Using Think Alouds to Remove Bottlenecks in Mathematics

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Preliminary Research Report

Abstract

Think alouds are a research tool originally developed by cognitive psychologists for the purpose of studying how people solve problems. The basic idea being that if a subject can be trained to think out aloud while completing a certain task then the introspections can be analyzed and may provide insights into misunderstandings as well as higher thinking. This talk is a preliminary report of a think aloud conducted with calculus students to understand their difficulties with work problems in integral calculus.

Keywords: Calculus, cognitive science, classroom research, think alouds

Students in integral calculus often face difficulties in problems involving applications to physics like work and pressure problems. It is unclear whether their difficulties are due to lack of understanding of the definition of the definite integral as a limit of Riemann sums or whether it is difficulty in actually applying the concept to a physical situation like a work problem. This study is a preliminary report of an investigation conducted using the think aloud (or protocol analysis) technique with second semester calculus students at a two year campus of the University of Wisconsin.

Think alouds are a research tool originally developed by cognitive psychologists for the purpose of studying how people solve problems. The basic idea being that if a subject can be trained to think out aloud while completing a certain task then the introspections can be analyzed and may provide insights into misunderstandings as well as higher thinking. Schoenfeld(1) has used verbal transcripts and protocol analysis to study mathematical problem solving .

The goal of this study was to answer the question "Why do calculus II students have difficulty solving Work problems?". This was a qualitative study. Six students of varying abilities were selected to participate in the study. Students were first trained to think aloud by being asked to solve simple linear equations. They were then given a series of work problems ranging from the simplest kind with a fixed force and a fixed distance to the more involved that had a variable force and/or a variable distance. The sessions were video recorded. During the sessions the students were not prompted in any way and nor were any interventions introduced. The only comments made by the instructor were to request the student to verbalize their thoughts if and when the student fell silent. The recordings were transcribed and screen shots of the diagrams were taken. The transcriptions were coded and analyzed.

A coding scheme (see Appendix 1) was developed to code the verbalizations using Polya's four step problem solving process. Next we needed to identify the codes that would enable us to answer our question. We highlighted three codes, strategy, mathematical argument and logical inference. These codes reflect the thought process of a mathematician while problem solving. Each problem was then assigned a rating from 1-5 for each of the selected codes using a rubric (see Appendix 2 for the rubric), with 5 representing the score of a mathematician and 1that of a novice problem solver. These ratings helped to identify possible bottlenecks in the problem solving process.

Preliminary analysis indicates that the students who got "1" under strategy had a common trait. They headed straight for the fluid slice in the pool type problems but were then confused as to what to do next. Therefore one possible bottleneck is students memorizing a fragment of the instructor's strategy without understanding the underlying connections. Four of the six students seemed to have a significant strategy but were unable to solve the problem correctly due to mistakes in calculating the volume of a generic slice or incorrectly calculating the weight of the slice. This suggests that students do not have a good handle on the basic mathematical tools that are considered essential at this level.

During regular assessment students often erase their wrong work, so we only see the end product which doesn't always help us to identify the bottleneck. With a think aloud we are able to see much more of the problem solving process, the students' struggles in formulating strategies and mathematical arguments and thus make the thinking process more visible.

Questions for discussion

1. Can we make our think aloud coding list portable for problem solving across the university mathematics curriculum?
2. As an intervention for lack of strategy, will a grading rubric which will ask students to actually write down the strategy (before starting the mathematics), encourage students to strategize more ?
3. Strategy, Mathematical Argument and Logical Inference are key thought processes of a Mathematician solving problems. Are there any others that should be considered in this analysis?

References

1) Lendol Calder and Sarah-Eva Carlson, "Using Think Alouds to Evaluate Deep Understanding", Handout at Faculty College 2009, Office of Professional and Instructional Development, University of Wisconsin System.
2) Alan Schoenfeld, "Mathematical Problem Solving", Academic Press, Inc, 1985.

Appendix 1: Coding Sheet based on Polya's four steps

A. Understanding the problem

I. Understanding the problem UP
II. Recall RE

B. Devising a plan

I. Initial Plan DP
II. Alternate Plan DAP

C. Carrying out the plan

I. Setting up the variable SV
II. Mathematical argument MA
III. Mathematical argument (In correct,) MA_I
IV. Questioning (Q)
V. Guess (G)
VI. Recognizing limits (H)
VII. Narration (N)
VIII. Uncategorized (X)
IX. Strategizing (ST)
X. Strategizing with Reflection (ST_R)
XI. Inference (geometry) I_G
XII. Inference (Reflexive) I_R
XIII. Inference (Previous) I_P
XIV. Rearranging Terms R_T
XV. Calculation (C)

D. Looking Back

XVI. Revision (R)
XVII. Reflection on concept (R_C)
XVIII. Reflection(Rf)

Appendix 2: Mathematical Argument Rubric

Representative Thought	Points
Focal mathematical Argument executed(correct) early, i.e. Mathematical procedures applied correctly at the appropriate steps to solve the problem correctly	5
Focal mathematical Argument executed(correct) , late Or Focal Mathematical Argument executed (correct)early except for non-conceptual mistakes	4
Mathematical argument with some focus correctness but has significant mistakes Could not completely carry out mathematical procedures	3
Mathematical argument , with little focus with minor parts which are correct	2
Unfocussed Mathematical argument containing a few correct components.	1

Strategy Rubric

Representative Thought	Points
Focal Strategy achieved early (professional)	5
Focal Strategy achieved	4
Focal Strategy achieved with uncertainties	3
Indication of a strategy in the problem solving process but is not the focal strategy nor does it contain parts of the focal strategy	2
Unfocussed /unsignificant strategy , i.e.No evidence of a strategy or procedure	1