Eric A. Pandiscio
University of Maine
eric.pandiscio@umit.maine.edu

Geometric Constructions to Activate Inductive and Deductive Thinking Among Secondary Teachers

Preliminary Research Report

For this preliminary research report, I have two goals: a) to present initial findings from the pilot study, and b) to use feedback from the session to design a more robust follow-up study.

The following research question formed the basis of the pilot study:

> To what extent can students improve their abilities in geometric reasoning and proof through learning experiences that combine dynamic geometry software and traditional compass and straightedge constructions?

Students were provided an inquiry-oriented, construction-based experience dealing with Euclidean geometry topics. Researchers hoped to demonstrate that such an experience can gain increase students' ability to write deductive proofs. A learning environment was created that involved extensive work with constructions using traditional compass and straightedge techniques as well as with dynamic geometry software. A major piece of the work was a rigorous program of "deconstructions" whereby participants gave written and oral validations of each construction. A pre-test/post test consisting of formal, written proofs served as one assessment instrument.

Building on the work of Hollebrands (2003; 2004) and Galindo (1998), written response analysis was used to determine students' understandings and attainment of reasoning abilities related to transformational geometry. I propose in the follow-up study to use a mixed-method approach, adding task-based interviews.
A case study methodology, as described by Gall, Borg, and Gall (1996) and Lincoln and Guba (1985) will be particularly helpful in producing a detailed description of a phenomenon. In this instance, the specific phenomenon under study is geometric reasoning and proof, situated within the context of a technological learning environment. Since part of the goal of the research is to describe the interaction of traditional and modern construction techniques as they related to concept development, case study is particularly useful (Yin, 1994).

In addition, a quantitative analysis will be utilized. The construct of "normalized gain," initially described by Hake (1998) is appropriate in this setting. Widely duplicated (e.g. Fagen, Crouch & Mazure, 2002), normalized gain has been shown to provide meaningful data on changes in student achievement. A small number of students in the pilot completed a pre-test/posttest inventory (Usiskin, 1982) designed to assess how students' levels of geometric reasoning is affected by particular, targeted instruction. This inventory is supported by a large reliability and validity database. Preliminary results will be reported, although the overarching goal is to administer such an instrument to a larger number of participants in the future.

This proposal is for a work in progress. Data from the pilot study have been collected, but not yet fully analyzed. I anticipate analyzing these data shortly. A follow-up study is planned for late spring or summer of 2011.

The need for studying geometric learning is great. Clements (2003) points out in compelling detail the generally poor performance of students in the field of geometry. Other researchers have found limited success in improving particular aspects of student work. Pandiscio (2002) found that participants can fundamentally alter their view of proof as it relates to inductive reasoning through targeted instruction. Hiebert (2003) describes how students "learn what they have the opportunity to learn," citing the success of focused outcomes aligned with particular instructional settings. Other studies have determined that proportional and conceptual reasoning can be enhance through curricular modifications (Ben-Chaim, et. al., 1998). Battista and Clements (1995) posit that alternatives to axiomatic approaches may lead to greater success in students' proof and reasoning abilities, and cite Geometer's Sketchpad as viable platform. With the development of GeoGebra (Hohenwarter, 2002) we now have a freely available, open source software that combines many features of dynamic geometry software and computer algebra systems into a single package. Hohenwarter and Jones (2007) have posited great potential for helping students to visualize mathematics through GeoGebra. Based on this, a rationale exists to examine the stated research questions.

Preliminary questions for the audience:
1. Does the essential research question have enough merit to justify a follow-up study?
2. Does anyone know of empirical evidence to support the intuitive suggestion that by deliberately strengthening inductive reasoning, students will increase their proficiency at formal geometric deduction?
3. How do I address the concern that it is unlikely to have more than 10-12 students participate in a follow-up study that requires a substantial commitment of time on the part of the students?
4. Are there well-designed instruments to measure student's ability at writing formal geometric proofs?
5. How likely is it that task-based interviews will reveal student reasoning about geometric ideas

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