SIGMAA on RUME

FARIBORZ MASEEH DEPARTMENT OF MATHEMATICS + STATISTICS

On behalf of the Mathematical Association of America, Portland State University's Fariborz Maseeh Department of Mathematics & Statistics, and the RUME Conference Planning Committee, we welcome you to the 15th Annual Conference on Research in Undergraduate Mathematics Education! We have assembled an outstanding program of talks and events for this year's conference. We hope that you find the conference intellectually stimulating, productive in making connections with colleagues, and socially rewarding.

All talks in this year's program have online links to their accompanying short papers, which we refer to as the ***RUME Conference Reports***. Authors who present at the conference also have the option of submitting a 15-page paper for peer-review consideration for publication in the ***RUME Conference Proceedings***, to be published online later this spring. Guidelines for the RUME Conference Proceedings long papers are available on the conference webpage. Submissions to the RUME Conference Proceedings are eligible for the Best Paper Award.

On behalf of the RUME Executive Committee, we would like to thank the RUME Conference Planning Committee for their hard work and their efforts towards making this a successful conference. My sincere thanks go to Karen Marrongelle, Sean Larsen, Jason Belnap, Hope Gerson, Mike Oehrtman, Vicki Sealey, Tim Fukawa-Connelly, Allison Toney, Nicole Engelke, Joe Wagner, Evan Fuller, Karen Allen Keene, and Aaron Weinberg, and Craig Swinyard.

This year we received and reviewed over 100 preliminary, contributed, and theoretical report proposals. This work required the time and energy of over 40 reviewers, who prepared evaluations and feedback for authors. We could not carry out the conference without their contribution. Thank you to the 2012 reviewers!

If you have comments and suggestions about the submission or review process, please feel free to share them with members of the planning committee during the conference or send them to Stacy_Brown@pitzer.edu.

Local organizers will be wearing yellow "hospitality" ribbons on their name badges. Please feel free to ask anyone wearing a yellow ribbon questions you may have about the conference or Portland. Should you have any questions, concerns, or compliments about the conference, please do not hesitate to talk to any of us. Please also be sure to complete the Conference Evaluation form, as we're always looking for ways to improve. Enjoy and have a wonderful conference!

Sincerely,

Karen Marrongelle
Local Organizer

Stacy Brown
Program Committee Chairperson

SIGMAA on RUME

RUME 2012
Conference Proceedings and Best Paper Award
IMPORTANT INFORMATION

All authors who present at the 2011 conference will have the option of submitting a 15-page paper for publication in the RUME Conference Proceedings.

All long-papers will be:

- reviewed by at least 3 reviewers
- considered for best paper
- considered for pre-journal review, unless otherwise requested.

All long papers must follow the long paper formatting guidelines (available on the conference website under the "Submission Guidelines" tab) and will be reviewed according to the proposal review criteria.

LONG PAPER REVIEW CRITERIA

- Does the proposal explore a significant issue/question relevant to RUME?
- How does it relate to prior research on related topics/issues?
- Is the theoretical perspective clearly outlined?
- Is there an appropriate choice of research methodology?
- Is it clear what the conclusions/main claims are?
- Are those supported by data?
- Does the research contribute to teaching practice/theory development?

LONG PAPER FORMATTING GUIDELINES are available on the conference website under the "Submission Guidelines" tab.

Submission of a long paper is not required of participants but is strongly encouraged. Acceptance to the RUME Conference Proceedings is dependent upon reviewers' recommendations. All long papers are **due Friday, March 24, 2012**. Long papers should be sent as an attachment by e-mail to Stacy_Brown@pitzer.edu, the Chair of the Program Committee, with the subject line: RUME 2012 Long Paper Proceedings.

Thursday, February 23

8:00 AM – 12:00 PM RUME Working Group Meetings
12:00 PM – 1:00 PM Registration Near John Steinbach Room
1:00 PM – 1:30 PM Session 1 – Contributed Reports
1:40 PM – 2:10 PM Session 2 – Preliminary Reports
2:20 PM – 2:50 PM Session 3 – Contributed Reports
2:50 PM – 3:20 PM Coffee Break
3:30 PM – 4:00 PM Session 4 – Preliminary Reports
4:10 PM – 4:40 PM Session 5 – Contributed Reports
4:50 PM – 5:20 PM Session 6 – Preliminary Reports
5:30 PM – 6:30 PM Break Embassy Suites Complimentary Happy Hour – Arcadian Garden
6:30 PM – 8:00 PM Plenary Talk *The Challenges of Observation* *Alan Schoenfeld* *University of California at Berkley* Colonel Lindberg Ballroom
8:00 PM – 9:00 PM Reception Mezzanine

CHIEF POKER JIM

Research on Using Technology as a Research and Formative Assessment Tool in the Calculus Classroom

Organizers:
Nicole Engelke, California State University-Fullerton
Gulden Karakok, University of Northern Colorado
Aaron Wangberg, Winona State University

The primary purpose of this working group is to bring together researchers interested in investigating how students develop their understanding of calculus concepts using technology. We will share the possibilities of utilizing the open source online homework system WeBWork, and discuss and develop ideas for improving it as a research tool, an instructional tool, and a formative assessment tool within calculus courses. We will keep the discussion open on further extending this system or a similar system. Further, we are open to discussions about how this technology may be extended beyond calculus. The working group is intended for active scholars in undergraduate mathematics education, new researchers who are interested in investigating teaching and learning calculus, and expert researchers and practitioners who have been using or are planning to use technologies similar to WeBWork in their research or courses.

We will share our enhanced WeBWork system that we have been using as a research tool and also present our future development ideas on utilizing it to facilitate meaningful discussions in classroom instruction. The participants will be asked to share their experience teaching with technology, and discuss possible research ideas with other participants using such systems.

Participants do not need to present anything in order to participate and anyone who wishes to take part in the discussions is welcome to attend.

MARSHALL JOFFRE

Working Group for Research on Novice Teachers of College Mathematics

Organizers:
Jason K. Belnap, University of Wisconsin-Oshkosh
Jessica Deshler, West Virginia University
Shandy Hauk, WestEd and University of Northern Colorado
Natasha Speer, University of Maine

The development of professional skills in the area of college mathematics teaching, particularly for graduate students and beginning faculty, has become an increasingly visible topic in the mathematics community over the past three decades. To optimize the quality and effectiveness of such professional development requires a research base. However, the area is relatively young and many of the researchers are also early in their careers. Consequently, there has been a need for a venue in which researchers could interact with others interested in basic and applied research on novice college mathematics instructor development as well as a space to provide and receive feedback on developing ideas. These needs were left unfilled by existing conference structures so we created a working group for those who share this research interest. The working group pursues three primary goals: (a) to provide critical, informed support and feedback for those conducting research on or about novice teachers of college mathematics; (b) to help mathematics educators connect and collaborate on work about the experiences and professional development of novice college mathematics instructors; (c) to consider over-arching issues related to individual research agendas and to endeavor to contribute to these common concerns. The organizers invite all those who do research or are interested in developing a research program in the area of novice college mathematics instructor development to participate.

