Contributed research report

Title: On exemplification of probability zero events.

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Abstract: In this study the example space of pre-service secondary teachers on probability zero events is examined. Different aspects of such events as perceived by the respondents are discussed and their perception of impossible events versus improbable is studied. The examples are categorised in terms of the type of sample space and once again categorised in terms of how do they fit the classic definition of probability. The role of measure theory to approach probability is briefly looked at via the examples. Meanwhile the participants' understanding of "more complicated" is explored and different ways they add complexity to their examples are analysed.

Keywords: example space, classic probability, impossible events.

Background

Extended attention to probability and statistics in school curriculum resulted in renewed interest in these topics in mathematics education research. Despite the variety of studies that explore understanding of probability concepts among students and teachers, (Lester 2006), little or no attention has been paid to zero probability events. My study aims at addressing this deficiency.

Theoretical framework

The importance of experiencing with examples has always been dealt with in theories and frameworks for describing the learning of mathematics. Watson & Mason (2005) define a concept as being aware of dimensions of possible variation and with each dimension, a range of permissible change within which an object remains an example of the concept. They also develop the idea of example space as collections of certain types of examples and suggest this idea as central in teaching and learning. Another study highlights that when invited to construct their own examples, learners both extend and enrich their personal example space, but also reveal something of the sophistication of their awareness of the concept or technique (Bills, 2006).

Goldenberg and Mason shed more light on the construct of example space and on how it can inform research and practice in the teaching and learning of mathematical concepts (Goldenberg, 2008).

Methodology: Participants and Task

The participants of this study were pre-service secondary school teachers (n=30), holding Majors or minors in mathematics or majors in science. There were asked to respond in writing to the following task. The time for completing the task was not limited.

Give an example of an event with probability zero.

Give an example of a more complicated event with probability zero.

The task was followed by classroom discussion around the general notion of probability zero events and the given examples.

The research questions were:

How do pre-service teachers interpret and exemplify probability zero events in variety of situations?

What is their personal example space with regard to events with probability zero?

Data analysis: First examples

The data were first analysed in terms of the respondents' perception of probability, which appeared to be in accord with the classical interpretation. The reasoning behind the given examples made it explicit that their common perception of probability is of a fraction (the ration of favourable events to all possible events). According to the different ways a fraction could be equal to zero (exactly or approximately), the examples were categorised into three groups:

- Zero divided by a non-zero: this type of examples was called "logically impossible" events. This category dominated the participants' example spaces. (Example: Rolling a 7 with a standard die for instance). Number of examples in this category: 50.
- A non-zero number divided by a constant large number: this type was referred to as "estimated to be zero" probability events. (Example: Flipping 10 coins all sitting in heads, an event with a probability 0.00097, which is estimated to be zero). Number of examples in this category: 3.
- A nonzero number divided by a sequence of numbers tending to infinity: this group of examples was called "events with probability converging to zero at limit". (Example: tossing a fair coin infinitely many times, all of them sitting in head). Number of examples in this category: 5

From a theoretical account a fourth type of examples was introduced as "measure-theoretically explainable probability zero". (Example: picking a certain number from a given interval of real numbers.) Two examples could fit this category, however, no evidence to a reference to measures in the sense that distinguishes a set of countable points versus a set of uncountable points was given. However, the classroom discussion suggested that this type of probability zero events could be understood from the point of view of each of the three aforementioned categories.

Moreover, the examples were examined in terms of the probability generators used to make a random experiment. The impact of classical textbook objects for teaching probability on the example space of the teachers is conspicuous.

From 60 examples, 32 use dice, 14 use coins, 8 use marbles in a bag (or equivalent variations of it), one uses a spinner, one uses a deck of cards, 2 use picking random numbers and 2 use real life objects such as vending machine and street crossway.

Data analysis: Second examples

Watson and Mason (2005) discuss the "give another example" strategy as a powerful instructional tool. From the examination of second examples in this study it turned out that in 24 out of 30 cases the first and second examples fell in the same category. I further examined how the participants have made their second example "more complicated".

It turned out that combining is quite a popular technique to get more complicated events. A total of 20 examples out of 30 were combining two events in order to give an example of a more complicated event.

Three different types of combination have been recognizable from the data:

The impossible-possible combination:

In this type of examples the impossible event described in first example is frequently used as the impossible component; first example is rolling a 7 with a fair die while second example is asking for rolling a 5 and then rolling a 7 with a fair die.

The impossible-impossible combination:
Some participants have conceived "more complicated" as an event even less likely to happen than their first impossible event. The second example is a combination of two probability zero events.
First example: Getting infinitely many 1's when rolling a fair die infinitely many times[1].
Second example: Getting all faces when flipping a coin infinitely many times while getting infinitely many 1's when rolling a fair die at the same time.
The possible-possible combination with empty intersection:
Another way to get a "complicated" event was to combine the possible events in the sample space such that their intersection is empty, which at the same time makes the event logically impossible. The frequent example of this type was getting both 3 and 4 at the same time when rolling a fair die once.
As a second technique to add more complexity, some participants have used generalization; the second example is a generalized form of the first, so it is perceived to be both a zero probability event and a more complicated one. First example: rolling two dice and getting (6,7), second example: Rolling two dice and getting (i,j) such that i+j=13, for example. As Watson & Mason suggest, leading the learners toward generalization is one of the merits of asking for another or for a more complicated example.

Data Analysis: Number treatment
Any task designed for different research questions that deals in a way with numbers could reveal some by-product facts about people's perceptions on numbers and part of their real number sense. The task described in this proposal is no exception. One of such interesting by-products is the different treatment of numbers found in two of the examples: in both examples the random experiment wsa to pick a random number from a real interval and the probability zero event was to pick a certain pre-determined number, 4.3275 and 1.0000097 respectively. It could not be helped but notice that the examples are of the same nature: they provide "safe" examples of numbers that are not likely to be picked. However, both respondents were aware of the fact that picking any number has the same probability zero, but they might feel that the numbers like 0,1,2 or 1/3 are not safe enough to mention. My conjecture is that the reason for such preference may be in that frequently students are asked to locate integers and simple fractions on the number line, but they are never asked to locate 1.0000097. The first numbers are then analogous to big bold dots or thick dashes on the number line; they are 'exposed' numbers as opposed to 'anonymous' numbers living safely in the oblivion of atom-size inseparable habitants of real line.

An interesting issue related to the definition of probability zero event surfaced in a discussion with participants and will be included in my presentation.

Summary:
The example space of 30 pre-service secondary teachers on probability zero events was studied through the examples they were asked to generate. Their example space was found to be rather limited and dominated by the standard probability teaching examples. Also the 'expert' example space appeared to be missing. 'Complicatedness' of examples was mostly represented in combination and generalization.

References

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