Extending a Local Instruction Theory for the Development of Number Sense to Rational Number
Preliminary Research Report

Abstract

We report on results of the implementation of a local instruction theory for number sense development in a course for prospective elementary teachers. Students involved in an earlier teaching experiment developed improved number sense, particularly in the form of flexible mental computation. The previous research was informed by a conjectured local instruction theory and informed the refinement and elaboration of that local instruction theory. The present study concerns a recent iteration of the classroom teaching experiment, in which the local instruction theory guided instructional planning. In the recent iteration, the local instruction theory was extended from the whole-number portion of the course to the rational-number portion. Envisioned learning routes that were developed in the context of mental computation and estimation were applied to reasoning about fraction size. In this way, the application of the local instruction theory was extended from whole-number sense to rational-number sense.

Keywords: Local instruction theory, number sense, prospective teachers, rational number

We report on results of the implementation of a local instruction theory for number sense development in a mathematics content course for prospective elementary teachers. Our previous research showed that students involved in an earlier teaching experiment developed improved number sense, particularly in the form of flexible mental computation (Whitacre, 2007). The previous research was informed by a conjectured local instruction theory and informed the refinement and elaboration of that local instruction theory (Nickerson & Whitacre, 2010). The present study concerns a recent iteration of the classroom teaching experiment, in which the local instruction theory guided instructional planning. In the recent iteration, the local instruction theory was extended from the whole-number portion of the course to the rational-number portion. In particular, envisioned learning routes that were developed in the context of mental computation and estimation were applied to reasoning about fraction size. In this way, the application of the local instruction theory was extended from whole-number sense to rational-number sense.

Instruction

The course was intended to foster the development of number sense. In particular, the broad instructional intent was for students to come to act in a *mathematical environment* in which the properties of numbers and operations afforded a variety of calculative strategies, as opposed to one in which mathematical operations map directly to particular algorithms (Greeno, 1991). The mathematics content course is the first course in a four-course sequence. There are multiple sections of the course, and a common final exam is used. Topics in the curriculum include quantitative reasoning, place value, meanings for operations, children's thinking, standard and alternative algorithms, representations of rational numbers, and operations involving fractions. We have adapted the course in such a way as to engage students in authentic computational reasoning throughout the semester. This includes such activities as mental computation, estimation, and reasoning about the relative magnitudes of fractions and decimals.

By authentic, we mean that students encounter the need for computational reasoning in the process of their work on larger tasks. For example, in the process of solving a story problem, computations naturally arise, and students are expected to solve these mentally. Particular students' strategies are discussed by the class, and a set of established strategies gradually builds.

Local Instruction Theory

A *local instruction theory* refers to "the description of, and rationale for, the envisioned learning route as it relates to a set of instructional activities for a specific topic" (Gravemeijer, 2004, p. 107). Note that a local instruction theory (LIT) is distinct from a hypothetical learning trajectory (HLT). Gravemeijer (1999) identifies two key distinctions between these related constructs: (1) an LIT tends to describe an instructional sequence of longer duration; (2) an HLT is situated in a particular classroom, whereas an LIT is not.

We have described elsewhere our local instruction theory for the development of number sense (Nickerson & Whitacre, 2010). Here, we briefly list the three major goals around which this LIT is organized: (1) Students capitalize on opportunities to use number-sensible strategies; (2) Students develop a repertoire of number-sensible strategies; (3) Students develop the ability to reason with models. In the proposed session, we focus primarily on the second of these goals.

Design Research

We conduct design research in the form of classroom teaching experiments, which are reflexively related to theory building (Cobb & Bowers, 1999). In this case, our LIT was developed and refined in the context of a classroom teaching experiment in a course for prospective elementary teachers. The previous experiment focused on mental computation and estimation, primarily with whole numbers. In the present iteration, the LIT guides instruction throughout the same content course, including instruction concerning rational numbers. We describe how the LIT has guided instructional planning for a sequence concerning reasoning about fraction size. By the time of the conference, we will be able to report on how this sequence played out in the classroom.

Applying the LIT to Reasoning about Fraction Size

We focus on the goal of students developing a repertoire of number-sensible strategies, particularly those strategies involved in comparing fractions and otherwise reasoning about fraction size. The framework of Smith (1995) informed our thinking about these strategies and influenced the planning of the instructional sequence. Our pilot and pre-instruction interviews, as well as the findings of other researchers, such as Newton (2008), suggested that these students would come to the course with standard algorithms for comparing fractions. Smith describes the strategies of converting to a common denominator or converting to a decimal as belonging to the Transform Perspective. We also expected students to come to instruction with a meaning for fractions as indicating a number of parts of a whole (e.g., so many slices of a pie). In Smith's terms, this is an example of the Parts Perspective.

Tasks were designed and sequenced so as to begin with students' current ways of reasoning and to provide opportunities for reasoning about fraction size in new ways. In particular, we sought to engage students in reasoning about fraction size from Smith's Reference Point and Components perspectives. The Reference Point Perspective involves reasoning about fraction size on the basis of proximity to reference numbers, or *benchmarks* (Parker & Leinhardt, 1995). The Components Perspective involves making comparisons within or between the two fractions, as in coordinating multiplicative comparisons of numerators and denominators. The design and sequencing of tasks involved consideration of these perspectives relative to number choices, contexts, and anticipated student reasoning, and guided by the envisioned learning routes described in our LIT. Although Smith (1995) does not describe the perspectives or particular strategies belonging to his framework in a hierarchical way, we view the Reference Point and Components perspectives as generally more sophisticated categories of reasoning about fraction size. Thus, in our instructional sequence, we aim for these more sophisticated strategies to be used by students and established for the class by mathematical argumentation.

Questions for Discussion

By the time of the conference, we will be prepared to report on how the instructional sequence played out, although formal analysis of collective activity will be ongoing. Questions for discussion specific to the fraction-size sequence concern productive models for reasoning about fraction size, as well as our conjectured trajectory of strategies. We welcome suggestions concerning tasks and ordering of tasks in the instructional sequence. More broadly, we are interested in discussion local instruction theories, their relationship to hypothetical learning trajectories, and the notion of extending an LIT to new topics or implementing it in new contexts.

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