ERIC HAUSER

Working Group on Research on Community College Mathematics

Organizers:
April Strom, Scottsdale Community College
Ann Sitomer, Portland Community College
Vilma Mesa, University of Michigan
Mark Yannotta, Clackamas Community College

We seek to bring together researchers interested in investigating questions of mathematics teaching and learning at community colleges. This is a key area that is emerging as an important player in the scholarship of post-secondary education. The main goal of the working group is to continue the rich discussions, which began at our working group session at RUME 2011. We aim to continue our focus on setting a research agenda to understand issues of mathematics teaching and learning that are particularly significant when focused on issues unique to community colleges. The working group is intended for active scholars in undergraduate mathematics education, new researchers who are interested in investigating teaching and learning in this context, and researcher-practitioners (i.e., community college faculty who have earned, or are in the process of earning, a doctoral degree) who see research in context and teaching at community colleges as their primary focus. The activities for the Working Group session include the following: (1) providing a summary of the work done at RUME in 2011 and over the course of the year; (2) engaging in a discussion with an invited guest about strategies for moving a research agenda forward; (3) discussing a research grant proposal focused on collecting data of community college teaching and learning led by members of this group, (4) brainstorming future research projects based on the proposed strands in the current research agenda, and (5) planning the next steps for work during the upcoming 2012 year to continue furthering the work of this group. We anticipate the discussion outcomes to generate a report for the RUME Proceedings, a report for the research subcommittee of the American Mathematical Association of Two-Year Colleges, and a paper for a major journal, outlining this agenda. Participants will also have an opportunity to provide input towards the design and development of a grant proposal (to be submitted to IES in June 2012) focused on investigating the landscape of community college teaching and learning of mathematics.

SAM HILL

Infinity and Limits in Undergraduate Mathematical Learning

Organizers:
Robert Ely, University of Idaho
Timothy Boester, Wright State University

In our third meeting of the infinity and limits group, we will continue to develop and refine research goals pertaining to the undergraduate learning of infinite processes (including series and sequences), infinite sets, limits of real-valued functions, and other domains that significantly incorporate limits and infinity. While existing research has substantially charted out misconceptions and obstacles to infinity and limits, there is still great need for further understanding of the functioning of student knowledge of limits and infinity in context, the development of instructional goals and curricular implications, and the building of these ideas into learning trajectories. With these needs in mind, we have organized our research goals along four general themes: (a) the interaction of function and limit, (b) how limits manifest themselves throughout the calculus curriculum, (c) formalization of limit, and (d) mathematizing the "..." Although we plan to pursue these themes, we welcome participants interested in any areas related to limits and infinity. Part of the working group time will be for presentations of completed, ongoing, or preliminary work, with ample time for discussion, analysis, and planning of participants' research. There will also be time for more informal discussion, so we encourage anyone wishing to discuss their research to contact the working group organizers for time to be reserved in the schedule. Our primary goal for the working group is to nurture collaborative studies and publications based on the vision of all participants in the undergraduate learning of infinity and limits.

Session 1 – Contributed Reports
Thursday, 1:00 PM – 1:30 PM

ERIC HAUSER

Discursive Approach to Calculus Students' Thinking About the Derivative

Jungeun Park, University of Central Arkansas

This study investigated characteristics of university calculus students' discourses on the derivative using a communicational approach to cognition. The data were collected from a survey and student interviews from three calculus classes at a large public university in Midwest. During the interview, students were asked to explain their solution processes on the survey problems. The analysis of interviews focuses on students' descriptions about the derivative and the relationships between a function, the derivative function, and the derivative at a point. The results show that their descriptions were closely related to how they think about the derivative as a number and as a function. A common description of the derivative as a tangent line, which is a point-specific object but also a function defined on an interval was identified. This description was closely related to their use of word, "derivative" for both "the derivative function" and "the derivative at a point."

CHIEF POKER JIM

Expanding Toulmin's Model: The Development of Four Expanded Argumentation Schemes from Analysis in Linear Algebra

Megan Wawro, Virginia Tech

In this presentation, I define four types of argumentation schemes that are expanded versions of Toulmin's (1969) model of argumentation. These expanded schemes—Embedded, Proof by Cases, Linked, and Sequential—developed out of necessity when the original 6-part Toulmin scheme proved inadequate while analyzing argumentation of an inquiry-oriented linear algebra classroom community. Aspects of these four expanded schemes were adapted from and are compatible with those presented by Aberdein (2006, 2009). Within this presentation, I investigate how Toulmin's is used within mathematics education research, as well as other fields of research, propose how the expanded schemes provide needed detail when analyzing complex argumentation, and provide examples of each from the whole-class discussion of the introductory linear algebra course.

JOHN STEINBACH

Role of Faculty Professional Development in Improving Undergraduate Mathematics Education: The Case of IBL Workshops

Marina Kogan, University of Colorado-Boulder
Sandra Laursen, University of Colorado-Boulder

Professional development opportunities might provide the tools that faculty need to transition from lecture to the research-based student-centered instructional methods. With data from two offerings of faculty professional development workshop aimed at educating faculty in the use of Inquiry Based Learning (IBL) instructional techniques, we examine what changes in the instructional practices can—or cannot—be accomplished through the means of professional development workshops. Faculty participants report strong immediate benefits in the form of increased knowledge, skills, and motivation to use student-centered instructional methods. The long term outcomes are also very encouraging. The vast majority of respondents to the follow-up survey report that they had implemented IBL techniques to some degree and about half of these achieved full implementation. This self-reported implementation rate is even more encouraging, given considerable changes in practice indicated by respondents on the follow-up survey and corroborated in the interviews. The shifts in practice indicate that workshops have an impact not only on participants' learning, but also on their application of methods learned.

Session 2 – Preliminary Reports
Thursday, 1:40 PM – 2:10 PM

JOHN STEINBACH

Using Community College Students' Understanding of a Trigonometric Statement to Study Their Instructors' Practical Rationality in Teaching

Vilma Mesa, University of Michigan
Elaine Lande, University of Michigan
Tim Whittemore, University of Michigan

This preliminary research report documents work in progress from a study that seeks to generate conversations about instruction among community college mathematics faculty through the use of students' understanding of mathematical notions. Students' answers to one set of tasks were used to prompt discussions among two community college faculty. We describe the task, the students' responses, teachers' anticipations of students' difficulties, and their reactions and interpretations of students' understanding of the task. The activity provides insights into the nature of the community college mathematics curriculum and the demands that it imposes on teachers and students when pressures for increasing transfer rates are high. As a preliminary research report, we seek guidance from the audience on furthering the analyses of teachers' data.

ERIC HAUSER

Raising Calculus Students' Calculus Understandings to the Surface in Multivariable Calculus

Aaron Wangberg, Winona State University

This presentation will share initial observations and data collected from a pilot classroom study in which groups of multivariable calculus students made physical measurements and drew actual curves on real, tangible surfaces to construct geometric mathematical objects fundamental to the course and discover their properties. Students completed short group activities focusing on relationships between functions and level curves, properties of gradient vectors and directional derivatives, and solutions to optimization problems before other symbolic representations or procedures were discussed in lecture. Initial self-reported data suggests working with the surfaces helped students visualize functions. It also appears the activities helped students develop strong connections between the geometric, symbolic, and verbal representations of multivariable calculus concepts. Collected data suggests the surfaces helped uncover students' single variable misconceptions which hindered their new understandings. The goal of this presentation is to receive feedback for the design of a rigorous phase two study of this project.

MARSHALL JOFFRE

A Guided Reinvention of the Definitions of Ring, Integral Domain, and Field

John Paul Cook, University of Oklahoma

Within the last decade there has been a sizable increase in studies which develop innovative methods of teaching abstract algebra. In particular, the work of Larsen (2004, 2009) develops instructional theories to promote student reinvention of basic group theory concepts such as group, group isomorphism, and quotient group. Despite this increase in creative approaches to the subject, nearly all of the relevant literature pertaining to the teaching and learning of abstract algebra is confined to group theory, leaving ring and field theory relatively untouched. To this end, this talk presents some preliminary results from a study which creates and evaluates an original approach to teaching ring and field theory by using the ideas of Realistic Mathematics Education. In particular, I present preliminary results from a developmental research project designed to test and revise an instructional theory which supports the guided reinvention of the definitions of ring, integral domain, and field.

