An Analysis of Examples in College Algebra Textbooks for Community Colleges: Opportunities for Student Learning

Contributed Research Report

We report on an investigation of the quality of instructional materials available to students in community colleges as part of a larger research study that seeks to characterize mathematics instruction in community college with courses that prepare students to take a calculus sequence. One of such courses is College Algebra.

The rising costs of higher education have made the community college a natural, and in many cases, the only, option for completing postsecondary studies (Dowd et al., 2006). This makes analysis of the resources used in mathematics instruction timely. We focus on College Algebra, because the number of students taking this class is large, the cost of teaching the course is relatively low, and many programs have College Algebra as a prerequisite to other mathematics courses and to courses outside of mathematics (Gordon, 2008; Katz, 2007, Lutzer, Rodi, Kirkman, & Maxwell, 2007). Also, the influence of introductory courses such as College Algebra is significant to a student's life-long attitude to mathematics (Barker, Bressoud, Ganter, Haver, & Pollatsek, 2004).

Textbooks are an important resource for students and instructors in community colleges. We focus on textbooks because they portray information that is presumed to be relevant for learning about the subject matter (Herbel-Eisenmann, 2007; Howson, 1995; Love & Pimm, 1996) and because they are a source of examples and exercises for instruction; as such, however, they afford probabilistic rather than deterministic opportunities to learn mathematics (Mesa, 2004; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002). Thus what we infer are possibilities rather than definitive influences on teaching and learning.

We focus on the examples within the textbook for three reasons. First, they are usually intended to be representative of the work that students need to do–they correspond to those portions of the textbook "that demonstrate the use of specific techniques" (Watson & Mason, 2005, p. 3)—and as such, they are most likely to contain explicit information that will help students in solving similar problems. Second, instructors use examples in the textbook as part of their lectures, some times changing them slightly, so that students have later access to more than one illustration of how to solve a given problem. Third, instructors indicate that students, rather than reading the exposition, rely primarily on examples in order to work out homework problems (Mesa & Griffiths, 2010). Thus, if examples are to be used by students as models of thinking through problems, we ask, what the cognitive demands of the examples are, to what extent can they assist students in learning strategies for controlling the solutions to mathematical problems, what are the types of answer students are expected to generate, and what are the connections made among different representations. This study explores what textbooks offer and what they omit. We believe that by understanding the characteristics of textbooks, instructors can gain a clearer perspective on how this resource supports their practice.

Methods

We focus on seven College Algebra textbooks used by at least twelve large community colleges used in a Midwester state. We concentrated on three topics--transformation of graphs, exponential functions, and logarithmic functions--because they are both foundational for further study of calculus and emerge as key in solving many mathematical and "real world" applications. All text identified in the corresponding sections of each textbook as examples was analyzed using four frameworks. Cognitive Demand captures the level of complexity of tasks using four categories, Memorization, Procedures Without Connections, Procedures With Connections, and Doing Mathematics (Stein, Smith, Henningsen, & Silver, 2000). The second framework, Controlling the Work, examines explicitness of solutions, and focused on only four aspects: Further Elaboration, Correctness, Suggestion to Check, and Plausibility or Interpretation (Mesa, 2010). The third framework is Types of Representation and we sought to determine how frequently different representations (symbols, tables, graphs, numbers, and verbal) are given in statements of the examples and in their solutions. The last framework we applied is Types of Response. We sought to characterize the type of answer expected: Only Answer, Answer and Mathematical Sentence, Answer and Graph, Explanation or Justification, and Making a Choice (Charalambous, Delaney, Hsu, & Mesa, 2010). These analyses were chosen, because as a group they speak about the complexity of mathematical activity that is offered to students in the examples. The inter-rater reliability between two coders ranged from 74% to 96% across analyses.

Results

1. Cognitive Demand (N=348)	
Memorization	0 (0%)
Procedures Without Connections	312 (90%)
Procedures With Connections	34 (10%)
Doing Mathematics	2 (1%)

2. Controlling the Work (N=348)	
Further Elaboration	33 (9%)
Correctness	32 (9%)
Suggestion to Check	8 (2%)
Plausibility or Interpretation	0 (0%)

3. Types of Response (N=348)	
Only Answer	176 (51%)
Answer and Mathematical Sentence	53 (15%)
Answer and Graph	102 (29%)
Explanation or Justification	21 (6%)
Make a Choice	6 (2%)

4. Types of Representation (N=348)		
	In the Statement	In the Solution
Symbols	254 (73%)	145 (42%)
Tables	17 (5%)	42 (12%)
Graphs	36 (10%)	130 (37%)
Numbers	107 (31%)	248 (71%)
Verbal	68 (20%)	31 (9%)

Figure 1. Results from the four analyses with the seven textbooks.

