Concrete Materials in Mathematics Education: Identifying "Concreteness" and Evaluating its Pedagogical Effectiveness

A Preliminary Research Report

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A growing body of research suggests cognitive difficulties associated with the use of concrete learning materials. I argue that this research program may benefit from a critical examination of its underlying assumptions. Thus, this report was motivated by a concern that extant approach to evaluating the pedagogical effectiveness of "concreteness" in education is by-and-large undertheorized, resulting suboptimal interpretation of reform-based philosophy and recommendations, ultimately to the detriment of students. I hope to open up a space for a discussion of a more nuanced conceptualization of both (1) "concreteness" as a concept and (2) the observed cognitive difficulties evident in classroom implementation of concrete materials.

Keywords: cognitive research, theoretical perspectives, concrete problems

Introduction

Researchers and educators are likely to agree that concrete materials help student reason and solve problems (Burns, 1996); however, a consensus is growing that certain criteria must be met in order for concrete materials and problems to be effective (for one recent overview, see Brown, McNeil, & Glenberg, 2009). Furthermore, there is a sense that certain implicit (or background) assumptions should be made available to public scrutiny. This report was written in hopes of helping to lay bare the background assumptions implicit to the aforementioned research program.

Before I start, however, it is worth emphasizing that I do *not* intend to critique the research on concreteness itself; if anything, I am grateful for this work and hope my contribution may be of use to the overall research program.

The Issue of Definition: What is Concrete?

What is "concrete," anyway; and what is "abstract"? Perhaps unsurprisingly, this simplistic dichotomy is highly prevalent, and thus a useful starting place for our investigation. In the literature, "concrete" frequently refers to actual stuff in the "real world," (another concept which deserves the quotation mark treatment, see Lave, 1992) and "abstract" is typically synonymous with symbolic. This definition appears troubled beyond the first glance: one need only consider the actual experience of mathematicians. I posit that, to a professional mathematician, the Klein four group found in abstract algebra is no more abstract—or rather, no less concrete—than the number of pens on her desk (for an longer discusson on this topic, see Wilensky, 1991). Another approach to this issue is to consider whether the increasingly ubiquitous virtual worlds are concrete. "Concreteness," I argue, ought not to be viewed as a property of a given object but rather the quality and richness of connections constructed by an individual to said object.

This is not a purely academic argument. Labeling an object as concrete tends to come with certain baggage, some of it dangerous to the learner. That is, when an object is perceived as concrete, there is a temptation to assume that, because we can actually *point to it*, its meaning is shared among the interlocutors. Yet this need not be the case. Indeed, research has identified confusion arising from concrete objects as diverse and seemingly harmless as chalkboard drawings of cookies, marbles, and even fingers (Abrahamson, 2009; Saxe, 2004; Saxe & Esmonde, 2005)! Labeling an object as concrete, then, does not exempt it from ambiguity—yet may lead such ambiguity to remain hidden.

Questions related to the issue of definition:

When solving a mathematical problem, so you experience a sense of "ah, this problem is concrete" or "hm, this problem is abstract"? Is your experience in line with traditional interpretations of these words?

Wilensky argues that abstract mathematics is concrete to mathematicians versed in the area. Do you agree?

What, if anything, is the pedagogical value of deciding a problem is concrete *before* giving it to a student?

Are Cartesian graphs concrete? (Or, are virtual worlds concrete?) Why or why not? What kind of implicit confusion might arise if a teacher considers graphs concrete yet the students do not?

The Issue of Conflation: Not All Mistakes are Created Equal

It is not uncommon to see the following argument against concrete materials in contemporary research literature made either implicitly or explicitly (see, e.g., Kaminski, Sloutsky, & Heckler, 2009, for one explicit version of this argument):

(1) Concrete problems convey extraneous information.

(2) This information distracts the student from perceiving the underlying mathematical structure, reducing the chance of solving the problem.

(3) Thus, concrete problems interfere with learning.