CHIEF POKER JIM

Learning to Play Projective Geometry: An Embodied Approach to Undergraduate Geometry Learning

Ricardo Nemirovsky, San Diego State University
Brooke Ernest, San Diego State University/University of California-San Diego
Molly Kelton, San Diego State University/University of California-San Diego

Research is beginning to investigate the role of embodiment in undergraduate mathematics education. This study builds on this intellectual trend by developing embodied theories of tool use in the context of an undergraduate geometry course. During this course, students explored the mathematics of projective geometry through (a) a designed object called Alberti's Window, (b) a Geometer's Sketchpad sketch, and (c) an art project inspired by the geometry of linear projection. This preliminary report relays early analyses of classroom and interview video recordings. The research team is analyzing the data from the emerging theoretical construct of a mathematical instrument, a material or semiotic object together with a diverse collection of perceptuomotor activities involved with its use. We explore how students come to coordinate multiple mathematical instruments in order to 'play' projective geometry. Audience discussion will address questions related to perception, embodiment, and mathematical art in relation to undergraduate geometry education.

ROY YATES

Improving Undergraduate Novice Proof-Writing: Investigating the Use of Multiple Drafts

Sharon Strickland, Texas State University
Betseygail Rand, Texas Lutheran University

Because proof-writing involves both understanding mathematical ideas related to the theorem, as well as structural norms of formal proofs, we hypothesized that students could improve both content and structure of their proofs using the drafting techniques common to English Composition research. Our research question is, "Does proof revision lead to improved proof-writing skills?" The intervention group revised their proofs and turned in up to three drafts of each formal proof. This pilot led to the development of a coding tool to categorize the types of student individual errors. In this proposal, we share the coding tool as well as the ongoing analysis of two sets of Linear Algebra student proofs. Preliminary results suggest the drafting group engaged with the work more often than the control group during the semester and on the final, while the control students were more likely to skip proofs rather than attempt them.

SAM HILL

A Longitudinal Study of Mathematics Graduate Teaching Assistants' Beliefs about the Nature of Mathematics and their Pedagogical Approaches toward Teaching Mathematics

Debasree Raychaudhuri, California State University-Los Angeles

The purpose of this research study was to explore mathematics graduate teaching assistants' (GTA) beliefs about the nature of mathematics, their pedagogical approaches toward teaching mathematics and how these evolve over a span of a year. The GTAs participated in four open-ended interviews designed around the planning, performing and assessing framework of Speer and Kung (2009). Our preliminary analyses revealed hierarchical stages of GTA knowledge of their students as well as a separation between their ontological and pedagogical stances.

Session 3 – Contributed Reports
Thursday, 2:20 PM – 2:50 PM

JOHN STEINBACH

The Search for the Normative Identity in a College Algebra Class

Alexandria Theakston, Michigan State University

This paper presents results from a qualitative study examining talk about classroom norms from two students and the instructor from a single college algebra classroom. Three cases are presented and interpreted using an interpretive scheme developed by Cobb & Hodge (2007). This study considers the following questions: (a) How do participants talk about the normative identity? and (b) How do participants talk about themselves with respect to the normative identity? Results include participants' observations, interpretations, and evaluations of students' general and specifically mathematical obligations. Results indicate that participants were aware of classroom norms and were able to describe several students' obligations in rich detail. Further, participants described their roles in negotiating these classroom norms, illuminating issues of authority and agency for all participants. Taken together, the results provide unique insights into classroom activity and culture, which have implications for future work on identity and for teaching at the collegiate level.

CHIEF POKER JIM

Making Jumps: An Exploration of Students' Difficulties Interpreting Indirect Proofs

Stacy Brown, Pitzer College

This paper reports findings from an exploratory study that examined undergraduate mathematics students' proof preferences, as they relate to indirect proof. While many have suggested that undergraduate students dislike and have metatheoretical difficulties with indirect proof, findings from 15 proof-preference surveys provide evidence that students' preferences are in fact more nuanced that previously anticipated and that in certain contexts students' prefer indirect proofs. Building on this work, 5 clinical one-hour interviews were conducted with a sample of survey participants, so as to better explain the discrepancy between these findings and those of other researchers. Data from the clinical interviews with advanced students are used to show: (1) aspects of indirect proofs, which others have argued are the root of students' dislike, were not salient to the students, and (2) students' difficulties recognizing secondary statements, rather than their metatheoretical difficulties, may account for their dislike of indirect proof in the form of proof by contradiction, but not of indirect proofs in the form of proof by contraposition.

ERIC HAUSER

A Characterization of Calculus I Final Exams in U.S. Colleges and Universities

Michael Tallman, Arizona State University
Marilyn Carlson, Arizona State University

The final exam in a mathematics course is one source of information about the nature and level of student learning that is expected in the course. In this study, a three-dimensional framework was developed to analyze post- secondary calculus I final exams in an effort to determine the skills and understandings that are currently being emphasized in college calculus. Results indicate that Calculus I final exams generally require low levels of cognitive demand, seldom contain problems stated in a real-world context, rarely elicit explanation, and do not require students to demonstrate or apply their understanding of the course's central ideas. Data from a survey that investigated instructors' beliefs about teaching, the role of exams and homework in learning, etc. was completed by the same instructors and was used to investigate instructor beliefs that correlate with exams that are more and less conceptual in their focus. We found that there is a misalignment between the nature of calculus final exams and instructors' perceptions of their exams relative to the extent to which students are asked to explain their thinking and the proportion of exam items that focus on skills and methods for carrying out computations.

Session 4 – Preliminary Reports
Thursday, 3:30 PM – 4:00 PM

MARSHALL JOFFRE

The Effects of Three Homework Systems on Student Learning in Intermediate Algebra: A Comparative Study

Jerome Trouba, Ferris State University

Online homework systems are designed to engage students with course topics while providing immediate feedback. Though other studies have evaluated ALEKS (Assessment and LEarning and Knowledge Spaces) as an online homework tool, there is a lack of research comparing it with other homework systems. Additionally, ALEKS is typically used alongside other classroom activities. This study seeks to add to the research examining ALEKS by investigating the performance of students taking Intermediate Algebra using either WebAssign, ALEKS, or traditional homework and by using ALEKS as the sole method of instruction. Each instructor participating in the study taught at least one course with each homework system. Preliminary results indicate no significant difference in student learning between students using WebAssign or traditional homework. ALEKS data is currently being collected and suggests students are developing a thorough understanding of specific Intermediate Algebra topics.

CHIEF POKER JIM

Instructional Influence on Student Understanding of Infinite Series

Brian Lindaman, Montana State University

Many studies have documented the nature of student conceptions for various topics in college calculus. Of interest in this study are the sources of these understandings. In particular, the instructor's discourse seems to significantly impact students' views and conceptualizations of a topic. The nature and scope of this influence, on a particularly troublesome topic, infinite series, is the object of study in this research. Research questions:

1. What are the sources of students' misconceptions about infinite series?
2. Are there student misconceptions about infinite series which arise from the classroom discourse in college calculus?

The data collected consisted of survey responses, transcripts from interviews, and videotapes of instruction. Student conceptions regarding the convergence of a series agreed in key areas, and evidence indicated that this understanding was fostered during classroom discourse. Further study will reveal the extent to which the instruction influenced other aspects of students' conceptions of infinite series.

