The Construction of Limit Proofs in Free, Open, Online, Help Forums

Carla van de Sande & Kyeong Hah Roh
Arizona State University

Abstract: Free, open, online, help forums are found on public websites and allow students to post queries from their course assignments that can be responded to asynchronously by anonymous others. Several of these forums are tailored to helping students with mathematics assignments from various courses, and Calculus, in particular, is a heavily trafficked area. Students use the forums when they have reached an impasse, either in constructing or understanding a solution to an exercise that they have encountered, or to seek verification of their own reasoning. The queries posted by students include both computational tasks as well as proof constructions. In this project, we examine threads on limit proofs for single-variable functions from two popular online forums. Our goal is twofold: to characterize the help students are receiving as they wrestle with using the formal definition of limit, and to compare the construction of proof to other tasks in online forums.

Keywords: computer-mediated discourse; limits; online help; student understanding of proof

Introduction

As students tackle assignments or struggle to understand their coursework, they seek access to a large and varied set of resources. One such resource that has emerged fairly recently and appears quite popular is found on the Internet. Free, open, online, help forums are located on public websites and allow students to post queries from their course assignments or materials that can be responded to asynchronously by anonymous volunteers. Help forums exist for many subject areas and grade levels, with mathematics, in general, (and Calculus, in particular) receiving much traffic. Students use the forums to post questions from exercises and examples that they encounter in their material classrooms. These include both computational or procedural tasks, as well as proofs. It is the help seeking for the construction and comprehension of proofs in the forums that has drawn our attention. Of the many proof types that surface on the forums, we focus on limit proofs for single-variable functions. This type of proof serves as students' first introduction to rigorous proof in Calculus, but is challenging and often poorly understood. Our goals are to characterize the help that students are receiving as they wrestle with examples and exercises of proofs using the formal definition of limit, and to compare help seeking on proof to that on other types of exercises in online forums.

Literature & Theoretical Perspective

In contrast to being cheat sites, there is evidence that many forums profess the intent to assist students rather than to do their exercises for them (van de Sande & Leinhardt, 2007). In such forums, students, instead of simply publishing the problem statement, are more prone to demonstrate understanding of the exercise (e.g., what they have tried) and contribute to the construction of the solution (e.g., by responding to helpers). Students also take more responsibility for initiating resolution by communicating how the interaction was helpful. In terms of the helpers, in some forums, they exhibit a strong sense of community, as they correct one another, work collectively to help individual students, and engage in collegial banter. The students and helpers who participate in these forums are using technology to engage with one

another on the mathematics that students are expected to perform. Proof is an example of one such activity that students face.

However, students have little experience knowing what counts as mathematical proof and how these differ from common sense proofs. Unlike the logical deduction used by mathematicians, undergraduates consider specific cases to constitute proof (Harel & Sowder, 1998). Even for students who possess an accurate conception of mathematical proofs, proof writing is challenging (Selden & Selden, 2008). One of the main difficulties occurs when students structure their proofs in terms of the chronological order of their thought process instead of rearranging them with careful consideration on proper implications (Dreyfus, 1999). It is also a challenge for students to transform informally written statements into mathematically formally structured ones in calculus (Selden & Selden, 1995). In addition, students have difficulty in relating conceptual ideas that can help them generate their proofs. Knowing formal definitions or theorems is insufficient for students to construct a proof (Weber, 2001). Research calls attention to personal heuristic knowledge of mathematical concepts (Roh, in press), and the connection of heuristic ideas about mathematical concepts to proof construction (Raman & Weber, 2006; Raman & Zandieh, 2009).

We hold that help seeking is an important strategy that can be instrumental in the development of autonomous skill and ability (Nelson-LeGall, 1985). In terms of constructing limit proofs, this ability requires not only deductive reasoning, but also abductive reasoning to extract a pattern from observations, measurements, or events in a holistic manner, and inductive reasoning to identify or synthesize common regularities across several events.

Research Methods

Our general methodology is one that has been applied in the context of open, online, help forum research (van de Sande & Leinhardt, 2007). It involves selecting an online forum(s) as a research site, searching the archives using keywords (such as "epsilon" and "prove"), and selecting threads from the search that match a particular target (here, limit proofs for single-variable functions). We have adopted this observational methodology for ethical reasons and because of the exploratory nature of the research.

For this project, we selected two online mathematics help forums, located at www.freemathhelp.com (FMH) and www.sosmath.com (SOS). Both forums allow any member to contribute (as a helper) to any ongoing thread (as opposed to restricting the set of helpers or assigning incoming queries to one particular helper). This participation structure allows multiple helpers to be involved and interact in a given thread. In addition, member status in both forums depends only on the number of individual threads to which one has contributed (as opposed to depending on others' ratings of one's contributions). Finally, both forums are functional and active in the sense of daily postings and membership, and have existed for about a decade.

Our search and selection process in the current FMH and SOS archives netted 73 threads involving limit proofs for single-variable functions: FMH, n=19, dated 8/30/05-6/13/10 and SOH, n=48, dated 6/1/03-5/26/10.

