The van Hiele Theory Through the Discursive Lens:
Prospective Teachers' Geometric Discourses

Sasha Wang
Michigan State University

Abstract
This project investigates changes in prospective elementary and middle school teachers' van Hieles levels, and in their geometric discourses, on classifying, defining and constructing proofs with geometric figures, resulting from their participation in a university geometry course. The project uses the van Hiele Geometry Test from the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) project, in a pretest and posttest, to predict prospective teachers' van Hiele levels (Usiskin, 1982), and also uses Sfard's (2008) framework to analyze these same prospective teachers' geometric discourses based on in-depth individual interviews. Additionally, the project produces a translation of van Hiele levels into a detailed model that describes students' levels of geometric thinking in discursive terms. The discussion will focus on studying college students' reasoning and methods of proof regarding geometric figures in Euclidean geometry.

Keywords: prospective teachers, Euclidean geometry, mathematical discourse, the van Hiele Theory

Over the past decade, there has been an increasing push in the mathematics education research community to study students' reasoning and understanding in the teaching and learning of mathematics, and to examine issues emphasizing the use of vocabulary and terminology in the mathematics classroom. In response, this project investigates the changes in prospective teachers' levels of geometric thinking, and the development of their geometric discourses, in the classification of quadrilaterals.

Theoretical Framework
In Sfard's (2008) *Thinking as Communicating: Human Development, the Growth of Discourses, and Mathematizing,* she introduces her commognitive framework, a systematic approach to analyzing the discursive features of mathematical thinking, including word use, visual mediators, routines, and endorsed narratives. To examine thinking about geometry, this project connects Sfard's analytic framework to another, namely the van Hiele theory (see van Hiele, 1959/1985). The van Hiele theory describes the development of students' five levels of thinking in geometry. The levels 1 to 5 are described as visual, descriptive, theoretical, formal logic and rigor. In addition, this project produces, on the basis of theoretical understandings and of empirical data, a detailed model, namely, *the Development of Geometric Discourse*. This model translates the five van Hiele levels into five discursive stages of geometric discourses with respect to word use, visual mediators, routines, and endorsed narratives at each van Hiele level.

Three overarching questions guide the project: (1) How do prospective teachers' familiarities with basic geometric shapes, abilities to formulate conjectures, and abilities to derive geometry propositions from other geometry propositions change as a result of their participation in a university geometry course? (2) What are the changes in prospective teachers' geometric thinking with regard to the van Hiele Levels? (3) How do the findings help to revise the proposed model of geometric discourse development?

Method

Guided by these research questions, the process of inquiry includes the data collection and analysis of a pretest and posttest, each of which is followed by the collection and analysis of interview data. Seventy-four college students who enrolled in a college mathematics content course for elementary and middle school teachers participated in the pretest and posttest. Twenty-one of these 74 students participated in the interviews. Data for this project comes from three resources: (1) Written responses to the van Hiele Geometry Test (see Usiskin, 1982) (from pretest and posttest), (2) Transcripts (from two in-depth interviews, the first interview conducted right after pretest, and the second right after posttest), (3) Other written artifacts (students' written statements, and answer sheets to the tasks during the interviews).

Data collection takes place in four phases: (1) the pretest is administered to all students during class time in the first week of the semester, (2) Student volunteers are interviewed a week after they participae in the pretest, (3) All students participate in the posttest at the end of the semester, (4) Students who participated in the interviews at the beginning of the semester are interviewed once more. All tests are collected and analyzed. All interviews are video and audio recorded. All interview data are transcribed and analyzed.

Results

Preliminary results suggest that most students in the project have moved one or two van Hiele levels, and the majority of the students' levels of geometry thinking are at van Hiele levels 2 or 3 after their participation in a college geometry course. However, when comparing a student's written response in the van Hiele Geometry Test with his/her interview response, it appears that the van Hiele level of a student determined by the written test is not always coherent with expected geometric discourse at the given level described in the model of the *Development of Geometric Discourse*. For example, after being assigned to van Hiele level 3 based on his/her written response in the van Hiele Geometry Test, the student is interviewed. Analysis of the student's geometric discourse with respect to his/her word use, routines, visual mediators and endorsed narratives shows that the student's van Hiele level is at level 2 instead of level 3. This result does not indicate that the van Hiele Geometry Test is inaccurate in determining students' van Hiele levels, but rather suggests that using a discursive lens to analyze students' geometric discourses at each van Hiele level provides additional information about the student's levels of geometric thinking, and detects information which has been missed in the van Hiele Geometry Test.

Educational Significance

The project provides a better understanding of what prospective teachers know about geometric figures such as triangles, quadrilaterals and their properties, and of prospective teachers' abilities in mathematical reasoning, conjecturing, and proving. The project also sheds light on prospective teachers' use of mathematical terminologies and definitions related to triangles and quadrilaterals, through their geometric discourses. This information about prospective teachers' competencies in geometry helps to improve the teacher preparation program with regard to their mathematical content knowledge. Additionally, the project produces, on the basis of theoretical understandings and empirical data, a detailed model, the *Development of Geometric Discourse*. This model helps to identify additional information that are missed, or not clearly presented, in the general description of van Hiele levels through the analyses of geometric discourse. In practice, distinguishing students' levels of geometric thinking helps to recognize obstacles faced by students, and provides information for instructors teaching prospective teachers, to help improve classroom interactions and instructions.

I am interested in feedback from the audience about (1) the issues of students' abilities in reasoning and proof in introductory undergraduate geometry courses; (2) students' ways of using mathematical terminologies, definitions and propositions in mathematics classrooms; and (3) comments on the model, *the Development of Geometry Discourses.*

Reference

Sfard, A. (2008). *Thinking as Communicating: Human development, the growth of discourses, and mathematizing*. New York: Cambridge.

Senk, S. L. (1989). Van Hiele levels and achievement in writing geometry proofs. *Journal for Research in Mathematics Education, 20*(3), 309-321.

Usiskin, Z. (1982). *Van Hiele levels and achievement in secondary school geometry* (Final report of the Cognitive Development and Achievement in Secondary School Geometry Project: ERIC Document Reproduction Service No. ED 220 288). Chicago: University of Chicago.

Van Hiele, P. M. (1959/1985). The child's thought and geometry. In D. Fuys, D. Geddes & R. Tischler (Eds.), *English translation of selected writings of Dina van Hiele-Geldof and Pierre M. Van Hiele* (pp. 243-252). Brooklyn, NY: Brooklyn College, School of Education.

Van Hiele, P. M. (1986). *Structure and insight: A theory of mathematics education*. Orlando, FL: Academic Press.