JOHN STEINBACH

Factors influencing students' propensity for semantic and syntactic reasoning in proof writing: A case study

Juan Pablo Mejia-Ramos, Rutgers University
Keith Weber, Rutgers University
Evan Fuller, Montclair State University
Aron Samkoff, Rutgers University

We present a case study of an individual student who consistently uses semantic reasoning to write proofs in calculus but infrequently uses semantic reasoning to write proofs in linear algebra. The differences in these reasoning styles can be partially attributed to his familiarity with the content, the teaching styles of the professors who taught him, and the time he was given to complete the tasks. These results suggest that there are factors, including domain, instruction, and methodological constraints, that researchers should consider when ascribing to students a proving style that have been ignored in previous research.

ERIC HAUSER

Examining Students' Mathematical Transition Between Secondary School and University – The Case of Linear Independence and Dependence

Natalie Selinski, Universität Kassel

To understand the mathematical transition students make between secondary school and the university requires an in-depth look at the mathematical topics students learn at the time of this transition and the contextual, institutional changes that simultaneously occur. This preliminary presentation explores how linear algebra students at both the secondary school and university in Germany understand vectors and linear independence and dependence in the course of video-recorded, think-aloud problem-solving interviews. Analysis of these interviews indicate not only differences in mathematical content and sophistication between secondary school and university students, but also in students' disposition, particularly towards new mathematical experiences. A look at more informal data about the various institutional environments, secondary school and university, provides a potential reason for these differences. This report concludes with a discussion on how to create a blended analysis of these individual understandings and dispositions and their relationship with the institutional context as a better means of understanding the transition to university-level mathematics.

ROY YATES

Exploring success of underrepresented groups in university mathematics courses

Angie Hodge, University of Nebraska-Omaha
Christina Weber, North Dakota State University

Although some research indicates that the number of women in science, technology, engineering, and mathematics disciplines have been growing (Astin et al., 1983), women and other minorities in mathematics classrooms that serve these disciplines are still largely absent (Pattatucci, 1998). Given the lack of women and minorities in the classroom, how can instructors develop equity and quality in mathematics programs and fields where mathematics acts as a gatekeeper? By utilizing data collected in a differential equations course, we engage in a discussion that explores what leads to students' success in mathematics. We were interested in the interrelationship between students' demographic backgrounds and classroom dynamics to see how we can better serve women, minorities, and those from rural and first generation university backgrounds.

SAM HILL

Investigating the Teaching Practices of Professors When Presenting Proofs: The Use of Examples

Melissa Mills, Oklahoma State University

This study uses ethnographic methods to investigate the teaching practices of mathematics faculty members when presenting proofs in class. Four case studies of faculty members at a large research institution who are teaching in different mathematics content areas are used to describe the ways in which examples are used in proof presentations in upper-division proof-based undergraduate mathematics courses.

Session 5 – Contributed Reports
Thursday, 4:10 PM – 4:40 PM

JOHN STEINBACH

Reinvention Six Months Later: The Case of Megan

Jason Martin, University of Central Arkansas
Michael Oehrtman, University of Northern Colorado
Craig Swinyard, University of Portland
Beth Cory, Sam Houston State University

This research is a part of a larger project to gain insights into how calculus students might come to understand formal limit definitions. For this study, a pair of students participated in a guided reinvention teaching experiment where they had to wrestle with and resolve many problems in creating a formal sequence convergence definition. Six months after the initial teaching experiment, the students returned for individual interviews in which they were both asked to reproduce their sequence convergence definition. In this paper, we highlight one student's activities to recreate this definition. The definition was not immediately recalled, but instead, particular phrases and relationships were remembered. Furthermore, problems were re-engaged, and we claim that because of prior experiences with these problems, solutions were more readily available and reconstruction was thereby quickened.

ERIC HAUSER

Providing answers to a question that was not asked.

Egan Chernoff, University of Saskatchewan

The purpose of this article is to contribute to research on teachers' conceptions of probability. To meet this objective, prospective mathematics teachers were presented two different answer keys to a 10 question multiple-choice quiz and were asked to determine and justify which of the two answer keys was least likely to occur. This article utilizes the attribute substitution model (Kahneman & Frederick, 2002) to account for certain normatively incorrect responses from prospective teachers. This new perspective provides evidence that certain individuals, when presented a particular question, answer a different question instead. Results demonstrate that participants substitute a variety of heuristic attributes instead of making the intended relative likelihood comparison of the answer keys presented. Through recognizing that there is more than one particular candidate for the role of heuristic attribute, results further demonstrate that certain participants substitute more than one heuristic attribute in their response justifications.

CHIEF POKER JIM

Dynamic Geometric Representation of Eigenvectors

Shiva Gol Tabaghi, Simon Fraser University

This study summarizes the five participants' exploration of dynamic representations of an eigenvector of a 2×2 matrix associated with a negative eigenvalue. I drew on the complementary use of the different theoretical constructs to study the role of an instrument of semiotic mediation in learners' developmental process of mathematical understanding. My data analysis reveals that the participants' use of the dragging tool (which became an instrument of semiotic mediation) enabled them to understand properties of eigenvectors and eigenvalues. It also suggests an integration of analysis of participants' gesture and speech to provide an insight of the participant's modes of thinking.

Session 6 – Preliminary Reports
Thursday, 4:50 PM – 5:20 PM

ERIC HAUSER

Opportunities to Develop Understanding of Calculus: A Framework for Analyzing Homework Exercises

Rachael Todd, University of Delaware

The purpose of this study was to construct and apply a framework to examine opportunities to understand calculus deeply, as informed by prior research. I applied this framework to analyze opportunities to learn derivatives in two calculus texts: Hughes-Hallett et al. (2009) and Stewart (2012). These tests were chosen to represent different points on a continuum between conventional and reform calculus materials. An analysis of both texts suggests that they are more similar than might be expected with respect to the amount of context given in problems, their attention to position, velocity and acceleration, and opportunities to use multiple representations – algebraic, numeric, graphical, and descriptive. There were differences between graphical and descriptive problems between texts and opportunities to make connections between representations. The framework presented here illuminated degrees of variation and similarity between opportunities to understand calculus in these texts and could have further utility for examining additional calculus texts.

MARSHALL JOFFRE

A First Look at How Mathematicians Read Mathematics for Understanding

Mary Shepherd, Northwest Missouri State University

As students progress through the college mathematics curriculum, enter graduate school and eventually become practicing mathematicians, reading mathematics textbooks and journal articles appears to comes easier and these readers appear to gain quite a bit from reading mathematics. Previous research has focused on what early college students do as they read and the difficulties they encounter that interfere with understanding what has been read. This preliminary study was designed to help us begin to understand how more advanced readers of mathematics read for understanding. Four faculty members and four graduate students participated in this study and read from a first year graduate textbook in an area of mathematics unfamiliar to each of them. The reading methods of the faculty level mathematicians were all quite similar and were markedly different from all the students the researcher has encountered so far, including the more advanced students in this study.

CHIEF POKER JIM

For Educational Color Work: Diagrams in Geometry Proofs

Allison Toney, University of North Carolina-Wilmington
Kelli Slaten, University of North Carolina-Wilmington
Elisabeth Peters, University of North Carolina-Wilmington

Historically grounded in Oliver Byrne's reworking of Euclid's Elements, and based on a student-generated proof, we investigate the use of coloring to enhance geometry proofs. Charlotte Knight, an undergraduate mathematics major enrolled in Modern Geometry, regularly employed coloring techniques as a tool in her proof-writing. We met for a single semi-structured, task-based interview to discuss Charlotte's use of coloring in her organization and understanding of geometry proofs. Preliminary results indicate that Charlotte's use of diagrams is closely related to her construction of a proof. In particular, her use of color serves several purposes: (1) as an organizational tool to connect her diagrams to the content of her proofs, (2) to enhance her understanding of the proof she is writing, and (3) to illustrate relationships within her diagrams and proofs. We feel this small study has particularly interesting pedagogical implications.