Preliminary Results

We analyze the types of query in terms of (Q1) functions (linear or nonlinear functions), (Q2) layers of quantification (e.g., in finding a value of δ for a given value of ε, or for any ε), and (Q3) student need (how to prove or why we prove what we prove). We also analyze the types of help in terms of (H1) representations (algebraic or graphical approach) and their connections, (H2) reasoning promoted by helpers (inductive, abductive, or deductive reasoning), and (H3)

pedagogical moves (lecturing, scaffolding, or hinting). Finally, we analyze levels of resolution from the student perspective.

Example: On September 30, 2009, a student pomar posted a problem on SOS "*Use the given graph of f(x) = √x to find a number δ that fulfills the following condition. If absolute value(x-4) < delta then absolute value(sqrt(x)) - 2) < 0.4, δ ="* with a message "I do not know how to solve this problem can you please show the steps to solve this problem and explain why you did what you did." Pomar received 3 responses from two different helpers, Jack and measurable, as follows:

- Jack: "It's easy enough to do directly: If $|x-4|<\delta$, then $|\sqrt{x}+2|\ |\sqrt{x}-2|<\delta$." (September 30, 2009)
- measurable: "It is obvious that $\delta = 10^{-10}$ would do." (October 1, 2009)
- Jack: "Expanding on my earlier answer... If $|\sqrt{x}-2|<0.4$, then $-0.4<\sqrt{x}-2<0.4$, so $1.6<\sqrt{x}<2.4$. Now, add 2 to get $3.6<\sqrt{x}+2<4.4$. The part I care about is the upper bound of 4.4. Since we are adding and $\sqrt{x} \geq 0$, then $\sqrt{x}+2<4.4$ is equivalent to $|\sqrt{x}+2|<4.4$, so $|\sqrt{x}-2|<0.4 \Rightarrow |x-4| = |\sqrt{x}-2|\ |\sqrt{x}+2| < (0.4)(4.4) = 1.76$. Thus $\delta = 1.76$ or any other smaller number will do, such as measurable's answer." (October 1, 2009)

We claim that this exchange exhibits the following characteristics:

- (H1) Focus on algebraic manipulations without connections to graphical meaning (Even the graph that the student mentions is never addressed.)
- (H2) Improper use of abductive reasoning (Jack) or inductive reasoning (measurable)
- (H3) Restricted range of pedagogical moves (Scaffolding and hinting are not in evidence.)
- Low level of resolution from perspective of student (Student need to know how and why we construct δ that way is not addressed.)

Implications for Further Research

Students are using open, online help forums to seek advice on comprehending and constructing proofs requiring the formal definition of limit. Our preliminary findings suggest that the help they are receiving from this resource is unsatisfactory based on our understanding of proof construction, the formal definition of limit, and student reasoning. This work points to the need to either develop new, or modify existing, theory-based approaches to teaching limit proofs in response to student queries in an online forum environment. Furthermore, the major limitation of our methodology, namely that using observations alone severely restricts the analysis of student and helper thinking, calls for experimental studies that link activity on the forum to mathematical understanding and performance.

Discussion Questions

1. Can you suggest relevant analytical frameworks for our observational research project?
2. How do you envision a trajectory from an observational to an experimental research program?

References

Dreyfus, T. (1999). Why Johnny can't prove. *Educational Studies in Mathematics*, *38*, 85-109.

Harel, G., & Sowder, L. (1998). Students' proof schemes: Results from exploratory studies. *Research in Collegiate Mathematics Education* III, 234-283.

Nelson-Le Gall. S. (1985). Help-seeking behavior in learning. *Review of Research in Education, 12,* 55-90.

Raman, S. M. & Weber, K. (2006). Key ideas and insights in the context of three high school geometry proofs. *Mathematics Teachers*, *99*, 644-649.

Raman, S. M., & Zandieh, M. (2009). The case of Brandon: the dual nature of key ideas in the classroom. *Nordic Studies in Mathematics Education, 14*, 29-48.

Roh, K. (in press). College students' reflective activity in advanced mathematics. To appear in *Proceedings of the 32nd Conference of the North American Chapter of the International Group for the Psychology of Mathematics Education, Columbus, OH, Ohio State Univ.*

Selden, A., & Selden, J. (2008). Overcoming students' difficulties in learning to understand and construct proofs. In M. Carlson & C. Rasmussen (Eds.), *MAA Notes. Making the connection: Research and teaching in undergraduate mathematics education* (pp. 95-110). Washington, DC: MAA.

Selden J. & Selden A. (1995). Unpacking the logic of mathematical statements. *Educational Studies in Mathematics, 29(2),* 123-151.

van de Sande, C., & Leinhardt, G. (2007). Online tutoring in the Calculus: Beyond the limit of the limit. *Éducation et Didactique, 1(2),* 115-154.

Weber, K. (2001). Student difficulty in constructing proofs: The need for strategic knowledge. *Educational Studies in Mathematics*, *48*, 101-119.