Putting Research to Work: Web-Based Instructor Support Materials for an Inquiry Oriented Abstract Algebra Curriculum

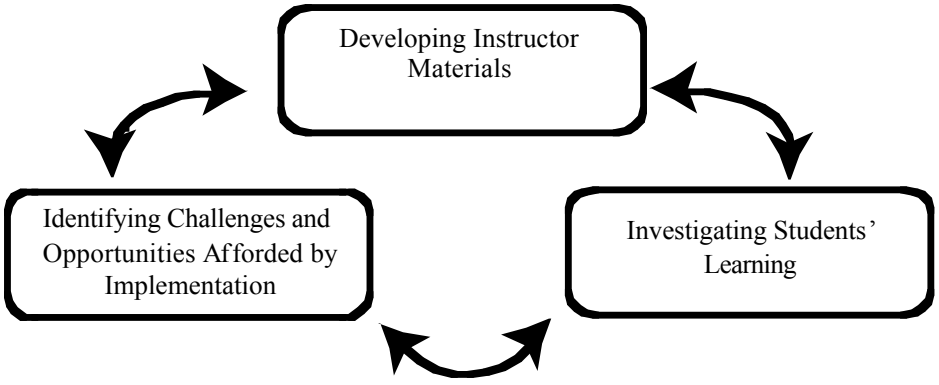
Contributed Report

Abstract

For several years we have been researching students' and instructors' experiences with an inquiry-oriented group theory curriculum. This research has resulted in a number of insights; include findings that may be significant only for instructors and students engaged with this specific curriculum as well as findings that appear to have broader significance. We are putting this research to work as we develop web-based instructor support materials to accompany the curriculum. These materials include 1) information about the rationale for each task/sequence, 2) insights about how student thinking related to the task/sequence, and 3) discussion of task/sequence implementation considerations. In this presentation we will share some of our findings (both general and specific) and illustrate how we have incorporated these findings into the web-based instructor support materials in the form of text, video-clips, and images culled from our research efforts.

Key Words: abstract algebra, curriculum, teaching, student thinking

Questions or issues explored by the research. This report comes from a three-year collaborative project involving mathematics educators, mathematicians, and community college faculty members. The larger project has three primary goals, all related to a set of inquiry-based group theory curriculum materials. These goals are to 1) identify the challenges and opportunities that are likely to arise as different instructors implement the materials, 2) develop instructor support materials to meet the challenges and take advantage of the opportunities and, 3) investigate how students' learning is enhanced by the curriculum materials. These three project activities are tightly interrelated with each informing the other.



In our presentation we will touch on each of these activities. We will describe some insights from our investigations into students' learning as they interact with the materials. We will discuss some challenges/affordances we have identified in our observations of instructors' efforts to implement the curriculum. And we will demonstrate the online materials that we have developed to support instructors by providing selected video-clips, images, and exposition culled from our research efforts.

Relation of this work to the research literature / Theoretical perspective. Our work draws from and builds on two kinds of previous research. First, our work is related to research on undergraduates' thinking and learning in the area of abstract algebra and related concepts including functions and operations. In particular both our early design work and our ongoing investigations of students' learning have been informed by previous work on students' learning of the specific concepts such as isomorphism (e.g., Leron, Hazzan, & Zazkis, 1995) and quotient groups (e.g., Asiala, Dubinsky, Mathews, Morics, & Oktac, 1997). Second, the work is part of a growing effort to develop undergraduate mathematics instruction that is consistent with the design principles of the instructional design theory of Realistic Mathematics Education (e.g. Rasmussen & King, 2000; Rasmussen & Marrongelle, 2006; Zandieh & Rasmussen, 2010). Like these researchers, we have been concerned with both developing instructional approaches that support students' reinvention of mathematics and with contributing to the ongoing development RME theory.

Research Methodology. Our research program has featured multiple methodological approaches. The original design of the curriculum was supported by small-scale teaching experiments conducted with pairs of students as well as whole-class teaching experiments. Data resulting from these teaching experiments include video-recordings, students' written work, and researcher notes. More recently we have video-taped mathematicians teaching with the materials and conducted video-taped debriefing/planning sessions with them. Typically our analysis of the video data been inspired by the iterative approaches described by Cobb & Whitenack (1996) and Lesh & Lehrer (2000). More specifically, during early design stages, initial passes through the data would feature a search for informal student strategies that anticipated the formal mathematics targeted for reinvention while in subsequent passes we would strive to understand how these ideas could be evoked and how they could be leveraged to support the reinvention of the formal mathematics. Our more recent analyses have been focused on the mathematician's instructional moves. In this case, early passes were conducted to identify instances where implementation differed significantly from what was anticipated while later passes were conducted to search for explanations for the deviations. The web-based instructor support materials feature video-clips, exposition, and images that have been drawn from all of these component research activities.

Results. We will share various results from our research efforts. Some of these will be of a general nature and have implications beyond the context of our specific abstract algebra curriculum. For example, a finding that is emerging from our

investigation of mathematicians' efforts to implement the curriculum is that what Ball, Thames, & Phelps (2008) refer to as *knowledge of content and students* is particular important for supporting the kind of *generative listening* (Yackel, Stephan, Rasmussen, & Underwood, 2003) required for successfully building on students' mathematical contributions (Johnson, Larsen, Rutherford, 2010). We will describe a couple of instances in which (in our analysis) mathematician's ability to listen generatively to their students was either supported or constrained by their knowledge (or lack of knowledge) of content and students. Then we will describe our general approach to providing information about students' thinking through the web-based instructor support materials and we will look specifically at our approach to addressing the issues involved in the shared episodes.

In addition to such general findings, we have accumulated a large number of smaller findings regarding how students will likely approach the various instructional tasks and what kinds of difficulties may emerge as they do so. We will share a selection of findings of this type and illustrate how they have informed the design of the curriculum itself and how they have been integrated into various aspects of the instructor materials.

Applications to/implications for teaching practice and future research. Our presentation is best conceptualized as a research-to-practice report. We will be describing our research program and some of our findings, but our main purpose is to share our efforts to develop the web-based instructor support materials. Our hope is that the presentation will contribute to ongoing efforts in our community to find ways to broaden the impact of instructional innovations emerging from our research.

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