Exploring the van Hiele Levels of Prospective Mathematics Teachers
Contributed Research Report

Carole Simard
Cal Poly, San Luis Obispo
csimard@calpoly.edu

Todd A. Grundmeier
Cal Poly, San Luis Obispo
tgrundme@calpoly.edu

This research project aimed to assess the influence of an inquiry-oriented, technology-based, proof-intensive geometry course on the van Hiele levels of prospective mathematics teachers. Data was collected in an upper division geometry course taught from an inquiry-oriented perspective. The course relied on technology (*The Geometer's Sketchpad*) to help students make and prove conjectures. Data was collected from classes in consecutive years, the first with twenty-one participants and the second with twenty-four participants. Most participants were prospective secondary mathematics teachers. Data collection included a pre- and a posttest of participants' van Hiele levels. Data analysis suggests similar results for both sets of participants in that the course had greater influence on the van Hiele levels of female participants. Results also suggest that the van Hiele test instrument used for this study operated well with university students.

Geometry, van Hiele levels, teacher preparation, secondary

This research was conducted as a quantitative study using a pre- and a posttest design with a convenience sampling as defined by Creswell (2005). The participants were from two geometry classes, in consecutive calendar years, in a four-year Master's granting university located in the central coast of California. Only one section of the course is offered per calendar year and every student enrolled was offered to participate in this study. Twenty-one students participated in the data collection from the first class and 24 students participated from the second class. Of the 45 participants from the two classes the majority had declared an interest in teaching secondary mathematics and some were considering teaching at the community college level. The course was taught each time over a ten-week period, and met four times a week for 50-minute sessions. The prerequisites for this geometry course included a course in methods of proof in mathematics, which focuses on instruction of logic and proof techniques. In addition, this geometry course is mandatory for mathematics majors in the teaching concentration while open to other students who have met the prerequisites. The purpose of this study was to assess whether a proof-intensive geometry course, taught from an inquiry-oriented, technology-based perspective, has any influence on the van Hiele levels of prospective mathematics teachers and whether the influence, if any, varies by gender.

In 1957, Pierre Marie van Hiele and Dina van Hiele-Geldof, mathematics educators in the Netherlands, developed a learning model for geometry as their doctoral thesis. They defined what are known as "the van Hiele levels of development in geometry", which, according to van Hiele-Geldof's thesis, are hierarchical (cited in Fuys, Geddes, & Tischler, 1984). Altogether, there are five van Hiele levels (VHLs): 1) visualization - students visualize geometrical figures as a whole and recognize them by their particular shape; 2) analysis - students recognize the geometric properties of the different figures and are able to analyze the figures separately, but do not yet make connections between figures; 3) abstraction - students recognize relationships between figures and between properties of different figures; 4) formal deduction - students can write proofs and should provide justifications for each step in the proof; 5) rigor - "… student understands the formal aspect of deduction… [and] should understand the role and necessity of indirect proof and proof by contrapositive" (Mayberry, 1983, p. 59), and students can understand non-Euclidean geometries. These definitions were gathered from several authors' interpretations of the five van Hiele levels (Burger & Shaughnessy, 1986; Mayberry, 1983; Mistretta, 2000). Exact definitions can be found in van Hiele-Geldof's doctoral thesis (Fuys et al., 1984), and a more detailed list of behaviors at each level can be found in Usiskin (1982, pp. 9-12).

As seen in past research, the van Hiele level or the level of competence in geometry of some teachers is not at the highest level (Mayberry, 1983, pp. 67-68; Sharp, 2001, p. 201; Swafford, Jones, & Thornton, 1997, pp.469-470), thus possibly hindering the learning of geometry of some students. A conflict may arise when there is a discrepancy between the van Hiele level of the teacher and the zone of proximal development (ZPD) (Vygotsky, 1987) of the student. We expect this conflict to be mitigated if a teacher is at VHL 5.

While it is ideal for all prospective teachers to be at VHL 5, gender differences favoring males are almost twice as large in geometry as in other areas of mathematics (Leahey & Guo, 2001). Furthermore, even though the findings reported in the literature suggest variations in gender differences, the differences are mostly in spatial visualization tasks (Battista, 1990). Senk and Usiskin (1983) studied high school geometric proof writing abilities, which they consider as a high-level cognitive task requiring some spatial ability. However, while overall geometry performance has not been analyzed by gender, they found no gender differences in achievement in geometric proof writing at the end of a one-year geometry course even though females started the year with generally less geometry knowledge (p. 193).

