Exploring Collaborative Concept Mapping In Calculus

Contributed Research Report

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Abstract

For the past 25 years, concept mapping has been considered primarily a solitary assessment instrument where individuals build an external illustration representative of some notion of held concept images. This study explored the role of concept mapping to collaborative settings and what discourse is generated as Calculus students engage with their individual concept maps to construct a map representative of the group's collective perceptions of calculus concepts. By using adjacency matrices to explore the structure of the concept maps, the study compared individual maps against one another, against aggregated maps and finally against the collaborative concept maps. In particular, the study identified differences in structure and emphasis across the students' maps and identified different discourse models generated by various methodologies employed to generate the collaborative maps. These observations were triangulated with student utterances during the collaborative concept mapping activity and reflections on both the individual and collaborative concept mapping activities.

Keywords: Concept mapping, Collaborative, Calculus, Concept Image

Objectives of the Presentation

This paper presentation will discuss the usage of concept mapping as a framework that supports research and learning. In particular, the study examined the feasibility of using collaborative concept mapping activities as an alternative assessment tool for exploring student calculus understanding. By using a three-phased investigation, the viability of collaborative concept mapping was explored to determine the level of individual and group engagement as well as the differences between individual and collaborative concept maps. In addition, the linguistic mechanisms as students generated their collaborative concept map were tangentially explored. This study provided further indication that as students interact with calculus concepts and face consolidating this information into their already present cognitive structures, they are forced to construct concept images. In addition, the further requirement to then come together and build a collaborative concept map excites further development of cognitive structures that were not evident in any of the individual concept maps. It is found that through the symbiotic transference of knowledge structures mitigated by the students' use of formal and informal language that the group generates a collaborative concept map that infuses the individual perspectives and provides a structure for deep discussions of mathematical linkages.

Perspectives

One model of mathematical understanding, namely understanding as generating concept images and concept definitions, as described by Tall, Vinner, and Harel provides context to this study (Harel & Tall, 1991; Tall, 1991; Tall & Vinner, 1981; Vinner, 1991). According to Vinner

(1991), learners acquire concepts when they construct a *concept image* – the collection of mental pictures, representations, and properties ascribed to a concept. Tall and Vinner (1981) wrote:

> We shall use the term *concept image* to describe the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes…. As the concept image develops it need not be coherent at all times…. We call the portion of the concept image which is activated at a particular time the *evoked concept image*. At different times, seemingly conflicting images may be evoked. Only when conflicting aspects are evoked *simultaneously* need there be any actual sense of conflict or confusion. (p. 152)

Evidently, a concept image differs from a concept's formal definition, if one exists, since a concept image exemplifies the way a particular concept becomes viewed by an individual (Davis & Vinner, 1986). The concept image involves the various linkages of the concept to other associated knowledge structures, exemplars, prototypical examples, and processes. As a result, the concept image is the overall cognitive structure constructed by a learner; however, in different contexts distinct components of this concept image come to the foreground. These excited portions of the concept image comprise the *evoked concept image* that consists of a proper subset of the concept image. This distinction between the image and the evoked image permits one to explain how students can respond inconsistently, providing evidence of understanding in one circumstance and a lack of understanding in another. A learner's description of his or her understandings may supply other discrepancies. In particular, any concept image has a related *concept definition* – the form or words used to specify the concept. This concept definition, however, can differ from the formal mathematical definition of a concept since the concept definition is an individualized characterization of the concept.

In order to elicit these linkages and connections, researchers have begun to use concept mapping to evoke external representation of student internal linkages (Baroody & Bartels, 2000; Ferry, 1996; Francisco et al., 2002; Kinchin, Hay & Adams, 2000; Laffey & Singer, 1997; Laturno, 1994; McGowen & Tall, 1999; Novak, 1984; Park & Travers, 1996; Roberts, 1999; Ruiz-Primo et al., 2001a, 2001b; Skemp, 1987; Von Minden & Walls, 1998; Williams, 1998). The origin of concept mapping sprung from the work of Joseph D. Novak, a Cornell University researcher who pioneered this tool from David Ausbel's theories concerning the significance of prior knowledge in being able to learn new concepts. Concept maps have been "developed specifically to tap into a learner's cognitive structure and to externalize, for both the learner and the teacher to see, what the learner already knows" (Novak, 1984, p. 40) and has been widely adopted by science educators over the past thirty years but has been underutilized by mathematics educators. In particular, a concept map is a graphical representation of a learners' knowledge structure of a particular concept (see Figure 1). To construct a concept map, ideas first have to be described or generated and the interrelationships between them articulated. Concepts are then placed in a hierarchical order with more general concepts at the top and specific concepts towards the bottom. Linking a concept to another via a linking word or phrase identifies a relationship. Cross links, i.e. across-page connections between concepts, engenders a rich connectivity.

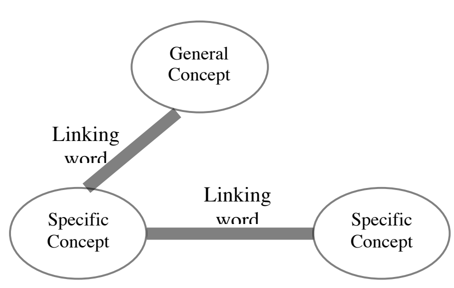


Figure 1: General format of a concept map

Hierarchical concept maps are not the only type of concept map that can be generated. In particular, a concept map can be drawn which exhibits a network, webbing, or circular pattern of concepts. Additionally, concept maps may also begin with a specific idea and work out towards a more general idea. Implementers of concept mapping activities have also employed the use of interpretive essays to accompany a concept mapping activity in order to force students to reflect and clarify their own thinking about mathematical ideas and situations (Bolte, 1997, 1998, 1999).