We found no memorization examples in these seven college algebra textbooks. However, nearly 90% of the examples were coded as procedures without connections and very few examples that would be categorized as more demanding. This trend was observed in all seven textbooks, with the percentage of examples requiring procedures without connections ranging from 75% to 100%. Only 2 examples were open enough to be coded as doing mathematics, and these appeared in one textbook.

Across textbooks, less than 10% of the examples modeled the four strategies that allow the solver to control their work. There were differences at the textbook level, with one textbook including seven Further Elaboration examples, but no Correctness examples and two textbooks accounting for seven of the eight Suggestion to Check examples in the corpus. Thus, in spite of these examples using real world applications, very little of that content was used to control the correctness of the solution.

We also found that 46% of the examples expected an answer only response. Examples asking for explanation or justification and asking for making a choice were relatively rare (from 0% to 15% and 0% to 6%, respectively) across all seven textbooks. All the textbooks had more Answer and Graph responses than Answer and Mathematical Sentence response.

Finally, symbols were most frequently used in the example statements, and numbers are most frequently requested in the solution. The second most dominant form of representation was numbers in the statement and the symbols in the solution. Tables, graphs, and verbal representations appeared less frequently in both the statement and the solution. With a couple of exceptions, the use of symbols and numbers in the statement and solution was common.

Discussion

In many cases, examples applied the concepts or formulas explained in the textbook. Although we found several examples that had features of reform-oriented tasks (e.g., 'real-world' contexts or use of technology), not all of those problems had high-cognitive demand tasks. While applying procedures without connections is an important activity, concentrating only on these less demanding examples can restrict students' perception on what mathematics is (Stein et al., 2000). One possible reason for these results might be that the reform movement for two-year colleges (Blair, 2006) has not yet influenced the textbooks, in spite of this document being available for authors. Another possibility is that instructors and colleges might prefer to adopt textbooks that are more traditional, or that they, in making decisions, focus on other aspects such as the number of problems for the students.

The analysis of strategies for controlling the work shows that in general the examples do not provide explicit information about the meanings of an answer or about ways to make sure answers are correct, which reduces students' opportunities to learn to use these strategies when solving problems. It might be possible that authors expect instructors teach these strategies during instruction; however, if it is true that students rely on examples to learn to do homework, an important opportunity is missed by not adding this information to the examples. Considering that students tend to go back to examples when they meet difficulties doing homework, increasing the frequency of further elaboration, checking or suggesting correctness, interpretation, and examining plausibility could be a way to enhance students' learning of these strategies.

Textbooks in this study have a large number of examples looking for only answer and few asking for explanations or justifications. Considering that students rely on examples to learn and practice the mathematics they are not yet competent with, having them paying more attention to explanation and justification will help students and instructors uncover misconceptions. Sometimes, students arrive at the right answers with erroneous understanding (Erlwanger, 1973). In addition it will be difficult to gain proficiency in explaining and justifying when the focus is mostly on finding an answer to a problem. Using problems that has explanation or justification as the type of response is also beneficial to teachers in terms of the acquisition of *pedagogical content knowledge* (Cohen, Raudenbush, & Ball, 2003). Attention to explanation and justification gives teachers a better access to their students' struggles, by making explicit the path students took to find the answer.

While the analysis of the expected response type contains some information about representations, our final analysis provides a better picture. The low percentage of tables, graphs, and verbal representation suggests that few connections are made across these types of representations. What is desirable for students is not only to become proficient at interpreting a situation with multiple representations, but also to generate models of real-life situation using multiple representations (NCTM, 2000). The ability of using numerical, graphical, symbolic, and verbal representations is expected to the students taking mathematics-intense courses in community colleges (Blair, 2006).

Our findings suggest that examples in these textbooks are not very demanding and that they do not assist students in developing strategies for controlling their work. Moreover, students are not asked to reflect their reasoning nor to actively make connections among different representations. While these findings are particular to College Algebra textbooks used in community colleges it would be important to know if that is the case for other college textbooks. Research is needed to find out whether this is the case for other textbooks for different topics at the community college level (horizontal) and different levels of algebra courses (vertical, at different institutions). How to introduce these changes needs to be researched, in order to provide instructors guideline for improving opportunities for students to learn college algebra in community colleges.

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