Supplemental Instruction and Related Rates Problems

Preliminary Research Report

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In this study, we observed first semester calculus students solving related rates problems in a peer-led collaborative learning environment. The development of a robust mental model has been shown to be a critical part of the solution process for such problems. We are interested in determining whether the collaborative learning environment promotes the development of such a mental model. Through our observations, we were able to determine the amount of time students spent engaging with the diagrams they drew to model the problem situation. Our analysis strove to also determine the quality of the student interactions with their diagrams. This analysis provided insights about the mental models with which the students were working. Engaging students with complex, non-routine problems resulted in the students spending more time developing robust mental models.

Background

Supplemental Instruction (SI) workshops, based on the work of Uri Treisman in the early 1980s and developed by the University of Missouri, Kansas City (UMKC), have been highly successful at campuses across the country and also in neighboring campuses with student populations like those at our institution. The UMKC SI workshop is a structured learning environment where students gain additional experience in the subject matter taught in the course to which it is linked (Bonsangue, 1994). From these models, we have developed a version of the SI workshops that meets our students' needs. Students do not simply review course material or do homework in SI workshops, but undertake additional, challenging problems or assignments to build confidence in their abilities and to gain self-reliance. They engage in active and cooperative learning activities, utilizing peer facilitators as resource persons. The peer facilitator attends each class lecture so that the workshop problems are relevant to course assignments. In doing so, the peer facilitator also serves as a role model for SI students and creates an increased culture of accountability in the classroom.

In the Spring 2009 semester, our institution began expanding our workshop program, particularly in calculus. We ran 3 successful pilot SI workshops for first semester calculus. The success of those courses led us to run 4 and 5 first semester calculus SI workshops, respectively, in the Fall 2009 and Spring 2010 semesters. Our workshop model is such that two calculus courses (usually) taught by the same instructor feed into one SI workshop. The workshop is an optional one semester credit hour course; it is hosted by a junior or senior level math major and graded credit/no credit based solely on attendance and participation. In each workshop session, the peer-mentor facilitates student group work on topics that have been recently presented in lecture. During the Fall 2009 and Spring 2010 semesters, we observed how students solve related rates problems in this peer-led instructional setting with the purpose of examining how students think about and solve application problems. We were particularly interested in whether the collaborative environment of the workshop setting promoted the development of the students mental model of the problem situation.

Theoretical Perspective

White and Mitchelmore (1996) studied students understanding of related rates and max/min calculus problems. Their study used differently worded versions of four problems, which ranged from a word problem that required the student to model the situation and come up with the appropriate relation to an almost strictly symbolic version that merely needed to be manipulated. It was found that students performed better when there was less need for translation from words to symbols.

White and Mitchelmores study showed that students have a tendency for a manipulation focus, in which they base decisions about which procedure to apply on the given symbols and ignore the meaning behind the symbols. Interview comments showed that manipulation focus errors were not just bad luck, but that students were actively looking for symbols to which they could apply known manipulations.(p. 88) The researchers further described two other forms of the manipulation focus: 1) the x, y syndrome, in which students remember a procedure in terms of the symbols first used to introduce the concept without understanding the meaning of the symbols; and 2) the students fail to distinguish a general relationship from a specific value.

In her studies, Engelke (2004, 2007a, 2007b) found that students fixated on procedural steps which prevented them from building a mental model of the situation. Without having a mental model of the situation, the students were less likely to engage in transformational (Simon, 1996) and covariational reasoning (Carlson, Jacobs, Coe, Larsen, & Hsu, 2002) about the problem situation. The ability to engage in transformational and covariational reasoning appeared to be necessary to identify relevant functional relationships. She suggested that a robust mental model facilitated being able to move flexibly between the geometric and algebraic representations which fostered being able to construct an appropriate formula in which one can think of each of the variables as a function of time.

Methods

In our study, the students are first semester calculus students that are coming from two or more standard lecture courses and have the option to participate in a Supplemental Instruction (SI) workshop. The SI sessions are hosted by junior/senior level math majors and are intended to further develop the students understanding of the material presented in the regular class sessions. During the SI sessions the students worked in small groups to solve problems provided by the SI leaders. During their time in the SI workshop, the students were filmed over several days, in which the students covered the chain rule, implicit differentiation, and related rates problems. Since the students were vocalizing and writing their ideas and thought processes, we were able to determine what types of reasoning structures the students were utilizing. These videos were transcribed for analysis, using pseudonyms for the students names. The coding process was done using atlas-ti software.

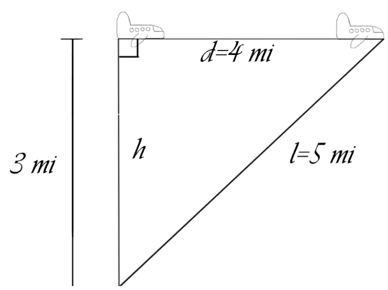
Results

The students were given the following problem: *A plane flying horizontally at an altitude of*

3 miles and a speed of 600 mi/hr passes directly over a radar station. When the plane is 5 miles away from the station, at what rate is the distance from the plane to the station increasing?