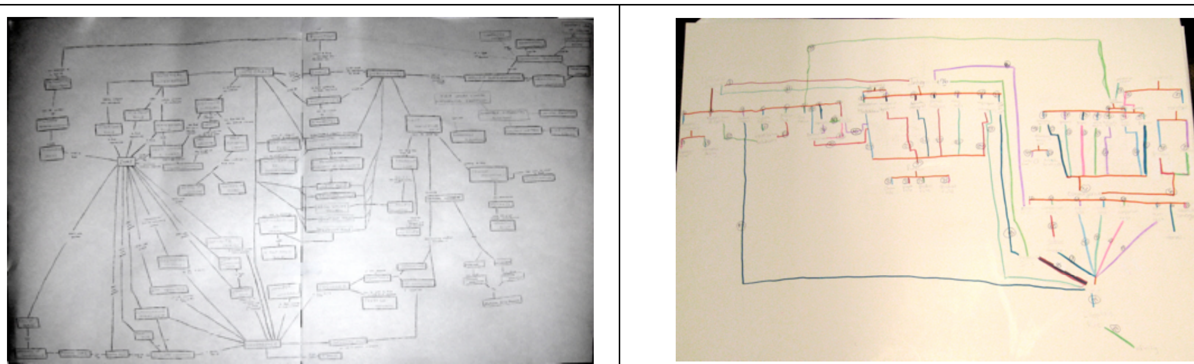
Methods, Techniques and Data Sources

This study was conducted in three phases with a single group of Calculus 2 students (n = 13) to from a Midwest, regional state university during Spring of 2007. The first phase asked participants construct a concept map containing a stipulated set of calculus terms spanning both calculus 1 and 2 concepts. Participants were asked to explain their concept map, address why they organized the map in the way that they did, and what they considered to be the main concept and how does it relate to the other concepts? In addition, they were asked to identify any concepts or links that they would add to their map and other relevant information necessary to understand their particular concept map. In the second phase of the study, participants were grouped alphabetically and asked to construct a collaborative concept map containing the same set of stipulated calculus terms while being photographed and their conversations were being audio taped. The final phase of the study involved the students reflecting on their experiences developing an individual concept map as well as the collaborative concept map.

Quantitative and qualitative data was analyzed using an open coding scheme, for concept maps and definitions of stipulated terms, and adjacency matrices to analyze structure, links, and usage of terms. In particular, each of the individual concept maps and collaborative concept maps were translated into adjacency matrices, using a technique similar to that used by McGowen & Tall (1999), to permit the analysis of linkages. These adjacency matrices then allowed one to examine changes in linkages, inactivated terms (i.e. stipulated terms not linked), added terms, major terms, and represent structural changes. In doing so, comparisons were made between each of the individual concept maps, the individual concept maps in a group and their collaboratively generated concept map and finally the four collaborative concept maps. This analysis when combined with student reflections and other qualitative data gave a rich tableau for triangulation that supported interpretation and conclusions.

Results and Conclusions

Beyond gaining glimpses into the students' mental organization of the concepts from calculus 1 and 2, it was found that the students selected various structures and methodologies to express the connections they held. These structural differences indicated differences in perceived importance of various concepts and the strength and structure of various linkages. For example, visual examination (see Figures 2a and 2b) revealed some structural differences between the individual concept maps that was confirmed by the analysis of the adjacency matrices. However, some of the students felt these were minor discrepancies since they used the same concepts as nodes. Another interesting analysis of the students' individual concepts was to aggregate the students corresponding adjacency matrices into a "master map" and then compare the individual maps against that master map. It was found that individuals may use the same basic set of concepts but those individual maps would contain from 70% to 95% of the links contained in that aggregated map.



(a) (b)

Figure 2: Two individual concept maps

Analysis of the collaborative concept mapping interactions, there were two types of methodologies employed by the students that impacted the level of connection to the students' individual concept maps. In one case, the group of students looked at the concept maps of the group members and then selected the best of their group and began by "replicating" the map but discussing the individual connections and whether they had similar or other connections. For instance, one student stated, "Most of our concepts were in a similar place, just some discrepancy on the smaller concepts so it was about the same as mine." The analysis of the adjacency matrices attested to this assertion since nearly 90% of the structure and connections could be directly seen in the individual concept map of one of the individuals in the group and then vestiges of the other individual concept maps were also evident. The alternative methodology applied by other groups was to use their individual concept maps as "talking points" but then engage in the activity of building a concept map that characterized their discussions. In those discussions, they rehashed their ideas of the big ideas and then set out to build the map. Each member of the group provided input and critiqued the generative process and the external representation. For instance, one student in a group that conducted their collaborative concept map in this manner stated, "Our group organized in the way that we did because it was easy to make connection from our big concepts. The main (middle) concept that we came up with was the limit then we went on from there to the Intermediate Value Theorem to Derivatives and Antiderivatives, and on from there to build… We built off the major concepts and then connected back to original concepts as well." Using this methodology took longer (build time was generally more than 15 minutes longer) but the conversations that were generated were richer and more diverse in the use of formal and informal language. Of particular interest was the negotiation of disagreements and consensus on how to represent various linkages. For instance, the presentation will discuss how one of the groups used the audiotape to characterize what was meant by a section of their concept map and then presented a condensed characterization on the physical representation.

Applications and Implications

As mathematics educators look to help support the exploration of student understanding, care must be taken to carefully examine the ability of those tools to provide reliable information. This paper discusses concept mapping, borrowed from science educators, but having connections to a particular model of mathematical understanding. What has become evident from this study is that collaborative concept mapping can serve as an effective summative assessment that engenders rich discussions amongst students that have already individually engaged with the concepts.

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