(Preliminary Research Report) Tracking and Influencing Concepts of Proof

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Abstract

"So the theorems in calculus aren't very scientific – they are not proved in an innocent way." This is a quote from a student in a first-semester calculus course at a research university after witnessing traditional limit-definition-based proofs of the 'Product Rule' and the 'Quotient Rule.' Here's a quote from a student in a junior-year course on Discrete Mathematics after writing on a white board a formal definition of the principle of strong mathematical induction and claiming that "If $n < 0$, then $P(n)$ is vacuously true since we know n must be a positive integer for the statement $P(n)$ to be true": "So, proof by strong induction doesn't require a base case." These, to us are interesting comments which, together with some quantifiable data, have spurred this research project. The remarks to us indicate the advent of certain perceptions and comfort levels with the notion of proof. In this preliminary research report, we attempt to track changes in understandings of, roles of, reasons for, and comfort levels with proof in four students from a first-semester calculus course and five students from a junior-year Discrete Mathematics course via video-taped interviews which are ethnographic in spirit – interviews unfold with minor direction on our part except when comments which we think are interesting are encountered. The interviews begin by asking the students to comment on strategically chosen results which are proved in class or solved in a homework assignment; for example, the "Product Rule" in the calculus course and the "cocktail party problem"[1] in the Discrete Math class. The strategy behind choosing the proofs for interview fodder is couched in our idea of what the proofs or problems represent: ways to verify a claim which is "known" to be true – all of the students in the calculus class are familiar with the "Product Rule" – or as a way to deal with a large number of cases at once – an exhaustive case-by-case consideration of the cocktail party problem would require examining $2^{15} = 32,768$ circumstances. The questions being pursued include: How does a student's conception of the role of proof evolve during a course in which proofs are used as the main tool for verification, primarily by the instructor (as in the calculus course), versus a course in which proofs are used to solve problems, primarily by the students (as in the Discrete Math course).

Keywords: Student conceptions of proofs, student respect for proofs, discourse analysis, teaching proofs.

There is a body of research on students' conceptions of proofs, [1, 6] for example, as well as why students have difficulties with proofs, for example [2, 3, 4, 5]. In this research project, we intend to document how or if students conceptions of proof, their role in Mathematics, their utility, their comfort with them, etc. evolve with respect to the the way proofs are used, and types of proofs given or required of the students. Similar to the way Selden and Selden in [4] identified interesting *key points* or *key problems*[2] which caused notable outcomes in their students, we attempt to anticipate moments which will have such notable effects and capture them in recorded interviews. To this end, two groups of students are identified and interviewed in an ethnographic spirit periodically throughout the semester in their respective courses, one group from a first-semester calculus course, the other from a third-year Discrete Mathematics course. The students from the calculus course were chosen because of their majors and their agreeing to participate in the interviews. Their majors are Physics, Nutrition and Food Science, Computer Engineering, and Mathematics Education. Those from the Discrete Mathematics course were chosen in a similar way, but their majors are not as diverse due to the nature of those required to take the course; their majors are: Mathematics Education, Math/Stat Composite-Teaching, Mathematics, Computer Science.

[1] The "cocktail party problem" is a result from *Ramsey Theory* and has an interpretation as follows: given six people at a party, there are either 3 people who did not know each other before the party or three people who knew each other before the party.

[2] These are our terms.

The nature of the calculus course is probably safely said to be typical of a university first-semester calculus course: The Mathematics and Statistics Department chose a textbook via a committee of department representatives and all sections of the course follow a common syllabus whose topics include limits, derivatives, integrals, and applications of them. The Discrete Math course is treated as a "bridge course," that is, as a course that serves as a transition to higher-level Mathematics courses, where the typical homework or even exam includes proofs. Consequently, the students of the Discrete Math course can be expected to be new to developing and writing rigorous proofs, but are expected to do so throughout the course. As for the style of presentation during class meetings, very few examples are presented; there is rather a focus on discussing proof strategies and why they are appropriate and the conceptual ideas involved in the current topic. An example topic which is fantastic fodder for proof technique exploration is Graph Theory and an example subtopic is that of domination graphs of directed graphs. After sufficient notation and terminology for directed graphs is presented, the definition of a domination graph is given and discussed and the domination graphs of a few examples are constructed. Students are then asked to prove some theorems from the theory of domination graphs. (See below for clarification.)

