The Physicality of Symbol-Use:
Projecting Horizons and Traversing Improvisational Paths Across Inscriptions and Notations

Contributed Research Report

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The way people use symbols and drawings has an intrinsic physicality. Viewed as an extension of gesture-making, symbol-use can give us insight into how symbol-users experience the mathematics at hand. Using a theoretical framework of embodied cognition, we explore this matter by conducting a phenomenological analysis of a 2-minute selection from an interview with a topologist about one of his published papers. We propose an interpretation of the mathematician's symbol-use in terms of two related constructs: *realms of possibility* in what the mathematician perceives as available to him and *paths* within and between these realms. Both of these are projected onto the writing surface and embodied through gestures, speech, eye gaze, and many other means. We explore the origins and relevance of these in our presentation.

Keywords: Embodied cognition, phenomenology, gesture, mathematicians

Traditionally, concepts have been understood in contrast to percepts (e.g. Kant, 1998). A growing body of research questions this division suggesting that the way we think about something, the way we perceive it, and the way we can and do physically interact with it are all inextricably intertwined (Noe, 2006). For instance, when we first learn to drive, the car feels like a foreign object and other vehicles suddenly seem much larger and more dangerous than when we were passengers. Yet as we become more familiar with driving, the car comes to feel like an extension of us and we learn to perceive and think about vehicles with a different sense of ourselves than when we were passengers or students of driving.

The split between concepts and percepts has been reflected in cognitive science as an opposition between modal (i.e. perceptual) and amodal (i.e. conceptual) systems (Barsalou, 1999). One way to argue for the inseparability between concepts and percets is to state there are no amodal systems in which we do logical reasoning, inference and so on, but instead that all activity is perceptuo-motor activity. Our reasoning about even the most highly abstract topics manifests through a partially covert sense of what we can do and perceive with the representations we use for said topics. It is in this sense that we say that cognition is embodied.

This leaves us with a question: How do those who are skilled in highly abstract forms of reasoning embody their thinking about those abstract topics? We know that this must occur through their interactions with the representations they use for the ideas in question, but how do those interactions contribute to the way in which the abstractions are understood and used?

We explore these questions in a case study of a mathematician explaining an aspect of his published work. We asked him to choose a paper he considered interesting or significant, did our best to understand the paper over the course of a few weeks, and then conducted a video-recorded unstructured interview (Bernard, 1988) in which we asked him to explain the paper as he thought of it. We watched the subsequent video several times to select segments for microanalysis (Erickson, 2004), choosing the segments based on which ones seemed most likely to give fruitful insight into the embodiment of abstract mathematics. With the 2-minute segment in question, we alternated between examining the microanalysis individually and discussing our examinations as a team. In our individual examinations, we would generate possible descriptions of the mathematician's actions based on what we knew about his background, the demands of ongoing circumstances (e.g. his reacting to the interviewer's questions), and the multiple unintended contingencies arising moment by moment. In our collective discussions we would share each other's examinations and explore the implications of one another's observations in light of the data on hand, with the goal of generating compelling and viable accounts of this mathematician's experiences allowing us insight into the nature of how abstract thought can be embodied. While this is an case study of a single subject, a microethnographic analysis has the potential to broaden our perspectives and to suggest new interpretations which may enrich our understanding of how anyone grapples with mathematical problem solving.

Our analysis has highlighted two related constructs that we'd like to share in this presentation. The first we term *realms of possibility*. A crucial observation arising from numerous phenomenological investigations is that what we perceive is not given merely by light, sound, and so on but is also saturated with our anticipations of how we might be able to interact with and change that which we perceive (Gallagher & Zahavi, 2008; Husserl, 1913/1983; Merleau-Ponty, 1962). The collection of such anticipations often presents itself to the individual as being a kind of space, just as we have a sense of the space in which we could move a chair and sit ourselves upon it. But just as there are limitations to how you anticipate being able to move a given chair, these realms of possibility have a kind of boundary, which Husserl (1913/1983, p.52) referred to as a "horizon". We find that the mathematician in our study consistently defined these horizons between realms as he experienced them by creating gaps in the blackboard or drawing dividing lines on it and reinforcing them with his gestures, gaze, and placement and orientation of his body.

The second construct is that of *paths*, both within and between realms of possibility. In order to actualize his explanations, the mathematician has to “travel” within the realms he describes. This “travel” occurs via gestures, speech, gaze trajectory, inscription on the blackboard, and so on. Some of these paths follow the symbols and drawings in the order in which they were inscribed, whereas others get overlaid on an existing inscriptional surface along temporal sequences that differ significantly from the order in which they were generated. Both the travel along and redefinition of paths occurs through the mathematician’s physical interactions with the symbols, such as when he seemingly runs into a difficulty with his exposition, physically steps away from the blackboard to gesture an explanation that gets around the difficulty, and then physically returns to the blackboard and manually puts his explanation into the symbols already written. The accompanying speech makes a corresponding shift as well; in this particular example, the mathematician switched to the subjunctive (“If you wanted to…”) until he physically reconnected his talk and gesture back to the symbols on the board with which he was making his original point. This is just one of several different kinds and uses of paths that we’ve noticed as defining methods of travel within and between realms of possibility in this episode.

In exploring these matters, we hope to contribute to basic research that can help frame mathematical activity in ways that are both practical for researchers and consistent with the mounting evidence supporting the close connection between concepts, perception, and physical action. These theoretical constructs – realms of possibility and paths within and between them – provide us with a way of perceiving some of the bodily interactions that individuals can have with mathematical entities. Further exploration of these and related constructs has the potential to provide a rich account of how collegiate mathematics is practiced while remaining true to the inseparability of mind and body.

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