Linking Instructor Moves to Classroom Discourse and Student Learning in Differential Equations Classrooms

Introduction

In undergraduate mathematics classrooms where instructors are beginning to focus on more student-centered instruction, teachers' moves foster mathematical discourse among students and teachers as a way to further the mathematics. While some are studying these teacher moves in K-12 classrooms, there seems to be little research focusing on this in the university classroom. We define a pedagogical content move to be a discursive or inscriptive act by an instructor that is purposely used to promote or further the mathematical agenda in the classroom (Lee, Keene, Lee, Holstein, Early& Ely, 2009). In an earlier paper, we presented several of these moves as identified in our data collection and analysis. In this proposal, we further this research by answering the question:

What specific links can be described between university professors' instructional moves and the discourse and learning in a classroom about one particular mathematical concept?

We have chosen parametric curves as the specific content to embed our work for two reasons. First of all, it is an overarching and important mathematics concept which appears in mathematics from precalculus through university level mathematics analysis. Specifically, in differential equations, as students learn how to find solutions, they are often represented as parametric equations and visualized as curves in two or three dimensions. Secondly, the authors have previously reported on research about how student come to visualize curves that are parameterized over time (Keene, 2007).

Literature Review

Parameter and parmetric curves. We define the concept of "parametric curves" to be representations in 2 or more dimensions of functions defined by two or more equations with the same independent variable. Often in differential equations this variable is time, but it is not a requirement to be a parametric curve. Research about student learning of parameter and parametric equations is limited. Student understanding of parameter was studied by Drijver (2001) who discusses how students understand parameter as place holder, changing quantity, and as generalizer. Keene (2007) also discusses the notion of parametric reasoning with time as the dynamic parameter. She provides the notion that parametric reasoning includes students' making time an explicit quantity, using and connecting qualitative and quantitative reasoning, and imagining the motion. Engelke (2007) introduces a framework for student understanding of related rates (of change), which closely links to the idea of parameter.

The idea that parametric curves are important to many areas of advanced mathematics has not led to significant research in their understanding. Some publishing appears about how to teach parametric equations using technology (Drivjers, 2001) but how students learn them in a classroom situation is missing.

Teacher pedagogy, discourse, and student understanding at the undergraduate level. Some prior research has begun to focus on pedagogical issues related to mathematics instruction at the undergraduate level. For example, in studying the implementation of the same differential equations curriculum materials, Wagner et al (2007) analyzed the specific problems a professor encountered in facilitating mathematical discussions. The professor in that study had taught DEs for many years from a traditional perspective and was new to inquiry-oriented instruction. In particular they found that the professor struggled to respond to unexpected student responses during whole class discussions. This is similar to the work of Bartlo et al (2008) that shows that the mathematics knowledge that a professor brings to an abstract algebra classroom is broad in certain ways but that there are pedagogical situations when building content connections and understanding student thinking is a challenge.

Additionally, other researchers have focused on discourse practices and moves in the mathematics classroom in K-12, as well as some at the university level. While general practices such as telling or revoicing have been carefully analyzed for their effects on the mathematical discussion (e.g., O'Conner & Michaels, 1993) most do not focus on the ways in which the instructor draws upon specific content knowledge when making a discursive move. Rasmussen, Marrongelle, & Kwon (2008) have developed an IODM (Inquiry Oriented Discursive Move) framework to analyze mathematical discourse. We are interested in using and modifying this framework to identify and analyze moves used by an instructor to introduce such tools and the mathematical content understandings that drive the move, specifically in terms of parametric equations and their representations as curves.