The students in each course are interviewed periodically throughout the semester at what we deem to be key points or at least the topics of the interviews are what we deem to be key moments, events, or topics in the course. Here is an incomplete list of key events or key moments for each class and how they are treated in the interviews:

Calculus Key events may be the proof of a certain theorem in class or the introduction of certain topic.

- Key point A: The proof of the 'Product Rule'[3] theorem *If* $F(x) = f(x)g(x)$ *and* f *and* g *are differentiable, then* $F'(x) = f(x)g'(x) + g(x)f'(x)$.
- Interview A starting point: What did you think of the proof of the Product Theorem? Was it informative?
- Key Point B: The proof that *If* $f(x)$ *is differentiable at* $x = a$*, then* $f(x)$ *is continuous at* $x = a$.
- Interview B starting point: Did the proof that differentiability implies continuity shed any light on the notion of continuity? Was the proof necessary in the first place?

Discrete Math Key events may be the presentation of a proof in class or a the completion of an assignment which included a specific statement to be proved, or the discussion of a proof technique in class.

- Key event A: The homework problem in which a student is asked to prove that *If a loopless directed graph has more than* $(n-1)^2$ *arcs, then it has a spanning cycle.*
- Interview A: Do you think, in grappling with the problem that you grew as a Mathematician?
- Key event B: the discussion in class of the intractability of the cocktail party problem extended as follows: *How many people are necessary ar a party to guarantee that there are 6 people who know each other before the party or 6 people who don't know each other before the party?*

The presentation the to RUME Conference will include the following:

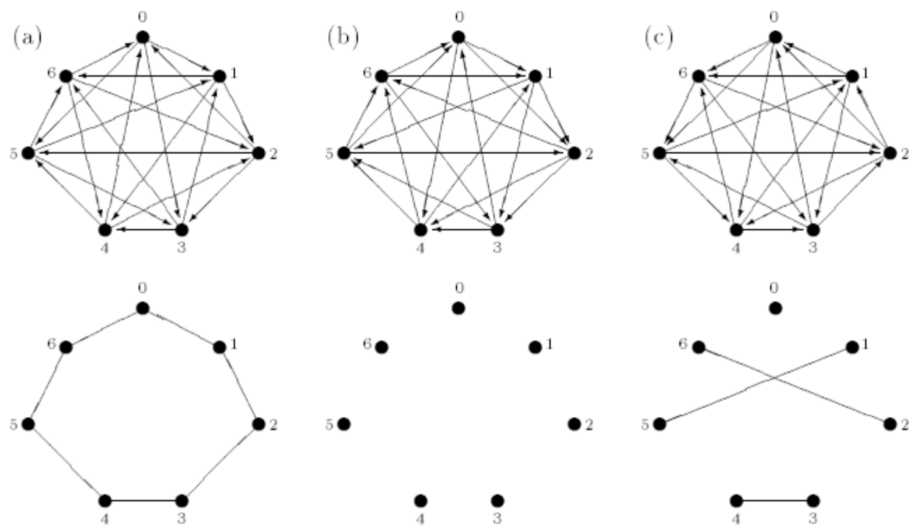
- Clips of what we deem as fascinating comments from the students.
- Clips of what we deem as statements indicating a change in perception of proofs or comfort with them.
- A more complete list of key events, proofs, problems, moments from both course.
- Reasons why the aforementioned moments were deemed as *key*.

[3] We note that we refer to the 'Product Rule' as the *Product Theorem* and insist that the students do too.

We will ask the audience for suggestions to further this study; in particular, if there are better understood reasons why students' proof conceptions and comfort levels change. We will ask for the audience's opinions and suggestions for further key proofs, problems, etc.

Definition of Domination Graphs:

Let $D = (V, A)$ be a directed graph. The *domination graph* of D is the graph on the vertex set of D with vertices x and y adjacent if and only if $\{u \in V : x \to u\} \cup \{u \in V : y \to u\} \cup \{x, y\} = V$. Conceptually, if the directed graph represents the results of a competition where $x \to y$ if x beat y in their defining game, the domination graph indicates pairs of players which together beat all the other players – so that pair may form a good doubles team if the directed graph is represents a tennis tournament. The domination graphs of the directed graphs in figures (a), (b), and (c) are given directly below them.



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