JOHN STEINBACH

Articulating Students' Intellectual Needs: A Case of Axiomatizing

Mark Yannotta, Clackamas Community College

This study uses qualitative methods to investigate how students' intellectual needs were articulated in an inquiry-based mathematics bridge course. A term-long teaching experiment was conducted in the bridge course using an RME-inspired curriculum for abstract algebra (Larsen, 2004). In addition to examining proof, the teacher-researcher encouraged progressive mathematizing to help orient the students toward more advanced mathematics. In a retrospective analysis of the teaching experiment, students' acts of mathematizing were examined and correlated with Harel's (2011) categories of intellectual need. A preliminary analysis of the data suggests that the act of axiomatizing a mathematical system—in this case, a group—can provide students with many opportunities to articulate and address a variety of intellectual needs.

ROY YATES

Calculus Student Understandings of Division and Rate

Cameron Byerley, Arizona State University
Neil Hatfield, Arizona State University
Patrick Thompson, Arizona State University

We have conducted a preliminary investigation of university Calculus students' conceptions of division and rate of change because these ideas are used to define the derivative. We conducted exploratory interviews focused on building models of student understandings of division and rate. Retrospective analysis revealed the students interviewed had a variety of meanings for these concepts. Difficulty thinking about division as multiplicative comparisons of relative size was observed in multiple students. Additionally a student who explained rate as an amount added in equivalent x-intervals struggled to determine if a quantity was changing at a constant rate over unequally spaced x intervals. We hypothesize that difficulty conceptualizing division as quotient, and quotient as a measure of relative size of two quantities, obstructs students' understandings of average and instantaneous rate of change. This research will further our goal of understanding student difficulties with derivatives.

SAM HILL

Student Understanding of Integration When Applied to Finding Volumes of Solids

Krista Toth, West Virginia University
Vicki Sealey, West Virginia University

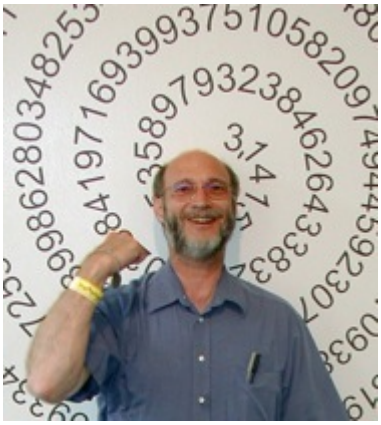
Past research has shown that students struggle when solving definite integral application problems, but little has been done to examine the sources of their difficulties. This study aims to more thoroughly examine student misconceptions about definite integrals and develop new curricula to address these issues. Participants were second-semester calculus students enrolled in a large, public university. Exam problems required students to sketch approximating slices of given solids, and set up a corresponding volume integral. Students' written work was analyzed for common mistakes and misconceptions. Although some students solved the problems correctly, a majority exhibited major deficiencies in their understanding of how to apply the definite integral. Most surprising was students' widespread failure to make a connection between the sketch and the set up of the integral. Further research is currently under way that aims to expose sources of students' faulty thought processes when using definite integrals to solve volume problems.

Plenary Talk
Thursday, 6:30 PM – 8:00 PM

COLONEL LINDBERG BALLROOM

The Challenges of Observation

Dr. Alan Schoenfeld
University of California at Berkeley



Suppose you think certain classroom actions lead to improved student understanding. How would you go about examining that hypothesis? Well, you'd build (or, preferably, steal) a measure that captures the frequency of the things you think support learning; you'd build (or, preferably, steal) a measure that captures the depth of student understanding; you'd sample lots of classrooms, and see if the correlation between "lots of good classroom things" and "doing well mathematically" holds.

Simple, right? Not in practice. Despite the fact that I have some pretty good ideas about teachers' decision making and how to model it, what counts in classroom activities, and student assessment, the closer we looked the more complicated things got. The purpose of this talk is to unravel the complexities and show just why such "simple" ideas are a real challenge.

Friday, February 24

Morning Session

7:30 AM – 8:30 AM Breakfast – Arcadian Garden Registration – Near John Steinbach Room
8:30 AM – 9:00 AM Session 7 – Preliminary Reports
9:10 AM – 9:40 AM Session 8 – Contributed Reports
9:50 AM – 10:20 AM Session 9 – Preliminary Reports
10:20 AM – 10:50 AM Coffee Break
10:50 AM – 11:20 AM Session 10 – Contributed Reports
11:30 AM – 12:00 PM Session 11 – Preliminary Reports
12:00 PM – 1:00 PM Lunch Colonel Lindberg Ballroom

Afternoon Session

1:00 PM – 2:00 PM Special Session *Sharing the Teaching Commons: SoTL and RUME* *Jacqueline Dewar* *Loyola Marymount University* Colonel Lindberg Ballroom
2:10 PM – 2:40 PM Session 12 – Contributed Reports
2:40 PM – 3:10 PM Coffee Break
3:10 PM – 3:40 PM Session 13 – Contributed Reports
3:50 PM – 4:20 PM Session 14 – Preliminary Reports
4:30 PM – 5:00 PM Session 15 – Contributed Reports
5:15 PM – 6:15 PM Plenary Talk *The engineers in your math classes: What are they thinking?* *Cynthia Atman* *University of Washington* Colonel Lindberg Ballroom
6:30 PM – 7:30 PM Embassy Suites Complementary Happy Hour – Arcadian Garden
7:30 PM Dinner On Your Own

Session 7 – Preliminary Reports
Friday, 8:30 AM – 9:00 AM

ERIC HAUSER

Genetic Decomposition of Integration

May Hamdan, Lebanese American University

In this paper I present a theoretical analysis (genetic decomposition) in the sense of APOS theory, of the cognitive constructions for the concept of infinite Riemann sums and the Fundamental Theorems of Calculus as a linking tool between the derivative and the integral, following Piaget's model of epistemology. This genetic decomposition is primarily based on my own mathematical knowledge as well as on my personal continual observations of students in the process of studying integration. I also present empirical data in the form of informal interviews with students at different stages of learning. The analysis of those interviews will later suggest a review of the initial genetic decomposition. Based on this analysis I also suggest instructional procedures that motivate the mental activities described in the proposed genetic decomposition. This study will shed new lights on the concept and make the connections more obvious between two key concepts in calculus.

MARSHALL JOFFRE

Assessing Proof Schemes: An Interesting "Proof" By Mathematical Induction

Madeleine Jetter, California State University-San Bernardino

Students face an array of difficulties when they learn to understand and write proofs by mathematical induction (MI). This paper describes the responses of students in an inquiry-based (IBL) number theory course when presented with a false proof by MI asserting that all humans are the same height. We use the theory of proof schemes developed by Harel, Sowder and coworkers as a lens through which to analyze student responses. Some were consistent with the misconceptions already in the literature on MI, while others may be especially revealing of IBL students' ways of understanding mathematical proof in general and MI in particular.

JOHN STEINBACH

First Semester Calculus Students' Understanding of the Intermediate Value Theorem

Vicki Sealey, West Virginia University
Jessica Deshler, West Virginia University
Krista Toth, West Virginia University

In our calculus courses, we often see students perform poorly on problems involving the Intermediate Value Theorem (IVT), despite being a fairly basic concept. Thus, we designed a study to analyze students' conceptual understanding of the IVT and their ability to express the theorem in their own words. Two groups of students were video-taped while working on an activity designed to guide their construction of an initial understanding of the IVT, and fifty-four students were later asked to state the IVT in their own words. Both video data and student responses on the written work were analyzed to identify common themes. It was found that even though students were able to understand the concepts behind the Intermediate Value Theorem, they were unable to correctly describe the IVT in their own words, largely due to confusing the independent and dependent variables and issues with the if/then structure in a theorem.

