Mathematicians' Pedagogical Thoughts and Practices in Proof Presentation

Preliminary Research Report

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Abstract:

Little is known about how mathematicians present proofs in undergraduate courses. This descriptive study uses ethnographic methods to explore proof presentations at a large comprehensive research university in the Midwest. We will investigate three research questions: What pedagogical moves do mathematics faculty members make when presenting proofs in a traditional undergraduate classroom? What do mathematics faculty members contemplate as they plan lectures that include proof presentations? To what degree and in what ways do faculty members engage students when presenting proofs? To pursue these questions, four faculty members who were teaching proof-based mathematics courses were interviewed and 6-7 observations of each classroom were conducted throughout the course of the semester. The data were analyzed to identify some of the pedagogical content tools that were used, to develop an observation instrument, and to understand how mathematicians think about the pedagogy of proof presentation.

Keywords: proof presentation, pedagogical content tools, teaching proof, ethnographic methods

Literature Review:

It has been well documented that students struggle with mathematical proof (Grassl & Mingus, 2007; Larsen, 2009; Larsen & Zandieh, 2008; Selden & Selden, 2003). The transition from computational mathematics to formal mathematics is a dramatic shift (Tall, 1997). Undergraduate level proof-based mathematics courses have been studied by mathematics educators for the past few decades. This research mostly comes in two flavors: investigating student thinking (Knuth, 2002; Larsen, 2009; Simpson & Stehlikova, 2006; Healey & Hoyles, 2000; Almeida, 2000; Selden & Selden, 2003) and developmental research projects (Gravemeijer, 1994) that focus on developing innovative ways to teach proof (Leron and Dubinski,1992; Larsen, 2009; Weber, 2006). These studies shed light on teaching and learning in the context of mathematical proof, but it is often difficult to translate these findings into widespread changes in teaching.

It is generally acknowledged that lecture is the norm in most university classrooms. The lecture style has been criticized by many, especially by those who propose alternative, more interactive teaching methods (Leron & Dubinski, 1995; Leron, 1985; Larsen, 2009). Leron (1985) called for a divergence from a linear proof presentation method in favor of "heuristic" presentations, which give the audience a better idea of how the ideas were constructed. The

"pure telling" lecture-style format has generally been contrasted with inquiry-oriented teaching (Rasmussen & Marrongelle, 2006), but personal experience tells us that many instructors are somewhere in between those two extremes. Little is known about how variations within the lecture style of proof presentation affect student understanding.

There are very few research projects directed at what is currently going on in a traditional university classroom. In the area of geoscience education (Markley, Miller, Kneeshaw & Herbert, 2009), a study was done to study the relationship between instructors' conceptions and practice in the classroom. There were interviews with the faculty members about their perceptions of teaching and learning, and then there were observations of their classrooms. The observation data focused on how the instructor interacted with students and whether or not the classrooms were student centered. In mathematics education, a recent study addressed the issue of proof presentation by interviewing nine mathematics faculty members to explore their pedagogical decisions concerning proof presentations (Weber, 2010). Fukawa-Connelly (2010) observed a mathematics faculty member over the course of a semester in a traditionally taught abstract algebra course. He analyzed classroom dialogue through the lens of pedagogical content tools, looking for instances in which the faculty member 'modeled mathematical behaviors.' This study gives an existence proof that university mathematics professors do not always use a "pure telling" method of proof presentation.

While some studies are beginning to address proof presentation, much more work needs to be done. Most of these proof-based courses are taught by working mathematicians, who are likely unfamiliar with current mathematics education research. Though an instructor identifies himself as traditional, he may still make efforts to involve and engage students in proof construction, but may not be familiar enough with the language of mathematics education to describe his pedagogical moves. This study will combine faculty interviews with classroom observations to explore not only how mathematics faculty members think about presenting proof, but also what they do in practice.

This study has several goals; one is to investigate how the faculty members' pedagogical ideas about proof presentations manifest themselves in the classroom. Another aim is to analyze the nuances of traditional teaching methods in regard to proof presentation, and to identify some of the tools that mathematics faculty members currently use to help students understand proof and write their own proofs. A final goal is to develop an observation instrument to simplify data collection and analysis. The video data will be useful both to develop an observation instrument and to minimize validity concerns, since the instrument is in the developmental stages.

Research Questions:

What pedagogical moves do mathematics faculty members make when presenting proofs in a traditional undergraduate classroom? What do mathematics faculty members contemplate as

they plan lectures that include proof presentations? To what degree and in what ways do faculty members engage students when presenting proofs?

Methodology:

Since the teaching and learning of mathematics can be viewed as an enculturation process, we will view the data through an interpretivist lens, which "looks for culturally derived and historically situated interpretations of the social real world" (Crotty, 1998, p. 72). The instructor is viewed as an expert in the discourse on mathematical proof, trying to help the "newcomers" enter into the discourse community (Sfard, 2008). This discourse can be analyzed through symbolic interactionism, because the language and other communicative tools that the professor is using to help the students understand will be studied (Crotty, 1998). Since the classroom is studied as a culture, pragmatism will be our theoretical perspective (Morgan, 2007), which is generally associated with the ethnographic methods that will be used.

The first phase will be semi-structured interviews with four faculty members at a large comprehensive research university in the Midwest. These faculty members are currently teaching undergraduate level math courses that emphasize mathematical proof. The interviews will address what the instructors do when they present proofs in class, why they make those choices, and what they do to help students understand their presentation of proofs in class. The interview data will be analyzed for emergent themes.

Throughout the semester, 6-7 observations of each classroom will be conducted and analyzed in detail. Three of the participants agreed to allow the observations to be video-taped, and for the fourth, we will analyze field notes collected with an observation instrument. Though much of the data analysis will be qualitative, some of the qualitative observation data can be quantified (Chi, 2007) to more easily see the trends that occur. The researcher is developing an observation instrument to collect data about proof presentations. The first draft of the instrument was based solely on the researcher's experience as an observer and as a student in proof based mathematics courses. The themes from the interviews will be used to modify the observation instrument, and as the observations occur throughout the semester, the observation instrument will evolve. Because the instrument is not in its final form, video data is crucial, because the researcher may need to go back to look at earlier observations.

Before the final data analysis, there will be an additional interview with the faculty members for a member check. At this time, the participants will be able to see the themes and trends that have emerged, and they will have the opportunity to give an insider's perspective into the data. Since the researchers are constructing their own knowledge about how proofs are presented in class (VonGlassersfeld, 1996), the input of the participants will be a valuable resource for data analysis.

Applications to Further Research:

As we work to describe how faculty members present proofs in class and what they think about the pedagogy of proof presentation, we hope to be able to identify more pedagogical content tools (Rasmussen & Marrongelle, 2006) that they use to train students in reading and writing proof, and to help students enter into the culture of mathematical proof. Once we are able to identify some of these tools, we hope to be able to design some studies that can investigate their value. The recent work of Mejia-Ramos, Weber, Fuller, Samkoff, Search, & Rhoads, (2010), has designed a model for proof comprehension with six different dimensions that can be assessed by a quiz. Future research will combine their method of assessment with the results of this study to evaluate the efficacy of different methods of proof presentation in a traditionally taught proof-based course.

Questions:

Do you have any suggestions about how to analyze the data from the classroom where I was not video-taping? Should that data be thrown out entirely?

Are there suggestions for the observation instrument? Have any of you used an observation instrument in the past?

We plan to design a study to evaluate the pedagogical content tools we have identified. Any suggestions about study design?

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