Mathematical Knowledge for Teaching: Exemplary High School Teachers' Views

1. Introduction and Research Questions

Mathematical knowledge for teaching (MKT) has been a central topic of recent research in mathematics education (e.g., Ball, Thames, & Phelps, 2008; Ma, 1999). However, most of this research has focused on elementary or middle school mathematics. Few researchers have investigated the specific content knowledge needed to teach high school mathematics. Although most high school math teachers complete an undergraduate major in mathematics, some researchers have argued that high school teachers should receive content preparation specific to teaching (e.g., Moriera & David, 2008). In order to determine exactly what this content preparation should be, more research is needed on high school mathematics teacher knowledge.

Ball, et al. (2008) suggested building a theory of teacher knowledge by beginning with classroom episodes and observations of effective teaching in order to analyze the knowledge teachers need for such endeavors. This study also begins with effective teaching to build theory, but rather than observing teachers, it seeks the perspectives of exemplary high school mathematics teachers. These teachers' voices can provide critical knowledge associated with practice that researchers may not recognize or understand (Cochran-Smith & Donnell, 2006).

In this proposal, we present an exploratory study on exemplary high school teachers' views on the subject matter components of MKT. *Subject matter components* are aspects of mathematical knowledge that are not necessarily pedagogical (Ball, et al., 2008). The following research questions guided this study: (a) What subject matter components of MKT do exemplary high school teachers believe are important in their practice? (b) When and how do these teachers believe that their MKT developed?

2. Related Literature and Theoretical Framework

This study draws on the model of Mathematical Knowledge for Teaching proposed by Ball, Bass, and colleagues (e.g., Ball et al., 2008). In this model, MKT is comprised of subject matter knowledge and pedagogical content knowledge. Both types of knowledge are specific to mathematics content, but pedagogical content knowledge is knowledge of mathematics pedagogy, and subject matter knowledge is knowledge of content that is not necessarily pedagogical. The purpose of this study is to explore aspects of the latter.

Research focusing specifically on the subject matter components of MKT has explored how teachers understand particular concepts. Even (1990) synthesized research on teacher knowledge and conjectured that there are six components of a teacher's understanding of a particular concept. These are knowing, with regards to the concept, (a) essential features, (b) different representations, (c) multiple perspectives and applications, (d) unique characteristics, (e) a basic repertiore of examples, and (f) a conceptual understanding. Even added that teachers must also have knowledge of mathematics as a discipline. Other similar frameworks have been developed in the content strand of geometry (Chinnapan & Lawson, 2005) and for both mathematics and science teaching (Kennedy, 1998). The goal of this study is to further explore the elements of teachers' subject matter knowledge by talking to exemplary teachers rather than from systematically reviewing the research literature.

Other researchers have been interested in secondary mathematics teachers' perspectives on the use of advanced mathematical knowledge (i.e., undergraduate-level matheamtics) in their teaching. Zazkis and Leikin (2010) surveyed 52 secondary matheamtics teachers and found that,

even for teachers who claimed to use their advanced mathematical knowledge often, they could rarely cite a specific example of the use of this knowledge. Recognizing the fact that teachers may have difficulty describing their subject matter knowledge and its use, this study takes into account both teachers' explicit statements of their subject matter knowledge as well as elements of their subject matter knowledge that are revealed through analysis of written lesson plans.

3. Methodology

3.1. Participants. Eleven high school mathematics teachers from one state participated in the study. These teachers received at least one of three prestigious honors in the state: Between 2000 and 2010, these teachers (a) were state or national finalists for the Presidential Award for Mathematics and Science Teaching (NSF, 2009), (b) were named County Teacher of the Year in their county, or (c) were National Board Certified Teachers in Adolescent and Early Adulthood Mathematics (NBPTS, 2010). The 27 teachers in the state who met these criteria were invited to participate, and 11 teachers accepted the invitation. Of these 11 teachers, four received the Presidential Award for Mathematics and Science Teaching, three were named County Teacher of the Year, and seven were National Board Certified. (Some teachers met more than one criterion.)

Eight of the participants taught at public schools and three taught at private schools or vocational schools. The eight teachers at public schools were well distributed among a range of schools in terms of socioeconomic status and student success rates. Similar statistics for the private schools were unavailable.

