*Contributed Research Report*

*Title:* Student Understanding of Eigenvectors in a DGE: Analysing Shifts of Attention and Instrumental Genesis

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*Abstract*: This study examines the potentialities of the theory of instrumental genesis and shifts of attention in analysing students' evolving understanding as they interacted with a dynamic geometry representation of eigenvectors and eigenvalues. Although the former theory provides a framework to analyse students' interactions with tools and transformation of tools into instruments, it makes an assumption about the role of instrument in cognitive development. According to Verillon and Rabardel (1995), the founders of the theory, the role of instrument in cognitive development is a sensitive point. I thus explore the complementary use of the theory of instrumental genesis with the theory of shifts of attention to enable an analysis of students' cognitive development in a digital technology environment.

*Keywords*: Technology, linear algebra, instrument and attention

The integration of digital technology in mathematics education has given rise to continuing research concerning mostly students' use of digital tools to develop an understanding of mathematical ideas and objects. Researchers have proposed several theoretical perspectives for analysing the interactions between tools and the student (Guin and Trouche, 1999; Kieran and Drijvers, 2006; Arzarello et al., 2002; Falcade et al., 2007). In this study my focus is on the potentialities of the use of the theory of instrumental genesis to analyse students' mathematical knowledge acquisition.

The theory of instrumental genesis (Verillon & Rabardel, 1995) draws on actions and procedures taken by a student to use a tool. The tool can be transformed into an internally oriented tool (instrument of semiotic mediation) by the process of internalization (Vygotsky, 1978) that occurs through semiotic processes. For example, given a specific task in a dynamic geometry environment, the dragging tool can be transformed into a sign referring to the idea of function as covariation between dependent and independent variables (Falcade et al., 2007). The development of instrumental genesis is a complex process that depends upon several factors such as potentialities and constraints of the tool, actions and procedures taken by the student, the student's knowledge of mathematical concept in the task, and also the student's awareness of the affordances of the tool. The two interconnected components of instrumental genesis, instrumentalization and instrumentation, are used to describe the processes involved in the interactions between the student and the tool. The instrumentalization process, directed toward the tool, involves the development of skills to use the tool, the personalization and the transformation of the tool. It is about what the student thinks the tool was designed for and how the student uses the tool. It therefore calls upon attending to tool use. The instrumentation process, directed by the tool, involves the constraints and potentialities of the tool that shapes the student's knowledge acquisition (Trouche, 2005). This involves a shift of attention from tool use to what the tool can do, so that the tool becomes not the object of attention, but something that focuses and directs attention in particular ways, a mediating tool. The two components are concerned mostly with processes involved in transforming a tool into an instrument, not the role of instrument in knowledge acquisition. As researchers point out, the role of instrument in cognitive development is a sensitive point (Verillon and Rabardel, 1995), and the theory of instrumental genesis has shortfalls in putting forward the potentialities of instrument in the development of mathematical thinking.

On the other hand, among theories on cognition, John Mason's theory of shifts of attention appears to be more descriptive in terms of revealing the developmental process of mathematical being. Mason (2008) believes in the power of awareness and its education. Awareness refers to what enables us to act, calling upon our conscious and unconscious powers, and sensitivities to detect changes and to choose proper actions in certain situations (Gattegno, 1987; Mason, 2008). To educate awareness is to draw attention to actions which are being carried out with lesser or greater awareness. Attention can be drawn not only to mathematical objects, relationships and properties, but also to manifestations of mathematical themes, and to heuristic forms of mathematical thinking (Mason, 2008). According to Mason, the structure of attention comprises macro and micro levels; what is being attended to is as important as how it is being attended to. At the macro level, Mason describes the nature of attention as follows: "attention can vary in multiplicity, locus, focus and sharpness" (p.5). At the micro level, he distinguishes five different states of attending: holding wholes, discerning details, recognizing relationships, perceiving properties and reasoning on the basis of agreed properties. Holding wholes is when a student gazes at a definition, collection of symbols and/or diagram. The student may not focus on anything in particular, while 'waiting for things to come to mind'. Looking at the wholes, the student may discern and identify useful sub-wholes or details. Discerning details is a process that participates in and contributes to subsequent attending. As the student discerns details, she may recognize relationships between symbolic and geometric representations of mathematical concepts. When she becomes aware of possible relationships in the particular situation, she may perceive these as instantiations of a property. As she continues attending, she can use the perceived properties as a basis for mathematical reasoning. It is noteworthy that the described states of attention are not levelled or ordered. They often last for a few micro-seconds and alternate among other states. Those that become stable and robust against alteration for varying periods of time may block further development of awareness (Molina and Mason, 2009).

As summarized above, Mason's theory provides a framework for analysing students' attention in a mathematical activity. However, given the important role of the digital tool in the DGE-based activity, I want to take into consideration the interaction between the student and the tool. In the context of using DGS then, how might the relationships between instrument and attention be conceived, and, in particular, what might the effect of instrument of semiotic mediation be on shifts of attention?

To identify the states of attending, one may analyse semiotic resources such as gesture and discourse used by a student in a paper-pencil environment. However, my empirical study shows that in a digital technology environment, the instrumental genesis also causes shifts of attention. This suggests combining the theory of instrumental genesis with the theory of shifts of attention to enable analysing cognitive development in a digital technology environment. In particular, my empirical data supports the conjecture that analysing processes of instrumentalization reveals evidence of shifts of attention.

I interviewed a total of eight undergraduate linear algebra students who had all successfully completed a Linear Algebra course. They were given a worksheet containing a formal definition of eigenvectors and eigenvalues. They were then given a sketch designed to enable exploration of eigenvectors and eigenvalues for given $2\times 2$ matrices using *The Geometer's Sketchpad* software (Jackiw, 1991). As shown in Figure 1, the sketch includes a draggable vector $x$, as well as a non-draggable vector $Ax$. As, vector $x$ is dragged about the screen the vector $Ax$ moves accordingly. The sketch also includes numeric values of the matrix-vector multiplication ($Ax$). The user can change the values of matrix $A$.

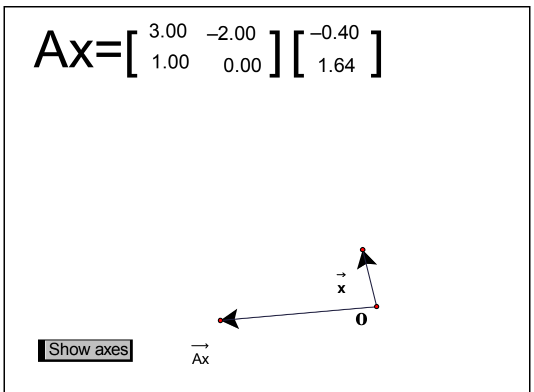


Figure 1. A snapshot of eigen sketch

In my analysis, I looked at students' actions with the sketch, different dragging strategies, and their ways of communicating orally about their interactions. Mason's theory of shifts in attention enables an analysis of students' interactions with the sketch as well as a description of their mathematical awareness. These shifts were made evident in part by the students' changing focus of attention from the definition to the sketch, but also in their different dragging strategies, which they used to identify eigenvectors and to explore the relationship between eigenvectors and eigenvalues.

In my presentation (and my extended paper), I will illustrate details of my analysis of students' actions with the dragging tool and argue that the dragging tool that was transformed into an instrument mediated students' conceptualization of the concept of eigenvectors and eigenvalues.

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