The Impact of Instruction Designed to Support Development of Stochastic Understanding of Probability Distribution

Preliminary Research Report

Darcy L. Conant
University of Maryland, College Park

Abstract:

Large numbers of college students study probability and statistics, but research indicates many are not learning with understanding. The concept of probability distribution undergirds development of conceptual connections between probability and statistics and a principled understanding of statistical inference. Using a control-treatment design, this study employed differing technology-based lab assignments and investigated the impact of instruction aimed at fostering development of stochastic reasoning on students' understanding of probability distribution. Participants were approximately 200 undergraduate students enrolled in a lecture/recitation, calculus-based, introductory probability and statistics course. This preliminary research report will discuss the framework used to develop the stochastic lab materials and preliminary results of an assessment of students' understandings.

Key words: Probability distribution, stochastic reasoning, technology-based instruction, instructional intervention.

Statement of research issue:

Large numbers of university students study probability and statistics (Moore & Cobb, 2000), but research indicates that many of these students exhibit difficulties in learning and applying probabilistic and statistical concepts (Garfield & Ben-Zvi, 2007; Shaughnessy, 1992, 2007). Inappropriate reasoning in probability and statistics is widespread and persistent across all age levels. After probability instruction, many post-calculus students demonstrate merely instrumental understanding (Skemp, 1976) and present notions about probability that are not aligned with formal probabilistic concepts (Barragues, Guisasola, & Morais, 2007). This study draws on constructivist and situated learning perspectives and assumes understandings are built through learning experiences, which are impacted by the learner, teachers, and the instructional material. The study assumes that: (1) teaching impacts learning and can facilitate learning with understanding; (2) effective teaching elicits students' pre-existing understandings and builds on that understanding; (3) effective teaching helps students develop deep knowledge connections in the context of a conceptual frame for the content domain (Bransford, Brown, & Cocking, 2000). This research was designed to evaluate the effectiveness of an instructional intervention that builds on students' initial understandings of probability and statistics and facilitates student understanding of content within a connected conceptual framework. The study seeks to measure and describe individual understandings of probability distribution.

The concept of probability distribution is a powerful springboard for the development of stochastic reasoning as it may facilitate making deep conceptual connections around probabilistic understandings related to variability, independence, sample space, and distribution (Liu & Thompson, 2007). Principled knowledge (Spillane, 2000) refers to an understanding of the ideas and concepts that support mathematical procedures. Principled knowledge of probability distribution not only refers to a conceptual understanding of the mathematical procedures used when solving probability problems, but also an understanding of connections between and within the constructs of probability, variability, and distribution.

This large-scale control-treatment study investigated the impact of an instructional intervention on post-calculus students' understandings of probability distribution. The treatment intervention consisted of lab materials designed to address stochastic reasoning and to support students' principled knowledge of probability distribution. The control lab materials reviewed prerequisite calculus content which students encounter in the course, thus controlling for quantity of instruction. This study addressed the following question: *What is the impact of an instructional intervention designed to support development of stochastic understanding of probability distribution of undergraduate students enrolled in an introductory calculus-based probability and statistics course?*

Summary of Related Research

Stochastic reasoning is grounded in conceptual connections between probability and statistics. To reason stochastically means conceiving of an observed outcome as but one expression of an underlying repeatable process that will produce a stable distribution of outcomes in the long run (Liu & Thompson, 2007). One reason why learners may experience difficulty with stochastic reasoning is because learning about random experiments through simulation or experimentation is not connected to learning about combinatorial schemes or tools such as tree diagrams in probability (Batanero, Godino, & Roa, 2004). Also, intuitive thinking based on experience with random generators appears to be disconnected to formal mathematical thinking about probability (Abrahamson, 2007). Making statistical inferences requires application of stochastic thinking for correct interpretation, and a stochastic conception of probability supports thinking about formal statistical inference (Liu & Thompson, 2007).

