Changing Mathematical Sophistication
in Introductory College Mathematics Courses
Contributed Research Report

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Abstract: The Mathematical Sophistication Instrument (MSI) measures the extent to which students' mathematical values and ways of knowing are aligned with those of the mathematical community based on eight interwoven categories: patterns, conjectures, definitions, examples and models, relationships, arguments, language, and notation. In this paper, we present the results of a study designed to explore whether students' scores on the MSI improved during their introductory college mathematics courses. A large sample of five sections of a first course for elementary education majors, five sections of College Algebra, and seven sections of mathematics for liberal arts majors completed the instrument both at the start and end of the spring 2009 term. Results showed that students in courses where instructors used inquiry-based pedagogies scored markedly higher on the instrument at the end of the semester than at the start. In courses where instructors used traditional pedagogies, only slight changes in scores were observed.

Keywords: inquiry-based pedagogy, teacher knowledge, mathematical enculturation, autonomy, mathematical sophistication

Background and Framework: In previous research (Seaman & Szydlik, 2007), we studied the ways in which preservice teachers learned mathematics by observing their attempts to understand ideas in arithmetic and number theory using a teacher resource website. Results suggested that our participants were profoundly mathematically unsophisticated; they displayed a set of values and tools for learning mathematics that was so different from that of the mathematical community, and so impoverished, that they were essentially helpless to create fundamental mathematical understandings.

Based on our comparison of the 2007 participants' mathematical behaviors and beliefs with those of mathematicians, we created a framework to define a construct that we termed *mathematical sophistication.* The construct is defined in terms of beliefs about the nature of mathematical behavior, values concerning what it means to know mathematics, avenues of experiencing mathematical objects, and distinctions about language and notation. Specifically, we proposed the following list of values and behaviors that indicate mathematical sophistication.

1) Seeking to understand patterns based on underlying structure.
2) Making and testing conjectures about mathematical objects and structures.
3) Creating mental (and physical) models and examples and non-examples of mathematical objects.
4) Using and valuing precise mathematical definitions of objects.
5) Valuing an understanding of why relationships make sense.
6) Using and valuing logical arguments and counterexamples.
7) Using and valuing precise language and having distinctions about language.

8) Using and valuing symbolic representations of, and notation for, objects and ideas.

Increasing students' mathematical sophistication became an articulated goal of the mathematics for elementary education sequence at our university. All instructors of those courses committed to using inquiry-based pedagogies; their students solved novel problems in small groups and then discussed their solutions, strategies, and reasoning as a class. Furthermore, instructors made the values of the mathematical community more overt. For example, making sense of definitions, discussing the value of pattern-seeking and generalization, and studying distinctions between inductive and deductive reasoning became explicit topics of that sequence. These goals are aligned with demands that teachers understand the rich connections among mathematical ideas; bridge gaps between students' use and standard mathematical use of notation and language; and model and request the mathematical behaviors of sense making, conjecturing, and reasoning (CBMS, 2001). (For a comprehensive overview of the literature on teacher knowledge see Hill et. al., 2007.)

In order to measure changes in mathematical sophistication in our students, we developed a twenty-five item, multiple-choice, paper-and-pencil Mathematical Sophistication Instrument (MSI) based on the above framework. Items were developed by, or in consultation with, eight mathematicians. Our attempt was to make the items substantially free of specific mathematics content. For example, consider the below MSI item designed to assess students' abilities to make sense of a new definition and the meaning of "or" in a mathematical context:

A number is called **normal** if it is less than 10 or even. According to this definition, of the numbers 5, 8, and 24,

a) Only 5 and 8 are normal.
b) Only 8 is normal.
c) Only 5 and 24 are normal.
d) All of these numbers are normal.

In Fall 2007 a large sample of students in their mathematics for elementary teachers courses completed the instrument during the first month of the semester. Twelve students (four who scored in the top quartile, four who scored in the middle half and four who scored in the lower quartile on the instrument) were interviewed to determine whether the level of sophistication shown by the students as they explained their thinking was reflected by their performance on the items. The MSI was revised based on that data.

In fall 2009 we assessed both the validity and reliability of the updated instrument with a large sample of undergraduates at a Midwestern comprehensive state university (Szydlik, Kuennen, & Seaman, 2010). In order to assess the validity of the instrument, course instructors rated the mathematical sophistication of their students based on our framework, and instructor ratings were compared with student scores on the items. Results suggest that the MSI is a valid measure of sophistication as defined by the eight categories. In pilot testing, the MSI has obtained Kronbach Alphas between .053 and 0.73.

Methods for the Current Study: In spring 2010 we sought to investigate whether students' scores on the MSI improved during their first course for preservice teachers: Number Systems. That semester Number Systems was taught by four different instructors, and all sections (116 students) participated. We formed two comparison groups for the research: a sample of 116 students taking a first liberal arts mathematics course (with four different instructors), and a sample 97 of students taking college algebra (with two different instructors). All three courses had the same prerequisite. The MSI was administered in classes at both the start and at the end of the semester in all participating sections. Almost all students present chose to participate.

Results: Each MSI item was scored 1 point for the most sophisticated answer (as determined by the mathematicians) and 0 points for all other response options. Cumulative pre-test scores ranged 1 to 19 (out of 25 points) and post-test scores ranged from 2 to 19 points. Students in all groups showed significant gains on the MSI during the spring 2010 term. This is not surprising; since the same instrument was used for both the pre- and post-tests, we expected gains. However, as shown in the table below, students in both Number Systems and the liberal arts mathematics course obtained important and highly significant gains on the MSI ($p < .0001$), whereas the students in College Algebra showed only modest changes. These results remained even upon deleting four MSI items with mathematics content explicitly addressed in one or more sections. For example, one instructor of liberal arts mathematics included a graph theory unit, and since two graph theory items appeared on the MSI to measure students' abilities to make sense of a new definition, those items were deleted (for everyone) in a reanalysis of the data.

Course	**MSI Score at the start of the term**	**MSI Score at the end of the term**	***p*-value**
Number Systems (n = 116)	Mean = 7.74 Stand. Dev. = 2.83	Mean = 10.01 Stand. Dev. = 3.65	$p < 0.00001$
Liberal Arts Math (n = 116)	Mean = 8.11 Stand. Dev. = 3.42	Mean = 9.12 Stand. Dev. = 3.60	$p < 0.0001$
College Algebra (n = 97)	Mean = 7.37 Stand. Dev. = 3.09	Mean = 7.90 Stand. Dev. = 3.17	$p < 0.04$

MSI Scores by Course

Conclusions: Because the instrument is substantially free of relevant mathematics content topics, we assert that gains on the MSI are due primarily to differences in the ways students in the courses *experienced* mathematics. According to an instructor questionnaire, and informal observations and discussions of teaching, inquiry-based pedagogies were used almost exclusively by instructors in all sections of Number Systems and were used on most days by the instructors of liberal arts mathematics. College Algebra was taught using traditional lectures.

This work suggests two conclusions. First, measurable changes in student sophistication can be affected during the course of a semester; and second, those changes appear to be the result of the students having engaged in mathematically authentic behaviors in the classroom. In our presentation we will share our instrument, and we will discuss possible connections between students' mathematical experiences in their courses and gains on the MSI.

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