This review of literature only found a few peer-reviewed published studies involving the level of content knowledge in geometry of prospective or practicing teachers. Among them, one study has been conducted on VHLs of prospective elementary teachers (Mayberry, 1983), one on VHLs of practicing middle-grade teachers (Swafford et al., 1997), and one on developing the geometric thinking of practicing K-7 teachers (Sharp, 2001), but none on the influence of an inquiry-oriented, technology-based, proof-intensive geometry course on VHLs of prospective secondary mathematics teachers.

After examining several documents written by the van Hieles and describing behaviors at each van Hiele level, Usiskin (1982) developed a 25-item test instrument to assess the van Hiele level of an individual. Although this instrument was primarily devised with high school students in mind, it has been used, with permission from the authors, for this study (S. Senk, personal communication, November 19, 2007). Whether the subjects involved would constitute an appropriate reference base for the study using Usiskin's test was considered since the subjects involved have all completed a one-year high school geometry course. However, even though the van Hiele levels have been defined while studying high school students, Pierre-Marie van Hiele, as quoted by Usiskin, believed that the highest level is "hardly attainable in secondary teaching" (1982, p. 12). Furthermore, Mayberry (1983), who devised her own test instrument, found that "70% of the response patterns of the students who had taken high school geometry were below Level III" (equivalent to level 4 in this study), and "only 30% were at Level III" (pp.67-68). Time constraints in preparing a VHL test and in-class time usage were also key factors in the selection of a test instrument. Burger and Shaughnessy (1986), as cited by Jaime and Gutiérrez (1994, p. 41), developed a test to assess VHLs, but its administration, through an interview, requires more time to conduct. Mayberry's (1983) 128-item test was discarded for the same reason. Usiskin's test was readily available and it is a timed-test limited to 35 minutes.

Finally, in 1990, Usiskin and Senk confirmed the validity of Usiskin's test even though they were aware of a better instrument, the RUMEUS (Research Unit for Mathematics Education at the University of Stellenbosch**)** test. Smith, as cited by Usiskin and Senk (1990), admitted that Usiskin's test was quicker and more convenient to apply in addition to being shorter than the RUMEUS test which he had used in a comparative study with Usiskin's test (p.245). It was thus decided to move forward with Usiskin's test to assess the van Hiele levels of development in geometry in a post-secondary setting.

Usiskin's test was administered during the first and last class meetings as a pre- and posttest. During class, students typically worked on inquiry-oriented activities using the dynamic geometry program *The Geometer's Sketchpad* (GSP) (KCP Technologies, 2006). The activities were generally completed in groups and provided the foundation for the inquiry-oriented, technology-based nature of the class as participants were expected to make and prove conjectures from their exploration with the dynamic geometry software. After students engaged with the activities, they were regularly asked to present their conjectures and proofs to the class, which often resulted in multiple avenues to prove the conjectures being explored. These activities, presentations, and class assignments make up the proof-intensive nature of the course.

Before analyzing the data with respect to our research purpose, we became interested in verifying the hierarchical nature of Usiskin's van Hiele test (1982) with our participants. We implemented a Guttman scalogram analysis similar to that of Mayberry (1983) to determine whether the VHLs as tested by Usiskin's test form a hierarchy. The scalogram analysis implied that Usiskin's van Hiele test operated adequately for both of our sets of participants in terms of the hierarchical nature of the VHLs.

To interpret the results of the pre- and posttests, each participant was assigned a raw score (out of 25) and a VHL similar to what Usiskin (1982) calls a "classical van Hiele level" (p. 25). The 4-item criterion (p. 24) was used since random guessing was not expected from the participants in this study and a higher mastery level was expected considering all the participants had completed a high school geometry course. Each group of five questions in Usiskin's test corresponds to a different VHL (questions 1 to 5 correspond to VHL 1, questions 6 to 10 to VHL 2, and so on). For a participant to be assigned a level, say *n*, at least four items must have been answered correctly at level *n* and at each preceding level. If a participant answered less than four questions correctly at level 1, then level 0 was assigned. The table below summarizes the raw scores and VHLs from both sets of data.

