathematics*. Vol. 68, No. 1, pp. 19 – 30.

Cobb, P.; & Steffe, L. (1983). The constructivist researcher as teacher and model builder. *Journal of mathematics behaviour*. 6, pp. 3 – 40. West Lafayette.

Gooya, Z. (1988). *Students' conceptual understanding of calculus*. Unpublished Master thesis. The University of British Columbia, Vancouver, Canada.

Harel, G. (2004). Perspective on "Concept Image and Concept Definition in Mathematics with particular Reference to Limits and Continuity". In T. Carpenter, J. Dossey, & J. Koehler (Eds.), *Classics in Mathematics Education Research. The National Council of Teachers of Mathematics*. pp. 98 – 108. VA.

Javadi, M. (2009). *Concept images and concept definitions of the concept of function*. Unpublished Master Thesis. Shahid Beheshti University, Tehran, Iran.

Jones, S. (1985). *Applied qualitative research*. Edited by Robert Walker. Great Britain.

Kilpatrick, J. (1987). What constructivism might be in mathematics education. In J. Bergeron, N. Herscovics, & C. Kieran (Eds.). *Proceedings of the Eleventh International Conference of Psychology of Mathematics Education*. Vol.1, pp. 3 – 27. Montreal, Canada.

Vinner, S. (1983). Concept definition, concept image and the notion of function. *The Journal of Mathematics Education in Science and Technology*. 14, pp. 293 – 305.

Walsh, C. (1988). Calculus in the secondary school. *Vector: Journal of the British Columbia Association of Mathematics Teachers*. 29 (3). Pp25- 31.