Calculus from a virtual navigation problem
Preliminary Research Report

Calculus appeared from the real world application, has a real world context, and is fundamentally a dynamic conception; this is why the framework of Realistic Mathematics Education (RME) should be the most efficient approach to teaching and learning calculus. The current study is devoted to investigation of the computer simulated bodily path optimization calculus. I adapted the conception of 'tacit intuitive model' for the particular calculus task of path optimizations. My hypothesis is that tacit mental modeling takes place with the allocentric frame of reference. I designed a paradigm in the Second Life virtual environment which allows simulating the navigational task of path optimization with two different mediums and with voluntary choice between allocentric/egocentric views. The reinventing the calculus problem of path optimization from the virtual navigation and its mathematizing would give a powerful intuitive link between the everyday real world problem and its symbolic arithmetic.

Key words: calculus, virtual navigation, egocentric/allocentric view, tacit intuitive model, Realistic Mathematics Education

Introduction

In the late 1980s the 'Calculus Reform Movement' began in the USA. The Calculus Consortium at Harvard (CCH) was funded by the National Science Foundation to redesign the Calculus curriculum with a view of making Calculus more applied, relevant, and more understandable for a wider range of students.

The didactical goal of the present study is to help learners to 'unearth' (Torkildsen, 2006) a calculus path optimization problem from the real world navigation problem simulated in the Second Life (SL) virtual reality. The learners would reinvent the calculus problem by controlling computer simulated body movements with either egocentric or allocentric views. The egocentric view provides the perception of 'being' within the virtual environment and seeing objects from the 'first person' view. The allocentric view is provided when the learner's avatar is present in the environment and the learner controls the avatar navigation: in this case the virtual reality objects are spatially related to the avatar. This enactive computer paradigm would allow the learners to explore mathematical ideas being engaged immediately into the optimal navigation problem. Since the designed virtual environment contains two different mediums, the task of path optimization should involve the intuitive anticipation of speed difference in different mediums: when being on land and when being in water; thus, the intuitively planned optimal path will be based on this speed difference anticipation. After a few trials of virtual navigating, the learner should reinvent the calculus path optimization problem and should try to mathematize it. When the problem has been mathematized the learner can connect and compare the intuitive understanding of the problem with its symbolic arithmetic. According to Tall (1991), "by providing a suitably powerful context, intuition naturally leads into the rigor of mathematical proof" (p.20). Since this paradigm gives a strong link between the everyday real world problem and its symbolic formal representation, it strongly relates to the theoretical framework of RME (Freudenthal, 1991; Freudenthal, 1973; Freudenthal, 1968).

The research goal of the study is to explore how egocentric and allocentric frames of references relate to different phases of optimal path problem solving, which, in turn, would provide better understanding of mental processes during the particular calculus problem solving.

Theoretical perspective and related literature

Navigation consists of two aspects: a topographic aspect and a procedural aspect that represents the trip itself (Besthoz, 2000). The topographic aspect is connected with a construction of cognitive map; the procedural aspect is connected with actual movements. Both topographical and procedural navigations include spatial orientation (ibid). Virtual navigation differs from real navigation: it doesn't involve vestibular, translation, or locomotor memory which, according to Berthoz (2000), is inherent to real space body navigation. In virtual environment the visual system plays the main and crucial role.

The new virtual paradigm of optimal path navigation is intrinsically of enactive nature. Tall (1997), in his turn, asserted that "the calculus concepts are starting from enactive experiences as an intuitive basis" (p.4). So, the optimal path virtual navigation paradigm is in accordance with his schematization of building of the concepts of calculus.

On the other hand, the computer simulation of body movements expressed either by an egocentric view of 'being' in the environment or by an allocentric view trough controlling the avatar navigation, provides an explicit perception of 'bodily' navigation which can be expressed in terms of embodiment. Tall (2007) categorizes mathematical thinking into three intertwined worlds: the *conceptual-embodied*, the *proceptual-symbolic* and the *axiomatic-formal*. He considers such categorization particularly appropriate in the calculus. According to Tall (2007), the conceptual-embodied world of mathematics is based on perception of and reflection on properties of objects. For the particular dynamic tasks of optimal navigation and taking into account the dynamic nature of calculus, I modify a conceptual-embodied world into '*procedural- conceptual-embodied'* world, reflecting embodied dynamism of body movement. This extended world is based not only on perception of and reflection on properties of objects, but also on an active body experience in its dynamism such as change of body position, speed, and acceleration.

For the navigational type of tasks, we first, mentally simulate the trajectory, and then we compare the actual movement with the predicted movement (Berthoz, 2000). For the mental simulation stage of navigation I adopted the conception of '*tacit intuitive model*' introduced by Fischbein (1989). The common characteristics of the tacit intuitive models are that they have structural entity; they are of practical and behavioral nature; they are mental, intuitive, and primitive; they are representable in terms of action; they are autonomous entity with their own rules; they are not perceived consciously by an individual. The important characteristic of the intuitive mental model is its robustness and its capacity to survive long after it no longer corresponds to the formal knowledge (ibid). For the case of optimal path navigation the last characteristic should be omitted and the tacit intuitive model should be modified. As Cazzato, Basso, Cutini, & Bisiacchi (2010) pointed out: people produce incomplete plans at the beginning of a route and continuously make decisions along the trajectory of navigation. So, the tacit intuitive model should be modified into a more flexible conception, reflecting dynamism and procedural nature of continuous adjustment according to the model's effectiveness. The term of '*tacit dynamics simulation'* would reflect both the procedural embodied world, on the basis of which the kind of tacit model is constructed, and flexibility and procedural character of such intuitive modeling.

