Students' Logical Reasoning in Undergraduate Mathematics Courses

Preliminary Report

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This narrative describes a pilot study (the author is conducting) to prepare for a proposed study to be implemented at Salisbury University in the academic year (2010-2011). As such, it is a Preliminary Research Report. The proposed study will be done in an attempt to describe undergraduate Salisbury University mathematics/computer science students' understanding of logical inference.

It is well-known that there is an extensive list of studies that have been conducted on proof and logical inference. Many of those studies will be cited and included in the literature review for the proposed study. For this brief report , only a few most relevant sources have been cited.

Theorems in mathematics are often stated in an "if-then" form, and this type of statement is often called a "conditional statement." If someone wants to become a student of mathematics, it is imperative to distinguish between this technical form of statement and that same form as used in everyday normal speech. In everyday speech one can glean from the context the meaning that is intended even if the form of the statement is not really correct. In mathematics, however, it becomes important to know precisely what the form of the statement implies (or does not imply).

In 1984, the author (Austin, 1984) conducted a study in an attempt to assess the level of understanding that undergraduate students at a state university seemed to possess in this regard. Responses were collected from 493 students enrolled in mathematics courses and from a sample of 219 students selected randomly from the student population.

In this study, students were given the following four items (reasoning patterns):

(1) Detachment (Modus Ponens):

If the couch is soft, then it is Linda's. The couch is soft. Is the couch Linda's?

(a) yes (b) no (c) not enough clues

(2) Conversion:

If the water is warm, it is dirty. The water is dirty. Is the water warm?

(a) yes (b) no (c) not enough clues

(3)Contraposition (Modus Tollens) :

If the shoes are big, they are John's. The shoes are not John's. Are the shoes big?

(a) yes (b) no (c) not enough clues

(4) Inversion:

If the man is old, he is sad. The man is not old. Is he sad?

(a) yes (b) no (c) not enough clues

Frequencies (and percentages) of correct responses were reported, and some observations were made. As one might expect, students fared best on detachment, and not so well on the other three reasoning patterns. Although responses from those students enrolled in mathematics courses were somewhat better than those in the random sample, the difference was not strikingly impressive.

The author also participated in another study that sought to describe pre-service elementary teachers' understanding of logical inference, and the qualitative portion of this study is available (Hauk, et.al. ,2008). The quantitative portion of the study is yet to appear. The qualitative study documents the fact that students appear to be at different levels in their understandings of logical inference, and based on the conclusions from this study, several implications for teaching mathematics to these learners are suggested.

The 1984 study was a descriptive one, and no attempt was made to discover why students thought (in the ways they must have) relative to their responses. Also, no real theoretical framework was used to interpret or shed light on any of the findings. In 1984 it seems that few, if any, researchers were using mixed methods in their research, and though much research was being done in mathematics education, there were not many research studies being conducted in undergraduate mathematics education. Since that time the research milieu has changed in that today many research studies employ mixed methods and undergraduate mathematics education research has become an active area of inquiry.

APOS (Action-Process-Object-Schema) theory describes a possible way that learners progress in their attainment of mathematical concepts (Dubinsky & McDonald, 2001). One of the salient aspects of APOS theory is that it posits that learners can be at different levels in their understandings of mathematical ideas/concepts. Like other theories of learning, this constructivist theory also embodies a progression from lower levels to higher levels of attainment. Although it is understood that it is probably not the case that everyone's learning journey fits into this model in a lockstep way, the APOS theory provides a means or lens by which or through which empirical results might be viewed or interpreted.

Balacheff's levels of proof understanding is another model with four stages (Balacheff, 1988). The four levels described in this model are (a) naïve empiricism or "proof by example" strategies; (b) crucial experiment (includes generation of counterexamples); (c) generic examples; and (d) thought experiment (learner arrives at structured deductive logical forms. Again, there is this progression from low levels to higher levels, and gives a background for assessing levels of understanding of proof.

This preliminary report describes a pilot study in which data has been collected on the four reasoning patterns (as the 1984 study) from students enrolled in one introductory statistics class (freshman class), two statistics laboratory classes (freshman/sophomore class), one class in a statistics class for mathematics and computer science majors (sophomore class), and a class in abstract algebra (junior/senior class). In addition to having the students give their responses to the multiple-choice items for the four reasoning patterns, students were asked to record their thoughts (thinking) as to why they chose the response they did. Students were asked if they found some items easier than others. If they did find items easier, they were asked why they thought they were easier? All of the data in the pilot study was collected by written information given by the students in a classroom setting.

The research questions for this pilot study were the following:

1. Are there observable differences between the responses for the four different classes? If so, how can these be characterized?
2. Can different levels of learning be detected (as posited by APOS theory)? How do these levels (if detected) compare to the level of the course? Do they match well or is there a mismatch between the two?

3. If the four reasoning patterns are ordered based on performance in each of the classes, is the order the same for all classes or are there different orders for different classes? How do the orders compare/contrast with the ideas of easiness? Does the performance on the four reasoning patterns match the ideas of easiness well or not?

Method

For the pilot study data were collected using a survey instrument and given to students enrolled in the classes. No formal instruction of logical reasoning had been given in any of the classes prior to the collection of the data. At this point, the data have not been analyzed, but findings from the pilot study will be presented at the RUME Conference (2011).

For the proposed study, data will be collected in the academic year (2010-2011) at Salisbury University using students enrolled in several undergraduate mathematics courses. Data will be collected using the refined survey instrument and by interviews. The author seeks suggestions and ideas from anyone at the RUME conference, especially for the interview protocols. Since the study is somewhat "fluid" at present, all suggestions for questions for the interview protocols will be welcomed and any ideas for conducting the study are solicited.

Questions:

1. What are your ideas for questions for the protocols?
2. What are suggestions you have for methodology?

References:

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