Research indicates that many post-calculus students, who are either currently enrolled in or have recently completed introductory probability and statistics courses, demonstrate probabilistic thinking and heuristical biases that are aligned with the thinking of novice learners in algebra-based classes and high school students (Abrahamson, 2007; Barragues, et al., 2007; Lunsford, Rowell, & Goodson-Espy, 2006). Even after instruction addressing probability, many post-calculus students still exhibit poor understandings of random phenomena and present mistaken conceptions of random sequences, insensitivity to sample size, and a deterministic bias. Research shows that post-calculus students who have completed a probability/statistics course still have difficulty with a modeling viewpoint and struggle to discriminate between empirical distributions and theoretical probability distributions (Barragues, et al., 2007). Research suggests that after completing an introductory, calculus-based probability and statistics course, most students are comfortable with formal mathematical manipulations of probability distributions and master algorithmic techniques, but they lack stochastic conceptions and deep conceptual understanding of probability distribution.

Research investigating development of post-calculus students' understanding of probabilistic concepts indicates that teaching is an important factor related to students' understandings of probability. Teaching which emphasizes procedures tends to result in instrumental understanding, whereas teaching which facilitates learner explorations of conceptual notions of probability as a distribution and its connection to mathematical theorems offers opportunities for students to build relational understanding (Skemp, 1976) in probability and statistics. A study of post-calculus engineering students' conceptions of probability found that conventional teaching can have a poor effect on students' probabilistic reasoning (Barragues, et al., 2007). Although not conducted in a classroom, the work of Abrahamson (2007) indicates that post-calculus learners can consolidate their intuitive notions of probability with their formal mathematical knowledge in the context of probability distribution. Still other research points to the promise of learners' engagement in tasks utilizing a computer-based dynamic statistical environment as a means towards facilitating development of notions of sampling distribution, variability, and inferential reasoning (Meletiou-Mavrotheris, 2003; Sanchez & Inzunsa, 2006).

Research Methodology:

This study compared the impact of differing instructional lab materials. The subjects were approximately 200 students enrolled in a calculus-based introductory probability and statistics course at a large, public university. The course setting consisted of two lectures with the same syllabus taught by mathematicians who covered the same content. Teaching assistants led accompanying recitations. One lecture had six recitations sections, and the other had four. Students were randomly assigned to a recitation section via their course registration. Each recitation section associated with a given lecturer was randomly assigned to either the treatment or control condition whereby a teaching assistant had both treatment and control recitations. This assignment balanced the treatment and control across lectures and recitation sites in order to mitigate confounding variables due to differences in teaching between the lecturers and between the teaching assistants. All students enrolled in a particular recitation received one type of lab material. The treatment group received lab materials designed to support stochastic reasoning, and the control group received lab materials which consisted of a review of calculus content used in the course. Students' understanding was measured via conceptual assessments in the form of an extra-credit quiz and course examinations. At the end of the study, selected students participated in interviews designed to provide insight into students' thinking and reasoning about conceptual assessment items.

Framework for Instructional Intervention:

The treatment instructional intervention implemented in this study consisted of six supplemental lab assignments aimed at the development of stochastic reasoning in the context of probability distribution. The design of these tasks was based on a hypothetical learning trajectory (Simon, 1995) of students' stochastic conceptions of probability (Liu & Thompson, 2007) which was adapted for use in the context of probability distribution. This study extended the research investigating the impact of bridging tools (Abrahamson & Wilensky, 2007) on college students' understanding of probability distribution into a classroom setting. Learners in the treatment sections engaged in technology-supported simulation tasks designed to elicit prior understandings of probability. These tasks required learners to consider juxtaposed constructs in the domain, such as theoretical versus empirical probability and independent versus dependent events. The approach was to have the learner decompose domain constructs into idea components and then use conceptual bridging tools to recompose the constructs using their intuitive and analytic resources. In order to control for instructional time, the control group received tasks which reviewed calculus content used in the course and covered topics such as integration using substitution and integration by parts. The instructional intervention material was designed to prepare students to learn from the lectures and therefore provide greater opportunity for students to make deeper conceptual connections (Schwartz & Bransford, 1998).

