**Title: The Internal Disciplinarian: Who is in Control?**
Preliminary Research Report.

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**Abstract**
A group of mathematicians and mathematics educators are collaborating in the fine-grained examination of selected 'slices' of video recordings of lectures drawing on Schoenfeld's KOG framework of teaching-in-context. We seek to examine ways in which this model can be extended to examine university lecturing. In the process we have identified a number of lecturer behaviours There are times when, in what appears to be an internal dialogue, lecturing decisions are driven by the mathematician within the lecturer despite the pre-stated intentions of the lecturer to be a teacher.

**Introduction**
In contrast with the manner in which a school teacher's Knowledge, Orientation and Goals (KOGs) determine their decision making, we present evidence that for mathematicians this decision making is additionally complicated by an inner argument between the lecturer-as-mathematician and the lecturer-as-teacher. Are there conflicting orientations and goals active in the decision moment? The way in which the decisions play out is thus a function not only of the lecturer's knowledge of mathematics but of the way they work mathematically.

**Research base**
This paper reports on an aspect of a project that explores how Schoenfeld's KOGs may be used to direct lecturers' attention to aspects of their decision-making in the lecture theatre as a professional development activity. The project is informed by research concerning how a teacher's knowledge, orientations or beliefs and goals impact on their teaching practice (Schoenfeld, 2007; Ball, Bass & Hill, 2004; Shulman, 1986; Speer, Smith & Horvath, 2010; Torner, Tolka, Rosken & Sriraman, 2010) However, these are studies of teacher practice in primary and secondary schools and similar work at the college level is 'virtually non-existent' (Speer et al, 2010, p 99). The project is also designed to build on the effectiveness of communities of practice (Lave & Wenger, 1999) and a culture of enquiring conversation (Rowland, 2000) for professional development. The project is described in more detail in Barton, Oates, Paterson and Thomas (to be published).

**Structure of project and data collection**
A group of four mathematicians and four mathematics educators are collaborating in the fine-grained examination and discussion of lecturer actions in video recordings of lectures (Kazemi, Franke, Lampert, 2009; Prushiek, McCarty, & Mcintyre, 2001). The theoretical approach draws on Schoenfeld's theory of teaching-in-context (Schoenfeld, 2002). The data for each lecture consists of videotape, an observer record, and a written lecturer-KOG (a statement by the lecturer of the knowledge used, orientation held, and goals, both specific goals intended for the lecture and more

general educational goals). A section of the lecture is chosen for discussion by the lecturer and is transcribed along with all discussion of the lecture.

The project aims to examine ways in which Schoenfeld's model can be used and extended to examine university lecturing and to support the professional development of lecturers (Van Ort, Woodtli, Hazard, 1991)) In the process we have identified a number of lecturer behaviours one of which is discussed in this paper.

**Observations and Discussion**

Schoenfeld (2007) argues that

> Teaching, …… depends on a large skill and knowledge base … its practice involves a significant amount of routine activity punctuated by occasional and at times unplanned but critically important decision making – decision making that can determine the success or failure of the effort. (p 33)

We have observed a number of instances of what appears to be an inner argument, or regulation by an inner voice, in the lecturer's communication with the class. In subsequent group discussion it has become clear that many lecturers are aware of this.

The example below is from a lecture to a general education first year course in which the students are introduced to the Fibonacci sequence and the golden ratio. In her personal KOG, written before the lecture, the lecturer stated:

> Knowledge I need includes: knowledge of the subject material, knowledge of the levels of the students.
>
> Orientation: I see this whole course partially as an exercise in 'public understanding of mathematics', and so try to treat the lectures as such – rarely going into much depth mathematically, and trying to keep everyone engaged and interested.
>
> Goals: for the students to appreciate the appearance of Fibonacci numbers in nature. To keep all the students engaged throughout the lecture.

In the lecture, once they have found the sequence of numbers, a recursive formula for the sequence, and arranged them in a table she says to the students:

> ML1: Then compute the ratio as you go down for each one. So for instance I have got 1divided by 1 is 1, 2 divided by 1 is 2 and this next one will be 3 divided by 2 which is 1.5. If you have a calculator you can calculate what they are otherwise you can leave them as fractions and I'll write down what they are in decimal notation."

After they have worked them out she continues:

> ML1: What's 5 divided by 3?
> Student: 1.6
> ML1: 1.6 recurring so I'll put 1.667
> (*looks at it a brief moment)* **dot dot dot**"

As a group we examined what caused her to pause and decided it appeared that she felt that the fact 5/3 was a recurring decimal had to be acknowledged. It led to the following exchange:

> ML2: I do this kind of thing all the time, I think it's really distracting because you've gone

out and tried to make your big point and then you get all flustered over some detail and you say oh sorry you know, you have to get it right and the students go "what the hell is going on and now I'm completely confused because it sounded really simple."
ML1: So should you just ignore that corner and just hope that it's not noticed but then is that bad because you've somehow told them something incorrect?

We saw another instance of the lecturer's need for rigor later in the lecture. Note that nowhere in her written KOG does she mention rigor, on the contrary she says, "rarely going into much depth mathematically".