The hypotheses of the research are: 1) *the tacit dynamics simulation* of finding the optimal path takes place with allocentric frame of reference even when the environment is viewed egocentrically; 2) the topographic phase of navigation also takes place with allocentric frame of reference, even if the virtual environment is viewed egocentrically; 3) the procedural phase of navigation can involve both frames of references in parallel, which is in accordance with Burgess's (2006) assertion.

Experimental design and methodology

The designed in SL virtual environment paradigm contains a big water pool with a platform, located at B (Figure 1). The paradigm is related directly to the calculus problem of finding the optimal path from an initial position A to the platform B under the condition that available paths must transverse two different mediums, involving different rates of speed (Figure 1).

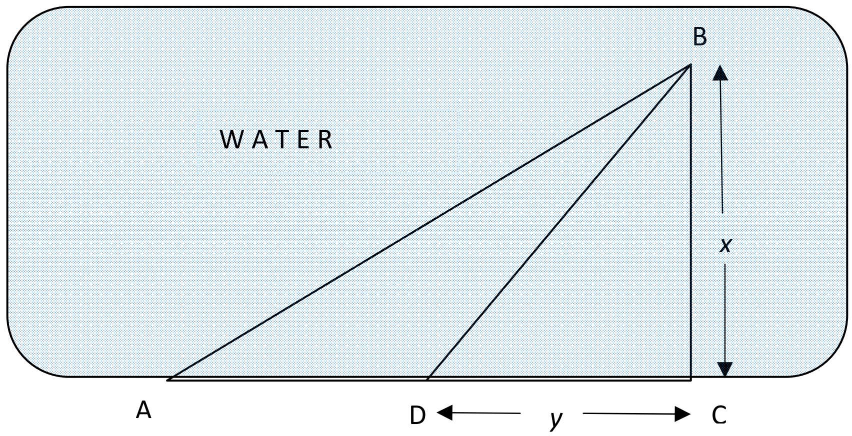

Figure 1: Paths to the platform (adopted from Pennings, 2003)

There are three phases in the experimental paradigm: 1) the exploration phase which allows the participant to learn how to control the avatar, and how to interchange between egocentric and allocentric views; 2) the topographic phase of staying on the platform and memorizing its location with the egocentric view; 3) the procedural phase of reaching the invisible platform from the beach position A as fast as possible; the participant can choose between the egocentric and the allocentric views; 4) repeating the topographic and the procedural phases with changed location of the platform B; 5) the problem mathematizing phase.

The last phase 5) includes the following reasoning. Let T(*y*) represents the time of reaching the platform. Let the participant decides to get into water at D, which is of *y* meters from C. Let *z* represents the entire distance from A to C; *r* is the running/ walking speed on land; *s* is the speed in water. To minimize *T*(*y*) means that $T'(y)=0$, then

$$T(y) = \frac{z-y}{r} + \frac{\sqrt{x^2+y^2}}{s}; \quad T'(y)=0, \text{ which gives } y = \frac{x}{\sqrt{\frac{r}{s}+1}\sqrt{\frac{r}{s}-1}}$$

The learners can see from the formula that since *r* and s are fixed, *y* is proportional to *x*. They can compare this result with their virtual navigation based on their intuitive mental simulation.
The measurements to be analyzed include: distance between B and C for every changed location of platform B, distance between A and D, choice of view (allocentric or egocentric) during the procedural phase of navigation, and after experiment interview data, which include the following questions: a) What view did you choose (allocentric or egocentric) and why ? b) What did you have in mind choosing your particular path to the hidden platform? c) How mathematics describing the process corresponds to your intuitively simulated optimal path?

Conclusion

This research is aimed to "un-earth" calculus from a virtual optimal path navigation problem. The study can have an important learning effect due to enactive nature of revealing the innate capacities of tacit intuitive simulation of optimal path. Mathematizing the problem has a certain didactical value as a particular case of RME. Choice of view at the procedural stage of navigation should serve as an indirect indication of what frame of reference is utilized while constructing cognitive map and simulating mentally the optimal path. The offered study can have an important learning effect from the viewpoint of developing intuitive understanding of the calculus problem due to active participation of learners in reinventing it from the real life situation.

Questions to the audience:

1) The SL virtual environment implies the same speeds in water and on land. What is better: to program different speeds in different mediums or let learners reveal themselves after a few trials that the speeds in the SL are the same and let learners explore this special case mathematically?
2) To what extent voluntary choice between egocentric and allocentric views in the SL virtual environment reflects corresponding frames of references as the brain encodings?
3) Do the number and quality of distant cues influence the path choice?

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