CHIEF POKER JIM

An Evolving Visual Image of Approximation with Taylor Series – A Case Study

Danielle Champney, University of California-Berkeley
Eric Kuo, University of Maryland-College Park

This paper will take a close look at the construction of a graphical image for reasoning with approximation in the context of Taylor series. In particular, it is a comprehensive case study of the genesis and evolution of an image created by one student, who draws extensively on other images and knowledge from calculus and physics to supplement gaps in his understanding of Taylor series and reason with Taylor series approximation tasks. His process resulted in a graphical representation that was leveraged to build knowledge and reason with the situation, even while lacking key considerations that are central to an understanding of Taylor series. The paper will speak to considerations both of students' understanding of particular content, as well as a detailed examination of the processes of constructing a visual image used for problem solving and obtaining and utilizing evidence to amend that visual image.

ROY YATES

Improving Student Success in Developmental Algebra and Its Impact on Subsequent Mathematics Courses

John Mayer, University of Alabama-Birmingham
William Bond, University of Alabama-Birmingham

One direction taken by course reform over the past few years has been the use of computer-assisted instruction, often applied to large-enrollment service courses in mathematics, and justified in part by cost-effectiveness. Elementary algebra is typically taken by undergraduate students who do not place into a credit course. The goal of such a developmental algebra course has been to enhance students' "algebra skills," for example, dealing procedurally with rational expressions. Higher-order thinking may be largely absent from such an approach. Our motivating question is "What approach maximizes the student's chance to succeed in subsequent courses?" In view of our theoretical perspective that an inquiry-based approach enhances learning, a subsidiary question is "Is it effective to blend a focus on skills development with a focus on problem-solving?" Results of the analysis, not yet complete, suggest that effectiveness is a matter of what student outcomes are valued, balanced against cost-effectiveness.

SAM HILL

The Role of Technology in Constructing Collaborative Learning Spaces

Brian Fisher, Pepperdine University
Timothy Lucas, Pepperdine University

Traditionally, research on technology in mathematics education focuses on interactions between the user and the technology, but little is known is about how technology can facilitate interaction among students. In this preliminary report we will explore the role that iPads versus traditional laptops play in shaping the learning spaces in which students explore concepts in business calculus. We will report on classroom observations and a series of small-group interviews in which students explore the concepts of local and global extrema. Our preliminary results are that the introducing the iPad, a portable device with intuitive applications, enhances collaboration by allowing students to transition back and forth from private to public learning spaces.

Session 8 – Contributed Reports
Friday, 9:10 AM – 9:40 AM

ERIC HAUSER

Teaching Eigenvalues and Eigenvectors with a Modeling Approach

Hilda Salgado, Instituto Tecnhologico Autonomo de Mexico (ITAM)
Maria Trigueros, Instituto Tecnhologico Autonomo de Mexico (ITAM)

This investigation reports a classroom experience in which eigenvalues, eigenvectors, and eigenspaces were taught using Models and Modeling and APOS Theory. Models and Modeling was used to design a problem in a realistic context and, with the genetic decomposition of APOS Theory, activities were designed to help students make the mental constructions needed to have a better understanding. The research question is: Is it possible for students to construct an object conception of eigenvalues, eigenvectors, and eigenspaces when they are taught using a didactical design based on Models and Modeling and APOS Theory? Using one team as a case study we describe work done by students and show that two students were able to construct an object conception of these concepts. We also show that both theoretical frameworks can be used in an integrated way and students learned the mathematical concepts in a more meaningful way.

JOHN STEINBACH

Experts' Reification of Complex Variables Concepts: The Role of Metaphor

Hortensia Soto-Johnson, University of Northern Colorado
Michael Oehrtman, University of Northern Colorado
Kristin Noblet, University of Northern Colorado
Lee Roberson, University of Northern Colorado
Sarah Rozner, University of Northern Colorado

Using a theoretical perspective of embodied cognition, we explored how six experts integrated metaphors to reason and communicate about arithmetic and analytic complex variables concepts. We found that experts who displayed evidence of reification of a complex variables concept or had a need to use a concept imparted their sense of understanding through dynamic representations blended with metaphors. These metaphors were often invented or reinterpreted, based on personal experiences and created to convey nuances of the experts' understanding to students. The experts appeared conscientious of using metaphors relevant to their own students. This research may support practitioners' efforts to create opportunities for students to reinterpret experts' metaphors into personally meaningful metaphors that both capture important mathematical concepts accurately and align with their own understandings, experiences, and culture. Further research may investigate how technology may serve as a tool for such an endeavor.

CHIEF POKER JIM

Identifying Developmental Students Who are At- Risk: An Intervention Using Computer- Assisted Instruction at a Large Urban Community College

Claire Wladis, City University of New York
Michael George, City University of New York
Kathleen Offenholley, City University of New York

Developmental mathematics courses can have completion rates as low as 25%, which can be a major barrier to degree completion. The primary motivation for the interventions described in this study was to increase student motivation levels and sense of self-efficacy in the developmental mathematics course sequence, with the aim of improving retention. Changes to the developmental mathematics course structure included: using a mandatory departmental midterm to identify at-risk students, and implementing a series of required intervention assignments using computer-assisted instruction in conjunction with regular class time for those students identified as at-risk. Significant gains in retention rates were obtained, with retention in some semesters as high as 50% greater than the semester prior to the intervention. In addition, in this study, at-risk students who spent at least twenty hours on intervention assignments obtained retention rates that were approximately twenty-two percentage points higher than the average remedial student.

Session 9 – Preliminary Reports
Friday, 9:50 AM – 10:20 AM

ERIC HAUSER

Operational and Structural Reasoning with Linear Transformations

Jessica Ellis, San Diego State University/University of California-San Diego
Francesca Henderson, San Diego State University
Chris Rasmussen, San Diego State University
Michelle Zandieh, Arizona State University

In this report we detail linear algebra students' interpretation of linear transformations as a core area of any linear algebra course. Data for this analysis comes from mid-semester, semi-structured problem solving interviews with 13 undergraduate students in linear algebra. We identified two main categories for student reasoning students in completing three tasks: 1) students who used structural reasoning with entries of the matrix, columns of the matrix, and orientation of the shape and 2) students who used operational reasoning through matrix and vector multiplication. Our preliminary results and initial analysis will examine how students employed various modes of reasoning in their solutions and how these tasks encouraged a transition from informal to formal reasoning, as well as concept development.

MARSHALL JOFFRE

Mathematical Activity for Teaching

Estrella Johnson, Portland State University

This work aims to establish a new theoretical construct, mathematical activity for teaching, and to investigate relationships between students' mathematical activity, mathematical activity for teaching, and teacher's instructional moves. Mathematical activity for teaching refers to the mathematical activity teachers engage in as they work to support students' mathematical activity. This construct represents one component of the Mathematical Activity for Teaching Cycle, a conceptual framework that guided my analysis of classroom interactions between mathematicians and their abstract algebra students. Through this analysis I was able to exemplify each component of the Mathematical Activity for Teaching framework and begin to identify relationships between teachers' mathematical activity and those of their students'.

CHIEF POKER JIM

What's the big idea?: Mathematicians' and undergraduates' proof summaries

Aron Samkoff, Rutgers University
Keith Weber, Rutgers University
Juan Pablo Mejia-Ramos, Rutgers University

In this study, seven mathematicians and seven undergraduates were asked to read and summarize mathematical proofs that they read to investigate which ideas they consider to be important in a proof. Mathematicians' ideas were generally a) important equations, theorems or facts used in the proof, b) general methods used in the proof, c) diagrams or graphs, or d) overarching goals of the proof. Additionally, mathematicians and students sometimes included details or computations in their summaries that were unfamiliar, subtle, or not routine for them.