Part (1) is largely noncontroversial; part (2) is arguable, with qualifications (compare Kaminski, Sloutsky, & Heckler, 2008; Koedinger, Alibali, & Nathan, 2008). The point of contention lies in the assumption implicit to (3) that, in order for students to *learn*, mathematical problems must be absent of competing interpretations and, ideally, allow for a clean interpretation. The concern here is that it may be important to distinguish between various types of extraneous information and the various types of mistakes such extraneous information induces. While some extraneous information may be of no pedagogical use, such as, say, the redness of counted apples, it does not follow that all extraneous information is pedagogically malignant. To that end, I present a recent study suggesting learning need not be hampered by competing interpretations arising from extraneous information.

In a recent study (for details, see Trninic & Abrahamson, 2010), fifty-one undergraduate students solved one of two phenomenologically disparate yet mathematically isomorphic problem situations.

For a familiar, concrete instantiation of the problem, we gave the following:

“You are attempting to open a door, but do not recall which key opens the lock. You have 5 keys and know that only one will open the lock. If you decide to try each key in turn, what is the chance that the fourth key you try opens the lock?“

The remainder of the students solved a mathematically equivalent yet generic urn thought experiment.

Participants were initially categorized by mathematical capability and then randomly partitioned. As we predicted from our previous pilot studies, and in line with much work on concrete problems, the generic group performed superior in terms of solving the problem (see Table 1).

Table 1.
Students' Responses on the Urn and the Key Problems

	Answer	
Problem	1/5	1/2
Urn	22	1
Key	17	6

In follow-up clinical interviews we found that the familiar, concrete setting (i.e., the extraneous information) of the key problem cued some students to misinterpret the key problem as a conditional probability (“Given that I am on the fourth try…”). Yet, despite their initial difficulty with interpreting the key problem, the interviewed students reported gains in both confidence and ability after resolving this conflict. I would argue that these gains occurred not merely *despite* students' initial difficulties, but rather, *because* of them. Abrahamson (2009) found similar results, concluding that powerful learning happens when children learn to explicitly reconcile their naïve intuitions with normative understanding (in his case, combinations versus permutations). In both studies, naïve understands were brought to bear precisely by “extraneous” information embedded in concrete problems. Thus the cognitive conflict elicited by extraneous information may provide a fertile ground for learning. The theoretical point is this: in line with the notions of productive failure (Kapur, 2008) and the need for pedagogically useful struggle (Stevenson & Stigler, 1992), students who fail to solve a particular problem because of its extraneous information may ultimately make greater learning gains than those who never experienced similar cognitive conflict.

Questions related to the issue of conflation:

Students frequently mistake permutations and combinations. What might be some arguments for and against problems which are likely to be misinterpreted as permutations or combinations?

When, if ever, should the student be exposed to real world problems which are messy or ill-defined?

Is there pedagogical value to “teaching mathematics so as to be useful” (Freudenthal, 1968)?

Is it ever useful for students to struggle with issues of *interpreting* the problem? Or, is it ever *not* useful for students to struggle with these issues?

Conclusion

Let us briefly consider the hypothetical case of a teacher faced with the selection of pedagogically effective mathematics problems. Suppose we find him wondering whether to present an "abstract" problem (e.g., qua a symbolic equation) or perhaps something more "concrete" (e.g., qua a physical manipulative). What does he choose?

While remarkable teachers can foster insights with close to nothing, so to speak, it stands to reason that not all problems are created equal and even exemplary teachers would agree that some problems benefit their instruction more than others. Indeed, teachers, school boards, and even national governments make decisions influenced by such considerations; and "math wars" are fought for less (see Schoenfeld, 2004; Tobias & Duffy, 2009). Yet some of these debates may prove misplaced, once their attendant dichotomies are deconstructed and implicated as secondary to what may be the critical factor: the nature of student engagement with these problems.

It is not enough to simply say that a problem is concrete and that concrete problems lead to difficulties—one must evaluate the nature of student engagement with such a problem.

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