	Data Set 1				**Data Set 2**			
	Male		***Female***		***Male***		***Female***	
	Pre	*Post*	*Pre*	*Post*	*Pre*	*Post*	*Pre*	*Post*
Raw Score	21.08	20.92	19.44	21.22	20.75	22.375	19.69	21.375
VHL	3.67	3.25	2.56	3.44	3.0	3.875	2.875	3.56

Beyond the analysis of raw scores and VHLs, we decided to look at the data by VHL to document changes, especially related to the proof-based nature of the course, levels 4 and 5. For both sets of participants, the females made statistically significant gains at VHL 4 and little change at all other VHLs. For the first group of participants, the males made statistically significant gains at VHL 5 with very little change at any other VHL. Similarly, although not statistically significant, the male participants in the second group made substantial gains at

VHL5 with their average increasing from 3.5 out of 5 questions correct to 4.25 out of 5 questions correct.

Some findings in this research are consistent with the findings of prior research. For instance, the results on the pretest are consistent with Leahey's and Guo's (2001) findings where male students did better than female students in geometry at the end of high school (p.721). As in Senk and Usiskin (1983), females and males performed (almost) equally well in geometric proof writing at the end of a geometry course. Additionally, as in this study where, in general, the females' performance has improved substantially, Ferrini-Mundy and Tarte, as cited by Leahey and Guo (p. 721), found that girls' performance improved after learning spatial-related strategies. This may correspond to the use of *The Geometer's Sketchpad* in this course and other teaching strategies used by the professor including the inquiry-oriented nature of the course. While the results of this research suggest a positive change in participants' VHLs, the small number of participants at VHL 5 continues to raise the question about the best manner to assess prospective teachers' preparedness to teach geometry.

References

Battista, M. T. (1990). Spatial visualization and gender differencs in high school geometry. *Journal for Research in Mathematics Education, 21*(1), 47-60.

Burger, W. F., and Shaughnessy, J. M. (1986). Characterizing the van Hiele levels of development in geometry. *Journal for Research in Mathematics Education*. *17*(1), 31-48.

Creswell, J. W. (2005). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research* (second ed.). Upper Saddle River, New Jersey: Pearson Prentice Hall.

Fuys, D., Geddes, D., and Tischler, R., Eds. (1984). *The didactics of geometry in the lowest class of secondary school*. New York: New York. (Original work by Dina van Hiele-Geldof published 1957).

Jaime, A., and Gutiérrez, A. (1994). A model of test design to assess the van Hiele levels. *Proceedings of the 18th PME Conference, 3*, 41-48.

KCP Technologies (2006). The Geometer's Sketchpad (Student Ed., Version 4) [Computer software]. Emeryville, CA: Key Curriculum Press.

Leahey, E., and Guo, G. (2001). Gender differences in mathematical trajectories. *Social Forces, 80*(2), 713-732.

Mayberry, J. (1983). The van Hiele levels of geometric thought in undergraduate preservice teachers. *Journal for Research in Mathematics Education,* 14(1), 58-69.

Mistretta, R. (2000). Enhancing geometric reasoning. *Adolescence, 35*(138), 365-379.

Senk, S. (1989). Van Hiele levels and achievement in writing geometry proofs. *Journal for Research in Mathematics Education*, 20(3), 309-321.

Senk, S., and Usiskin, Z. (1983). Geometry proof writing: A new view of sex differences in mathematics ability. *American Journal of Education*. *91*(2), 187-201.

Sharp, J. M. (2001). Distance education: A powerful medium for developing teachers' geometric thinking. *Journal of Computers in Mathematics and Science Teaching*. *20*(2), 199-219.

Swafford, J. O., Jones, G. A., and Thornton, C. A. (1997). Increased knowledge in geometry and instructional practice. *Journal for Research in Mathematics Education*, 28(4), 467-483.

Usiskin, Z. (1982). *Van Hiele levels and achievement in secondary school geometry* (CDASSG Project Report, 231 pages.

Usiskin, Z., and Senk, S. (1990). Evaluating a test of van Hiele levels: A response to Crowley and Wilson. *Journal for Research in Mathematics Education*. *21*(3), 242-45.

Vygotsky, L. S. (1987). *Problems of general psychology, Thinking and speech* (N. Minnick, Trans.) New York: Plenum Press. (Original work published 1934).