ROY YATES

Challenges and Tools in the Facilitation of Combined Professional Development and Research Sessions: The Case of Community College Trigonometry Instructors

Wendy Aaron, University of Michigan
Vilma Mesa, University of Michigan
Patricio Herbst, University of Michigan

We report on a preliminary analysis of conversations among community college mathematics faculty proposed with both professional development and research goals in mind, which used various artifacts representing teaching. The ultimate goal is to propose a framework for facilitating productive conversations, that is conversations that fulfill both professional development and research goals. We describe one set of tools, those associated with the questioning practices of the facilitator, and illustrate how those tools can overcome challenges to productive conversations.

SAM HILL

What do Students Do in Self-formed Mathematics Study Groups?

Gillian Galle, University of New Hampshire

While it is widely taken as understood that students should be spending additional time outside of the typical undergraduate mathematics classroom studying, little is known about how students spend that study time. Currently available research has investigated how much time students spend outside of the classroom studying, whether they work alone or with others, and what materials students keep on hand while studying. However all of these studies rely on self-reported data in the form of interviews or anonymous surveys. This ethnographic study undertakes to expand our understanding of what activities students are engaged in when they say that they are "studying" through direct observation, journal entries, and interviews. Particular attention is given to how students study together in groups and how students make use of the materials they bring with them for studying purposes.

JOHN STEINBACH

Learning trajectories and formative assessment in first semester calculus

Rebecca Dibbs, University of Northern Colorado
Michael Oehrtman, University of Northern Colorado

While research suggests formative assessment, assignments given for feedback rather than grades, raise student achievement, the literature lacks an explanation for how these assessments affect student learning. The purpose of this study of an introductory calculus class was to investigate how the formative assessment component of design research instruction influenced the learning trajectory for the class. The preliminary analysis of the formative assessments suggested that the assessments appeared to scaffold metacognition, self-reflection, and transfer of the instructional framework between units.

Session 10 – Contributed Reports
Friday, 10:50 AM – 11:20 AM

JOHN STEINBACH

Understanding how precalculus teachers develop mathematic knowledge for teaching the idea of proportionality

Kathryn Underwood, Arizona State University
Marilyn Carlson, Arizona State University

The purpose of this study was to better understand how a precalculus teacher develops mathematical knowledge for teaching the idea of proportionality. We were also interested in understanding what instructional supports might foster shifts in a teacher's thinking, and then how shifts in her thinking affect her classroom decisions. The teacher was using a research-based precalculus curriculum designed to help students acquire improved problem solving abilities, and deeper understandings and connections among the courses central ideas. The findings revealed that the research-based professional development and support tools for teaching precalculus can lead to improvements in the teacher's mathematical content knowledge and aspects of her teaching practice. We also observed that the teacher was still making new mathematical connections during her second year of using the materials. We also observed that shifts in a teacher's content knowledge do not always improve the teacher's ability to leverage her student's thinking during teaching.

CHIEF POKER JIM

Does a Statement of Whether Order Matters in Counting Problems Affect Students' Strategies?

Todd CadwalladerOlsker, California State University-Fullerton
Nicole Engelke, California State University-Fullerton
Scott Annin, California State University-Fullerton
Amanda Henning, California State University-Fullerton

Counting problems ask students to compute the number of ways a certain set of requirements can be satisfied, and they are important in such mathematical subjects as probability, combinatorics, and abstract algebra, among others. Students are often taught to solve counting problems by looking for specific clues to help categorize the problems and identify solution strategies. In this study, we investigate how the wording of certain counting problems, specifically whether or not "order matters"', affects students' solution strategies. In particular, we gave students questions involving explicit statements as to whether or not order matters, some of which were intentionally misleading, and questions that do not contain such an explicit statement. Data was collected in the form of written responses and student interviews. The results show that many students do, in fact, rely heavily on such explicit statements about whether order matters, even when such statements are misleading.

ERIC HAUSER

An Analysis of the Effect of the Online Environment on STEM Student Success

Claire Wladis, City University of New York
Kay Conway, City University of New York
Alyse Hachey, City University of New York

Both online and STEM courses have been shown to have lower student retention; however, there is little research indicating what effect the online environment may have on retention in STEM courses specifically. This study compares retention rates for online and face-to-face STEM and non-STEM courses to determine if the online environment affects STEM courses differently than non-STEM courses. In addition, different subcategories of STEM courses are compared to see if the effects of the online environment are different for different course subtypes. Each online course is matched with the same course taught face-to-face by the same instructor in the same semester to control for possible confounding effects. This study found that retention rates in STEM courses were more negatively impacted by the online environment than in non-STEM courses. In particular, the course types which had significantly lower retention online were lower level STEM courses taken as electives or distributional requirements.

Session 11 – Preliminary Reports
Friday, 11:30 AM – 12:00 PM

CHIEF POKER JIM

Authority dynamics in mathematics discussions

Rebecca Dibbs, University of Northern Colorado
David Glassmeyer, University of Northern Colorado
Michael Oehrtman, University of Northern Colorado
Craig Swinyard, University of Portland
Jason Martin, University of Central Arkansas

We employed grounded theory techniques to examine the evolution and influences of authority relationships in an undergraduate mathematics education research study. Our analysis focused on video data from a five day teaching experiment with two faculty researchers engaging two second-semester calculus students in a guided reinvention of formal limit definitions. We will discuss our model for authority in a mathematical discussion and characterize the patterns, influence and evolution of authority that we identified in the guided reinvention. Finally, we illustrate the need for researchers to be cognizant of authority patterns in group data collection settings, since such patterns can mask individual evidence of knowledge and reasoning.

JOHN STEINBACH

Interculturally Rich Mathematics Pedagogical Content Knowledge for Teacher Leaders

Shandy Hauk, WestEd
Michelle Chamberlin, University of Wyoming
Billy Jackson, St. Xavier University
Nissa Yestness, University of Northern Colorado
Kristin King, University of Denver
Robert Raish, University of Northern Colorado

We report on our work to build a theory about teacher leader development of interculturally aware mathematics pedagogical content knowledge (PCK) that is based on existing and continuing work on classroom PCK and intercultural competence development for teachers. This preliminary report seeks feedback from RUME-goers on two questions: Question 1: How do we identify and capture evidence of what might be called "teacher leader pedagogical content knowledge" in interculturally aware ways? Question 2: What question formats might be productive for eliciting information from teacher leaders about their awareness of/attention to the intercultural aspects of mathematics instruction? ... of mathematics itself?...of teacher leadership? This includes questions for written instruments as well as interview prompts and possible survey items.

ERIC HAUSER

Student note taking behavior in proof-based mathematics classes

Tim Fukawa-Connelly, University of New Hampshire
Aaron Weinberg, Ithaca College
Emilie Wiesner, Ithaca College
Sarah Berube, University of New Hampshire
Kyle Gray, University of New Hampshire

There is a need to explain the relationship between teaching (classroom activities) and learning. This study is one attempt to explore student note-taking as a form of mediation between teaching and learning outcomes. We will adapt the theoretical framework described by Weinberg and Wiesner (2010), who applied ideas of literary criticism to describe factors that impact the ways students read and understand mathematics textbooks. The two concepts we will use are those of the implied reader and reading models. We are investigating student note-taking in the context of a proof-based abstract algebra class that is taught, primarily, via a lecture. We are recording the lectures and creating a set of expert-notes that are then compared with the student notes. We then interview the students to better understand the decisions that they make vis-à-vis note taking and how they "read" the text of a lecture.