Methodology

Data collection was conducted in Spring 2008 in a college level Differential Equations class in the southeastern United States (enrollment of 25) using a classroom teaching experiment methodology (Cobb, 2000). Most students in the class were mathematics, science, or engineering majors, had finished Calculus III, and about one third of the students had taken at least one prior course with this particular mathematics professor. The professor had been using inquiry-oriented strategies in his other courses (e.g., Abstract Algebra, Mathematical Reasoning) for several years, but had only taught Differential Equations once about 7 years prior and was implementing an inquiry-oriented differential equations materials (Rasmussen, 2003) for the first time that semester. Prior to many teaching sessions, the professor met with one of the researchers to discuss the material to be taught and make a planned trajectory. They also met immediately after class for debriefing sessions to reflect on the lesson and discuss any issues or questions that arose that may affect the content and teaching strategies used for the next class.

The class was designed to be student centered and inquiry-oriented with each class session involving cycles of learning: whole class discussion, followed by small group discussion, followed by whole class discussion. The learning environment of the classroom established by the professor required students to discuss the mathematics they were learning, express their own ideas, and make sense of, and agree or disagree with others' ideas.

The data used for analysis for this paper was drawn from the videotaped class episodes, field notes from a non-participant observer, video/audio taped debriefing sessions held immediately after class and student work. To begin our analysis, we reviewed videotapes and field notes of class sessions throughout the semester. We identified episodes (short periods of classroom video) where it was noted that the class was discussing ideas about parametric equations, time as a parameter, graphing of parametric equations as curves or related ideas. Once these episodes where identified, we used a coding scheme that was both top-down (Miles and Huberman, 1994) and generative in nature (Strauss & Corbin, 1990). It was top-down in the sense that we used research in prior literature (Rasmussen, Marrongelle, & Kwon, 2008; Whitacre & Nickerson, 2009) to identify instances where the instructor was initiating a conversation, possibly using one of the typical discursive moves such as telling or questioning, interjecting something in a conversation using revoicing or using pedagogical content moves (Rasmussen & Marrongelle,2006;). The coding was generative in nature as we created and used codes of ways the instructor was drawing upon his own knowledge and what discourse the students and teacher participated in. After identifying these episodes, we used a comparison method to establish links in teacher moves and the discourse. One assumption we made in the analysis is that discourse is one lens on student thinking and that communication is thinking (Pea, 1993; Lampert & Cobb, 2003). After the links were established, we triangulated the analysis results with students' work we collected.

Results

The results of the analysis are not finished at this time; currently, we have linked at least two of the teacher moves to the discourse and student reasoning. By the conference, we plan to have more evidence to support these linkages and others.

First, the teacher focuses and uses student ideas and builds upon them in ways that allows the students in the classroom to understand. For example, if a student mentions in classroom conversation that they remember $x=f(t)$ and $y=g(t)$ when asked if they know about parametric equations, but cannot remember what they mean, then the teachers brings that idea to the front of the class (either himself or the student may speak, either might be appropriate). Because he knows that it is from a student, he then asks questions to either small groups or the whole class to elicit ideas. He then creates and asks a question (the teacher move) that engages students in thinking about this so they can reconstruct understandings and participate in the discourse around the concept.

Second, the teacher focuses on eliciting ideas from students that will allow them to build up their mathematical habits of mind. These habits of mind for this particular teacher involve developing an intuition to recognize when mathematical ideas are present in the current mathematical agenda that can connect on concepts from their earlier learning. We provide examples of this in detail and other results of the analysis.

Implications

By identifying one particular mathematical content strand that weaves through many areas of mathematics, this research is a good model for those interested in finding ways to strengthen student understandings across mathematics as a discipline. Additionally, offering ways that teachers can make explicit pedagogical moves in a university level classroom, whether it be student centered or more teacher centered, provides new ways to improve mathematics teaching at the undergraduate level. For example, if mathematics instructors think about specific ways they can order student answers that allows discourse and reasoning to move forward mathematically, this could be an important area for future research and professional development.

Additionally, another area for future research is pointed to by this report. Mathematicians are interested in how to assure that mathematics majors at the university have long lasting understandings that span the curriculum. If thinking deeply about one particular topic, and ways that teachers can support learning of that, is useful, then researchers may be able to use the technique in other mathematical conceptual areas.

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