3.2. Data collection. Two sources of data were obtained for this study: (a) lesson plans and (b) interviews. Researchers have argued that MKT may be tacit (e.g., Zazkis & Leikin, 2010). Hence, lesson plans were used as stimuli during interviews in order to help teachers recall and discuss aspects of their content knowledge (Meade & McMeniman, 1992). Each participant was asked to submit one lesson plan from a traditional high school course (i.e., not college-level courses such as AP Statistics or AP Calculus). Lesson plans were obtained approximately one week before the interviews in order to tailor interview questions to the lesson where appropriate.

The main data source was individual interviews with participants. Interviews were semi-structured and lasted approximately one hour. Participants were asked about their background in mathematics education, the specialized content knowledge that went into the lesson that they shared, and general aspects of their mathematical knowledge as it related to their practice.

3.3. Data analysis. All interviews were audiotaped and fully transcribed for analysis. A grounded theory approach to analysis was used in the style of Strauss and Corbin (1990). After listening to the interviews and reading through the transcripts, initial codes were assigned to episodes that pointed to teachers' specialized content knowledge. Like codes were organized to form categories using the constant-comparative method (Strauss & Corbin, 1990). Each transcript was then revisited individually. Categories were refined and new codes and categories were formed when appropriate.

Next, lesson plans were revisited. Elements of the lesson plan which pointed to MKT were coded according to the categories developed from interview analysis. In most cases, the analysis of lesson plans supported interview data. In cases where the lesson plans provided disconfirming evidence, codes and categories were revised to accommodate the data from the lesson plans or led to proposed explanations for why the disconfirming evidence existed (Creswell, 2007).

4. Results

4.1. Essential aspects of subject matter knowledge. The results of our preliminary analysis include five aspects of subject matter knowledge that the exemplary teachers in this study found to be essential.

First, teachers believed that connections between mathematical ideas were important for teaching. Teachers claimed to be able to help students see basic mathematical ideas within a complicated mathematical concept and connect different mathematical concepts to help students understand mathematics more completely. In addition, they were able to connect the topics they were teaching to higher-level courses, such as calculus or non-Euclidean geometry.

Second, teachers believed that knowing the key examples of a concept was an important piece of MKT. When presenting mathematical concepts to their students, these teachers considered all cases of a concept or challenged their students to consider for which cases the concept would hold. In addition, the teachers had a flexible knowledge of cases of mathematical concepts so that they could create interesting and intriguing examples when necessary.

Third, understanding where mathematical concepts could be applied was an important piece of MKT for these teachers. They were knowledgeable of applications of the concepts they were teaching that were relevant to everyday life (and hence the students they were teaching). Fourth, teachers were aware of many techniques for problem-solving that were sometimes unusual or unique. Fifth, teachers recognized several representations of a concept and understood the ways in which a concept could be interpreted through these representations.

4.2. Development of MKT. Teachers also discussed ways in which they believed their MKT developed. These were through (a) formal courses, (b) professional experiences, and (c) personal experiences. Although some teachers spoke of individual courses as being influential to their thinking, most teachers did not cite formal coursework as a main source of MKT. Teachers overwhelmingly felt that their experience teaching a variety of courses and working with a variety of students helped them to develop MKT. In addition, several teachers mentioned that they individually sought to improve their practice through reading, conducting research, or applying for National Board Certification, and these activities helped develop MKT.

5. Significance

The teachers in this study appeared to understand the mathematics that they were teaching in a deep way. Many of the elements of their subject matter knowledge aligned with Even's (1990) framework for understanding of a concept, but the teachers' emphasis on connections between mathematical topics is important to note. Teachers overwhelmingly indicated that their MKT was developed through practice, not formal coursework. Hence, these findings can inform design of undergraduate mathematics courses for teachers. An important open question is whether courses that aim to develop these elements of subject matter knowledge are more productive for future teachers than traditional undergraduate mathematics courses.

6. Questions for Discussion

What aspects of MKT might be tacit? What research methods might help in exposing this knowledge? Which aspects of MKT (if any) might be specific to high school teaching? What might mathematics courses for teachers look like if they were to help teachers gain a depth of understanding? What specific parts of undergraduate-level mathematics are relevant to teachers?

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