Preliminary Research Report

Exploring student's spontaneous and scientific concepts in understanding solution to linear single differential equations

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In this study, we use the zone of proximal development to characterize students' spontaneous and scientific concepts of rate of change, rate proportional to amount, exponential function and long-term behavior of solutions for a system of one and two linear autonomous differential equations. Our focus on the dynamics of the differential equation systems is to investigate how these spontaneous and scientific concepts are incorporated from a system one linear differential equation into a larger system of two linear differential equations. We use and adapt previously used instructional activities from an inquiry-oriented differential equation course to help us gather our data by doing semi-structured interviews with five students. We present only preliminary findings on student's thinking of solutions mainly for single differential equations, with some insights of student thinking of solutions on a system of two differential equations.

KEYWORDS: Differential equations, solutions, rate of change, zone of proximal development

Research on the teaching and learning of differential equation concepts has recently appeared within the last decade (Habre 2000, Rasmussen, 2000, 2001, 2003, Rowland, 2006, and Rasmussen and Blumenfeld, 2007). These last studies show that students have difficulties acquiring meaningful understanding of the concept of derivative even though students can manipulate formulas algebraically or geometrically. For example, students have misconceptions in understanding the different components of a differential equation, such as having difficulties describing what the rate of change, variable and solution means within the context of the differential equation (Habre, 2000; Rasmussen and King, 2000; Rasmussen 2001). Rasmussen and Whitehead (2003) reported that similar cognitive difficulties were observed when students have to interpret solutions or varying rates for a system of two equations. In 2007, Rasmussen and Blumenfeld describe a teaching experiment about a spring-mass problem where students invented their own intuitive method for finding the eigenvectors for a system of two linear differential equations with constant coefficients. The authors found that students used proportional reasoning to help them understand the concept of eigenvectors. However, studies at this level, with a system of two differential equations, are very scarce.

This paper presents the preliminary results of an ongoing research investigation about the difficulties and ways of reasoning associated with understanding the dynamics of solutions and long term behavior of the autonomous linear system with one or two equations within different context representations (numerical, graphical, and algebraic). More specifically, these are the two main questions we want to explore:

1. What are students' spontaneous concepts about rate of change, "rate proportional to amount", exponential function, and long-term behavior of solutions to differential equations for systems of one and two equations?
2. Given a specific set of activities[1], what is the interplay between spontaneous and scientific concepts (within our focus)?

Our main motivation for this study is to explore student thinking and reasoning of rate proportional to amount in systems of linear differential equation with one and two equations. We used different mathematical representations so that students not only algebraically solve differential equations, but also given the opportunity to understand different patterns and relationships, and to model and predict general and qualitative behavior about solutions. We want to investigate the interplay of student's spontaneous and scientific concepts about rate of change, rate proportional to amount, and exponential function in connection to solutions to differential equations within both dimensions.

The theoretical foundations for this study follows: Vygotsky(1987)'s zone of proximal development and Steffe and Thompson (2000)'s teaching experiments. We use the framework of teaching experiments (Steffe and Thompson, 2000) not only to see first hand how students are constructing knowledge, but also to follow and not necessarily test a learning trajectory. According to Vygotsky, a learner develops meaning and understanding from mental processes and from a concept system (or from a structured concept system of ideas), in which both spontaneous and scientific concepts follow an interdependence learning development of a concept. A spontaneous concept originates when a person first encounters the new concept within empirical situations, while a scientific concept originates when a person first encounters the new concept in its generalized form. Both concepts follow interdependent paths of development in associating an object to its associated generalized concept. One of the attributes of a scientific concept is that it has to exist within a concept system so that connections of mental process can be made and generalized. Another attribute is that the learner has to operate with conscious awareness and volition in their thinking process. In contrast to all of these main attributes for the scientific concept, a spontaneous concept exist outside a concept system, it lacks some level of conscious awareness, with no generality and voluntary control involved.

Going back to our research questions, the second question is a very important question because not only we want to investigate characterization of spontaneous concepts to scientific concepts but also their influence on each other. Both spontaneous and scientific concepts required further thinking and reflection, so we anticipate these instructional activities will helps us get at those instances in students' learning development. We want to investigate what students can do with these concepts so that concepts are used consciously and with purpose (i.e. voluntary control). With "what can students do" we are referring to those mental images students developed in their learning process mediated by the spontaneous and scientific concepts. Hence, we are interested in the characterization of the zone of proximal development when learning differential equations given a specific set of instructional activities, and how certain concepts

[1] This specific set of activities were mainly developed by professor Chris Rasmussen

(especially rate and rate proportional to amount) are later incorporated into a more advanced system consisting of two differential equations.

For this paper, we will present preliminary findings with respect to our first question and some insights with respect to question two. Our preliminary findings indicate that students use differential equations (with one equation), $dp/dt = f(p)$ as a mathematical tool to predict numerical or graphical solutions, as reported in other prior studies. (Habre, 2000, Rasmussen, 2001, 2003, Rasmussen and Stephan, 2002). In general we expect students to be able to solve linear differential equations algebraically and qualitatively, but maybe not be consciously aware of why methods worked or what the solution actually means. We expect to see more cognitive difficulties when students make the transition from working with single differential equation to a system of two differential equations. Our long term goal for the study is to explore the effect of focusing on these different concepts given a set of particular set of instructional activities, and how it can help in improving the teaching and learning of differential equations.

Question for discussion:

1. Why is it useful to characterize spontaneous or a scientific concept of solutions in a single differential equation or from a system of two differential equations?

2. What is involved in creating a coherent learning trajectory for student when learning solutions from a single differential equation into a system of two differential equations? And what do we mean with a coherent learning trajectory?

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