She is proving that the value for $f_{n+1}$ divided by $f_n$ is the golden ratio, $\psi$. There is some literal hand-waving as it is established that the values oscillate about 1.6 something and then she says

OK suppose you want to compute what this number actually is
And it seems to be converging – **and it does actually converge** (*who is she reassuring*?)
So you know that $f_{n+1}$ is bigger than $f_n$ so this is going to be a number that is bigger than 1
Right? (*sounds as if she hears herself and adds this*) **Or equal to 1**.
So ..If I am thinking about what this ratio becomes as n gets really, really big
**So, for any specific n these 2 things are going to be different**
Right?
Because for one thing it was 1.6 and for the next one it was 1.625
So for any specific n it's going to be different

In this interlude we see and hear her spending a lot of time emphasising that for particular values of *n* the values of $f_n$ and $f_{n+1}$ are different and under what circumstances they are justified in making the approximation:

But as *n* gets bigger and bigger and bigger these 2 things are going to get closer and closer together
As long as *n* is big enough
So we will assume that we are in a place where *n* is big enough then **we can make this approximation**

The highlighted language in this excerpt shows her need to be mathematically explicit; hand waving will not do even in a class that is 'an exercise in public understanding of mathematics.'

Further examples seem to indicate that the manner in which the inner argument manifests appears to differ depending on the research field of the mathematician. A pure mathematician in the group spent a long time disentangling notation to ensure that a proof held together effectively even while he had stated that he believed the students were capable of deriving it for themselves. When discussing these actions he spoke of "KOG dissonance" to refer to his actions in contradiction of his stated intentions. An applied mathematician had a similarly mathematician-inspired interaction with a Matlab generated display that did not show what he knew it should show about the number of bifurcations in a logistic equations.

## Conclusion

It is not our argument that schoolteachers do not have an inner mathematical voice but we contend that in their case their motivation is the elucidation of the content so that the students can understand it better. In the case of the research mathematicians in a university environment we argue that they are concerned that the mathematics be appropriately presented, or at least not be misrepresented, because their relationship to it and the way in which they work with it demands this. There is evidence that there are times when, in what appears to be an internal dialogue, decisions are driven by either the mathematician, or the teacher, within the lecturer persona. We suggest that these two personae may have different orientations and goals, affecting the decisions reached at critical points and consequently influencing student learning. This teacher-mathematician interplay might prove to be a productive construct to work with in the professional development of lecturers.

**Questions:**

Q1 As a mathematics lecturer are you aware of this tension? Have you caught yourself listening to an inner voice in the middle of a lecture?

Q2 Do you think it would be useful for your lecturing to consider this phenomenon explicitly as part of your professional development?

Q3 Do you have alternative suggestions for aspects of your lecturing to focus upon in professional development sessions?

**References:**


Ball, D., Bass, H., & Hill, H. (2004). *Knowing and using mathematical knowledge in teaching: Learning what matters.* Paper presented at the South African Association for Research in Mathematics, Science and Technology Education. Durban.

Barton, B., Oates, G., Paterson, J. &Thomas, M. (accepted for publication) The Datum Project. Community for Undergraduate Learning in Mathematics Newsletter, Auckland, New Zealand

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge: Cambridge University Press.

Kazemi, E., Franke, M. & Lampert, M. (2009) *Developing Pedagogies in Teacher Education to Support Novice Teachers' Ability to Enact Ambitious Instruction* In Roberta Hunter, Brenda Bicknell, Tim Burgess (Eds), Crossing Divides, Proceedings of the 32nd Annual Conference of The Mathematics Education Research Group of Australasia, Vol 1, pp. 11-29. Palmerston North, New Zealand: Mathematics Education Research Group of Australasia

Prushiek, J., McCarty, B., & Mcintyre, S. (2001). *Transforming professional development for preservice, inservice and university teachers through a collaborative capstone experience.* Education, 121 (4), 704-712.

Rowland, S. (2000) *The Enquiring University Teacher.* Open University Press, Philadelphia, PA

Schoenfeld, A. H. (1998). Toward a theory of teaching-in-context. *Issues in Education, 4*, 1-94.

Schoenfeld, A. H. (2002). A highly interactive discourse structure. In J. Brophy (Ed.), *Social Constructivist Teaching: Its Affordances and Constraints* (Volume 9 of the series Advances in Research on Teaching, pp. 131–169). Amsterdam: JAI Press.

Schoenfeld, A. (2007) *A theory of teaching and its applications. In Commemoration of Guenter Toerner's 60th Birthday* The Montana Mathematics Enthusiast**,** ISSN 1551-3440, Monograph 3, The Montana Council of Teachers of Mathematics, pp.33-38

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Speer, N.M., Smith, J.P & Horvath, A. (2010) *Collegiate mathematics teaching: An unexamined practice.* Journal of Mathematical Behavior , 29. pp 99–114

Torner, G., Rolka, K., Rosken, B. & Sriraman, B. (2010) Understanding a Teacher's Actions in the Classroom b Applying Schoenfeld's Theory Teaching-in-Context: Reflecting on Goals and Beliefs. In B. Sriraman and L. English (Eds.) *Theories of Mathematics Education. Seeking the New Frontiers* (pp. 401-420) Heidelberg/Dordrecht/London/New York: Springer

Van Ort S, Woodtli A, Hazard ME. (1991) *Microteaching: developing tomorrow's teachers.* Nurse Education, *(16)*