Using Animations of Teaching to Probe the Didactical Contract in Community College Trigonometry Classes

Preliminary Report

Representations of teaching can be seen not only as cases of practice but also as probes on the rationality that practitioners use as they teach (Herbst & Chazan, 2003). Herbst and Chazan have developed a new kind of representation of teaching—animations of classroom scenarios, deliberately designed to probe some of the unspoken norms of classroom practice. Herbst and Miyakawa (2008) provided some details of how those animations are produced to be prototypes of models of instructional situations: Instructional situations are identified and modeled by hypothesizing the norms or tacit responsibilities of classroom participants in a situation, then scenarios are created that fulfill some of those norms but breach others; finally those scenarios are prototyped in a cartoon animation. Herbst, Nachlieli, and Chazan (in press) have shown how such animations can elicit data that informs about the rationality of teaching.

We describe how we applied those ideas in designing a research instrument that would be used to elicit community college teachers' practical rationality apropos of the knowledge management demands when solving problems on the board. We use the situation of 'finding vales of trigonometric functions' as context for this inquiry into the rationality that sustain larger contractual norms. The animations are meant to be representations of trigonometry teaching that occurs in a hypothetical community college that is similar to other large community colleges in the United States. Trigonometry is one of the mathematical domains conventionally taught in community colleges, either as a separate course or incorporated into other courses that are prerequisites to calculus (Lutzer, Rodi, Kirkman, & Maxwell, 2007). The course can be perceived as a skills- and knowledge-building course, in which the purpose is to ensure that students demonstrate competence in solving standard problems of trigonometry and familiarity with the definition and properties of the trigonometric functions. In the college where we collected the intact classroom data, the course has a guiding textbook and a master syllabus that outlines the knowledge for which students and instructors are held accountable.

The core question that we want to answer with the tool that we designed is: How much and what kinds of student participation do instructors perceive as feasible to handle when they work through examples at the board in a trigonometry class?

Identifying Key Norms of the Trigonometry Contract

The following describes our observations of the didactical contract in community college Trigonometry courses. The instructor is responsible for presenting the material and solving examples on the board. Students are responsible for doing homework, showing up for class, asking questions whenever they do not understand something, taking tests, and participating in class as demanded by the instructor. Students work under the assumption that their teachers are there to help them gain competence with the material and in general expect that their teachers press them for doing challenging work and believe that they are capable of doing what it takes to be successful (Mesa, 2010). The instructors are aware of the multiple demands that their students have on their time due to work and family responsibilities and have learned to not take it personally when students stop coming to their class (Grubb & Associates, 1999; Seidman, 1985). Instructors are also aware of the "holes" that students have in their mathematical preparation that hinder their opportunities to learn the content. They are also conscious that they have limited amount of time to ensure students' development of competence with the material.

When examining intact lessons' excerpts in which exemplification occurs, we have noticed the following:

- Instructors rarely ask questions regarding the plausibility or correctness of a response or a final solution to a problem;
- Instructors engage the students by asking questions about how to apply known procedures but they rarely, if ever, ask them to decide what procedure to apply;
- Instructors offer as examples problems that admit only one solution.

We hypothesize that these observations respond to contractual norms, that is, to tacit rules of the didactical contract. The instructional situation that has been chosen as context to explore the normative nature of those observations deals with solving the following problem:

> Using Fundamental Identities, find the exact values of the remaining trig functions given
>
> $\sin x = -\frac{4}{5} \quad \cot x = \frac{-3}{4}$

The original transcript of the class where the solution of this problem takes place illustrates what we believe are norms regarding exchange, division of labor, and organization of time. Exchange norms refer to what needs to be done, what it counts as, and what is not done; division of labor norms indicate who has the responsibility to do what, and norms of organization of time establish when things need to be done and how long they take. This particular problem calls for making a decision regarding the quadrant where the angle would be located, which permits the determination of the appropriate sign for the value of cosine of the angle, which is used to derive the value for the secant of the angle. As the solution unfolds, asking for justification of the steps or whether the answers make sense is not done; it appears that there is no explicitly assigned responsibility for justifying steps in the process and that the instructor alone determines how the solution unfolds; we also believe that the swiftness with which the problem is solved is related to the need of conveying the idea that problems are easy and that the homework won't take long. To test whether these are reasonable hypothesis, we created alternative scenarios in which some of the hypothesized norms are breached and we seek input from instructors regarding those breaches.

Consider the following scenario:

Teacher: So we know sine and cotangent, **what do you think we should do now?**

Male1: We can draw the unit circle and put these ratios in…

M2: We could draw the graphs of sin and cot and see what *x* gives us those values…

Female1: Nah, I think it is simpler than that. We could use that thing about the quadrants and the signs of the functions…

F2: We could use a circle with radius 5 and, then sin -5 over 4 is saying that the opposite is -4… so the angle must be somewhere here [on Quadrants 3 or 4], then the adjacent is….

M1: The point must be (3, -4) because of the cotangent; that's quadrant four.

M2: **We could flip sine *x* to get -5/4 for cosecant**

F2: **and tangent would be flipping cotangent…**

M2: **so cosine is… is three over five.**

F1: **and cosecant is just flipping that one. We're done, we got them all.**

T: **that's OK, but,** …

This scenario is meant to address the issue of control over the solution process, with students answering the problem using 'old' rather than the current material ('fundamental [trig] identities'). We anticipate that teachers won't see it feasible to relinquish control for two reasons. First, teachers perceive students as expecting the instructor to be in control, showing how things are done, and with the responsibility of explaining the content; in principle students are perceived as capable of negative reactions to what other students have to say, because they do not see their peers as having authority of knowledge to do that (Cox, 2009). Second, there is too much material to cover and a very efficient way to handle it in reasonable time is for the instructor to illustrate the process so students can mimic it later (Grubb & Associates, 1999). In this scenario, the students have 'solved' the problem but it is of less value or import, because it does not use the content of the unit. The instructor will need to validate the solution given by the students or to reject it as inadequate for the expected use of the new content. Thus, if the teacher gives control of the solution to the students he or she risks loosing control of the exchange value of the problem/solution. In either case, we hypothesize, the instructors would make sure that in addition to the proposed solution, the students would also see how the new content is used.

With scenarios such as these we expect to be able to uncover the resources instructors have at their disposal for making decisions regarding how to manage similar situations. They would either align with or distance from the teacher in the scenario and in that process they would make explicit what they do that the animated teacher does not. The information that we gather in this way, will allow us to map out community college instructors' rationality in teaching trigonometry with examples, as we test these animations with groups of faculty.

During our presentation in the conference we want to share a preview of the animation illustrated above and get input from the audience regarding its use as a research tool; if available—the animations are being produced now—we will share preliminary data illustrating teacher's reactions to the animations, and how the analysis allows us to formulate more specific conjectures regarding the reasons for the level of student participation that instructors perceive as feasible to handle when they work through examples at the board in a trigonometry class.

Answering this question is fundamental to understand the extent to which calls for reform of undergraduate college math classes (Blair, 2006), in which students play a more significant role in the construction of knowledge, can be effectively carried out at the community college level.

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