As we see in the transcript excerpt below, the students began by drawing a diagram to represent the problem situation and labeled the sides of the triangle that is being formed by the plane and the radar station with the variables l, h, and d. It was also determined that they should use the Pythagorean Theorem to relate the variables in the problem.

Student 3: The distance over time. Its miles like the island...okay so a plane is flying horizontally at an altitude of 3 miles [labels with d], at a speed 600 miles per hour. Passes directly over the radio station then is 5 miles away from the radio station, 5 miles. [calling the diagonal distance $l = 5$ miles] what is the rate, at what rate is the is the distance from the plane to the radio station increasing? [Also wrote $\frac{dd}{dt} = 600$ mi/hr and $\frac{dr}{dt} =?$ under the diagram] Ok. Were going to call this h [the vertical height of 3 mi]. Ok, someone. [Passing the chalk as per instructions from the SI leaders] Wait, wait, I forgot to do...never mind, the equation, the equation we are going to use to relate it all is the Pythagorean Theorem exactly.



Student 2: Does that look good? [wrote $3^2 + d^2 = 5^2$ to fill in $d = 4$ on the diagram]
Student 1: It looks great; I am just trying to
Student 3: Ok, can we just use 3 squared plus d squared is x squared, yes.
Student 2: Should I do like, should I differentiate it? ...
Student 1: Let me know if I am doing this right. We are looking for $\frac{dd}{dt}$, dont we need a derivative? [starts differentiating the Pythagorean theorem which he wrote as $a^2 + b^2 = c^2$]
Student 3: Why do a, b, c? I think it should really be the letters we have.
Student 4: Do we have this? We don't have this. [referring to $\frac{da}{dt}$]
Student 2: What we have is the second one.
Student 3: This still...if this still says a, I feel that this should say h, right? Not a right?

As the students start to incorporate the Pythagorean Theorem into the solution, there is some debate on the notation that is being used. Student 3 used d, h, and l as the variables, and Student 1 is now using a, b, and c. Even after Student 4 starts working, Student 3 is still worried about the mixed up variables.

There are three different ideas of what the Pythagorean Theorem should look like at this point in the problem solving process. Student 3 starts using $3^2 + d^2 = x^2$, but then changes her mind and thinks it should use the variables they have in their diagram. Student 1 is using $a^2 + b^2 = c^2$.

The variables used to write down the formula for the problem appear to be based on what has been used in previous problems for Student 1 and Student 3. However, Student 3 shifted her thinking to wanting the variables used in the formula to match the variables used in the diagram to represent the problem situation. In the end, it was decided that the variables in the formula should match the variables they had used in their diagram to get $3^2 + d^2 = l^2$. They differentiated their formula to obtain: $2d\frac{dd}{dt} = 2l\frac{dr}{dt}$. This brings up the question, where did the $\frac{dr}{dt}$ come from? Recall that when Student 3 drew the diagram at the very beginning of the problem solving process, she labeled $\frac{dr}{dt} =?$ under her picture. This could be an instance of the x, y syndrome described by White and Mitchelmore as the students wrote this for every problem. The students seemed to associate $\frac{dr}{dt} =?$ with the unknown rate regardless of which variables they had used in their diagram, so when they differentiated their formula, they had to accommodate this notation. The students successfully solved the problem; reporting that $\frac{dr}{dt} = 480$ mi/hr.

In their diagrams, these students were inconsistent in their use of variables, frequently not representing any of the changing quantities with variables but only labeling quantities with numerical values. While diagrams are being drawn, the students seem to be choosing a formula for the problem based on keywords about the "shape" that is mentioned in the problem rather than on the relationships that exist between variables in the problem. The absence of a robust mental model that incorporates variable names for changing quantities could be why the students had no issues writing down $\frac{dr}{dt} =?$ for each problem they solved. This was merely the notation for what they wanted to find rather than a representation of a rate connected to a particular changing quantity. These observations support Engelke's (2004, 2007a, 2007b) results that students are not adept at building mental models that support the problem solving process when solving related rates problems. Based on this evidence, we suggest that more time should be allotted for engaging students in building robust mental models that incorporate the relationships that exist between the diagram that is drawn, the variables that represent changing quantities, and an appropriate formula. We will present some ideas about how to improve students' mental models.

Questions for the audience:

1. What qualitative studies have been done on peer-led instructional settings, in calculus?

2. Is the problem solving process that occurs in peer-led instructional settings reflective of what occurs in the classroom?

3. How do you promote students' building of robust mental models?

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