Implications of this research:

Given the evidence that many college students in probability and statistics classes are not learning with understanding, it is critical to investigate the effectiveness of approaches for teaching probability and statistics in ways that build on students' initial understandings and helps students understand the content not merely as facts to be memorized, but as connected concepts within a conceptual framework. Knowledge of whether instruction which is aimed at fostering stochastic reasoning impacts learners' understandings of probability distribution could inform future design of instruction and development of instructional materials in probability and statistics.

Discussion Questions:

- How might this framework (to be shared in the presentation) be extended for use when planning instructional strategies or in designing instructional material?
- How might this framework further inform the analysis of the conceptual assessment items? (preliminary findings and planned analysis will be presented)
- What are the further implications of these preliminary findings for instruction in probability and statistics?
- What are the advantages/disadvantages of utilizing technology-based instructional material to support lecture/recitation delivery of course material?
- What other issues related to student understanding of probability and statistics might be informed by this study?
- What are the implications of the degree of student understanding of prerequisite calculus procedures/concepts (as revealed in the control labs) for those teaching probability and statistics as well as for the teaching of calculus?

References

Abrahamson, D. (2007). *The real world as a trick question: Undergraduate statistics majors' construction-based modeling of probability.* Paper presented at the American Education Research Association, Chicago, IL.

Abrahamson, D., & Wilensky, U. (2007). Learning axes and bridging tools in a technology-based design for statistics. *International Journal of Computers for Mathematical Learning, 12*(1), 23-55.

Barragues, J., Guisasola, J., & Morais, A. (2007). Which event is more likely? Estimating probability by university engineering students. *Recherches en Didactique des Mathematiques, 27*(1), 45-76.

Batanero, C., Godino, J., & Roa, R. (2004). Training teachers to teach probability *Journal of Statistics Education, 12*(1).

Batanero, C., Tauber, L., & Sanchez, V. (2004). Students' reasoning about the normal distribution. In D. Ben-Zavi, & Garfield, J. (Ed.), *The challenge of developing statistical literacy, reasoning, and thinking*. Dordrecht, the Netherlands: Kluwer Academic Publishers.

Bransford, J., Brown, A. L., & Cocking, R. R. (Eds.). (2000). *How people learn: Brain, mind, experience, and school*. Washington, DC: National Academy Press.

Garfield, J., & Ben-Zvi, D. (2007). How students learn statistics revisited: a current review of research on teaching and learning statistics. *International Statistical Review, 75*(3), 372-396.

Liu, Y., & Thompson, P. (2007). Teachers' Understandings of Probability. *Cognition & Instruction, 25*(2/3), 113-160.

Lunsford, M., Rowell, G., & Goodson-Espy, T. (2006). Classroom research: Assessment of student understanding of sampling distributions of means and the central limit theorem in post-calculus probability and statistics classes. *Journal of Statistics Education, 14*(3).

Meletiou-Mavrotheris, M. (2003). Technological tools in the introductory statistics classroom: Effects on student understanding of inferential statistics. *International Journal of Computers for Mathematical Learning, 8*, 265-297.

Moore, D., & Cobb, G. (2000). Statistics and mathematics: Tension and cooperation. *The American Mathematical Monthly, 107*(7), 615-630.

Sanchez, E., & Inzunsa, S. (2006). *Meanings' construction about sampling distributions in a dynamic statistics environment*. Paper presented at the Seventh International Conference on Teaching Statistics, Salvador, Brazil.

Schwartz, D., & Bransford, J. (1998). A time for telling. *Cognition and Instruction, 16*(4), 475-522.

Shaughnessy, J. M. (1992). Research in probability and statistics: Reflections and directions. In D. A. Grouws (Ed.), *Handbook of research on mathematical thinking and learning* (pp. 465-494). New York: McMillan.

Shaughnessy, J. M. (2007). Research on statistics learning and reasoning. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning*. Charlotte, NC: Information Age Publishing.

Simon, M. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, *26*(2), 114-145.

Skemp, R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching, 77*, 20-26.

Spillane, J. (2000). Cognition and policy implementation: District policymakers and the reform of mathematics education. *Cognition & Instruction, 18*(2), 141-179.