Spanning set: an analysis of mental constructions of undergraduate students

Preliminary Research Report

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Abstract

In this study we use APOS theory to propose a genetic decomposition for the concept of spanning set in Linear Algebra. We give examples of interviews that were conducted with a group of university students who were taking an analytic geometry course and their analysis in relation to our genetic decomposition. We also comment on the nature of difficulties that students experience in constructing this notion. One of the results that are obtained in this research that is in line with previous results reported in the literature is the difficulty in distinguishing a spanning set from a basis. Another aspect is that students have varying levels of difficulty when working with different types of vector spaces. As was expected, the concept of linear combination plays a very important role in the understanding of the notion of spanning.

Keywords: Spanning set, APOS Theory

Introduction and research objective

In an earlier study about the construction of the concept of basis in Linear Algebra (Kú, Trigueros and Oktaç, 2008) we observed the difficulties that students have with the concept of spanning set and the coordination of the underlying process with the process related to linear independence. These difficulties seemed to interfere in a serious manner with the construction of an object conception of basis of a vector space. As a result we decided to carry out research in order to look at these concepts separately, so that we could offer an explanation about the construction of each concept and related problems.

Some literature published previously touch certain issues related to the learning of spanning sets focusing on task design, cognitive difficulties and suggestions for teaching (Nardi, 1997; Ball et al., 1998; Dorier et al., 2000; Rogalski, 2000). What we are interested in with this research is to offer a viable path that students may follow in order to construct this concept as well as explaining the nature of related difficulties while learning it. Informed by our theoretical analysis and empirical data, we also focus on making pedagogical suggestions.

Theoretical framework and methodology

APOS theory has been used successfully in explaining the construction of several concepts in undergraduate mathematics curriculum. Its use with Linear Algebra concepts is more recent (Roa-Fuentes and Oktaç, 2010; Parraguez and Oktaç; 2009; Trigueros, Oktaç and Manzanero, 2007). We continue with this line of research and study the mental constructions and mechanisms involved in the learning of spanning sets.

The steps that are followed in APOS-related methodology are given in Asiala, Brown, DeVries, Dubinsky, Mathews and Thomas (1996). In line with this methodology, our research starts with a theoretical analysis which consists in a genetic decomposition as a possible way to construct the concept of spanning set. This is done in terms of mental constructions (actions, processes, objects, schemas) and mechanisms (interiorization, coordination, encapsulation, assimilation) that students might employ when learning this concept. We then designed an interview that consists in 7 questions, in order to test the viability of our genetic decomposition. This instrument was applied to a group of 11 undergraduate students who were taking an analytic geometry course at a Mexican university. These interviews are analyzed according to our theoretical framework (we are at this stage of our research). We will revisit the preliminary genetic decomposition and make the necessary modifications. Finally we hope to make some suggestions as to the didactical strategies to be employed, in order to facilitate the construction of this concept.

In our design of the interview questions we took into account different aspects of a spanning set. We asked questions of the type whether a certain set spans a given vector space, but we also asked the construction type of questions, namely given a vector space identifying possible spanning sets for it. We also asked the students to compare the vector spaces generated by different spanning sets. By dealing with different aspects of the concept of spanning set in this manner, we hope to shed light on where the difficulties lie and verifying the mental constructions involved in its learning.

Some results

One of the results that are obtained in this research that is in line with previous results reported in the literature (Nardi, 1997) is the difficulty in distinguishing a spanning set from a basis. Another aspect is that students have varying levels of difficulty when working with different types of vector spaces. In particular, when the vector space is not R^n, the interpretation of a spanning set becomes problematic. On the other hand, as was expected, we confirmed that the concept of linear combination plays a very important role in the understanding of the notion of spanning. We are also exploring the connections that students seem to make among the concepts of linear independence/dependence, basis, linear combination, dimension, spanning set and generated vector space. Our analysis so far indicates that it will be necessary to make certain modifications in the preliminary genetic decomposition, but the general model is in line with data.

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