MARSHALL JOFFRE

A Case of Bridging Communities: Mathematicians and Preservice Elementary Teachers

Karen Keene, North Carolina State University
William Hall, North Carolina State University

This report focuses on an ongoing project that is developing a calculus course required for all preservice elementary teachers at a large southeastern university. In the process of designing and implementing the new materials, several research-based tasks have been developed, tested and refined. We discuss the results of the implementation and the refined tasks. We specifically focus on the task developed for the introduction and development of students' limit understanding. Preliminary results indicate that students in our classes have difficulty thinking about the big ideas of the calculus, including limit, and that participation in these tasks, although difficult, is providing a venue for preservice elementary teachers to think more like mathematicians and come to view mathematics as more than a set of procedures to be followed. We hypothesize this experience will provide students with a stronger foundation as they begin their careers as elementary educators.

ROY YATES

Reading Comprehension of Series Convergence Proofs in Calculus II

Lisa Mantini, Oklahoma State University
Melissa Mills, Oklahoma State University
Jesse Johnson, Oklahoma State University

This study examines the effect of activities and assessments concerning reading comprehension of series convergence proofs in Calculus II on students' exam performance. Two sections of Calculus II taught during a summer semester were compared. Both sections used traditional lecture methods, and one section was also given reading assignments with open-ended questions and in-class quizzes evaluating reading comprehension. We compare test scores and interview data from the two sections.

SAM HILL

Tackling Teaching: Understanding Commonalities among Chemistry, Mathematics, and Physics Classroom Practices

Samuel Pazicni, University of New Hampshire
Karen Marrongelle, Oregon University System
Warren Christensen, North Dakota State University

Education research in chemistry, mathematics, and physics tends to focus on issues inherent to the discipline, most notably content. At this time, little literature evidence exists that documents fruitful collaborations between education specialists across the STEM disciplines. This work seeks to unite the disciplines by investigating a common task: teaching. This study explores how discipline-specific practices influence the common act of reformed teaching pedagogy with a focus on the use of inquiry. We seek to identify commonalities among classroom teaching practices in these disciplines and contribute to the development of analytical tools to study STEM teaching.

Special Session
Friday, 1:00 PM – 2:00 PM

COLONEL LINDBERG BALLROOM

Sharing the Teaching Commons: SoTL and RUME

Dr. Jacqueline Dewar
Loyola Marymount University

The Scholarship of Teaching and Learning (SoTL) owes its name to Boyer (1990) and can trace its roots back even further (Cross, 1986). SoTL, as an evolving international and multidisciplinary movement, has had many, and sometimes distinctly different, descriptions. SoTL practitioners view the classroom as a site for inquiry. They ask and seek to answer questions about their students' learning both to improve their own classroom practice and also to advance the larger profession of teaching. SoTL converts teaching from "private to community property" (Shulman, 1993) and can assist with important initiatives such as assessment, accountability and student success (Hutchings, Huber & Ciccone, 2011). The boundaries between SoTL and RUME are not always sharply defined. After describing SoTL's history and giving examples of SoTL questions from mathematics, this talk will attempt to illuminate goals SoTL shares with RUME and explore ways that the two communities can jointly tend the "teaching commons" to their mutual benefit and that of their colleagues and students.

Session 12 – Contributed Reports
Friday, 2:10 PM – 2:40 PM

JOHN STEINBACH

The Unit Circle and Unit Conversions

Kevin Moore, University of Georgia
Kevin Laforest, University of Georgia
Hee Jung Kim, University of Georgia

The unit circle is a central concept of trigonometry. Yet, students and teachers are often tied to using right triangles to reason about trigonometric functions with only superficial connections to the unit circle. In an attempt to identify ways to better support student learning in trigonometry, we conducted a teaching experiment investigating two pre-service secondary teachers' notions of the unit circle. Initially, both students had difficulty relating circle contexts to the unit circle. The students' actions suggested that their calculations stemmed from memorized procedures, as opposed to reasoning about quantitative relationships. In an attempt to foster connections between novel circle contexts and the unit circle, we implemented tasks that developed the unit circle as the result of conceptualizing a circle's radius as a unit of measure. We report on the students' progress during these tasks and the subsequent improvements in their ability to apply trigonometric functions to circle contexts.

ERIC HAUSER

Student understanding of statistical symbols

Hyung Kim, University of New Hampshire
Tim Fukawa-Connelly, University of New Hampshire
Samuel Cook, Wheelock College

This study explores the levels of understanding of the symbolic representation system in statistics that students hold, and describes the relation between student understanding of the symbolic system and statistical concepts, that students develop as the result of an introductory undergraduate statistics course. The theory is drawn from the onto-semiotic research tradition and draws upon the notion of semantic function that link representations and concepts and seeks to expand the range of representations considered in exploring students' statistical proficiencies. Results suggest that students experience considerable difficulty in making correct associations between symbols and concepts and describe the relationship as seemingly arbitrary and that, without prompts, they are unlikely to understand either statistics or parameters as quantities which can vary. Finally, this study describes the students' need for particularly robust knowledge of preliminary concepts in order to understand the construct of a sampling distribution.

CHIEF POKER JIM

Teacher Change in the Context of a Proof-Centered Professional Development

Osvaldo Soto, University of California-San Diego

The case study reported here examines the development of proof schemes and teaching practices of one in-service secondary mathematics teacher who participated in an off-site professional development (PD) for two years. Two sources of data were examined: video footage of the teacher doing mathematics at an intensive summer institute and footage of her own classroom teaching. Previously, an analysis of the development of the participant's proof schemes (Harel and Sowder, 1998) was reported, indicating a shift from Empirical to Deductive proof schemes. The current report focuses on the development of one participant's teaching practices during the academic years following each summer institute characterizing the development of: the way the teacher solicited student ideas, handled students' ways of understanding, capitalized on student thinking, and in the type of questions she posed. The report also includes theoretical connections between the development of the teacher's proof schemes, teaching practices and the PD.

Session 13 – Contributed Reports
Friday, 3:10 PM – 3:40 PM

CHIEF POKER JIM

Notion of Reducing Abstraction in Teaching: The Case of Mathematics Instruction

Krishna Subedi, Simon Fraser University

Mathematics is an abstract subject. When teachers plan, one of their most important challenges is to figure out ways of translating abstract concepts into understandable ideas. Reducing Abstraction in Teaching (RAT) is one of the theoretical frameworks that provides a window for looking at how teachers deal with abstraction in teaching. By analysing teaching practices of two of the mathematics teachers in college preparatory course, this paper illustrates various tendencies of teachers dealing with mathematical abstraction. It also exemplifies some instances where 'reducing abstraction' seems to be an effective teaching strategy while in other cases it may go unsupportive for the development of students' mathematical understanding.

JOHN STEINBACH

Student Concept Images of Function and Linear Transformation

Michelle Zandieh, Arizona State University
Jessica Ellis, San Diego State University/University of California-San Diego
Chris Rasmussen, San Diego State University

As part of a larger study of student understanding of concepts in linear algebra, we interviewed 10 university linear algebra students as to their conceptions of functions from high school algebra and linear transformation from their study of linear algebra. Analysis of these results led to a classification of student responses into properties, computations and a series of five interrelated metaphors. We see this classification as providing richness and nuance to existing literature on students' conceptions of function. In addition, we are finding these categories helpful in describing the compatibilities and distinctions in student understanding of function and linear transformation.

ERIC HAUSER

How and why mathematicians read proofs: A qualitative exploratory study and a quantitative confirmatory study

Keith Weber, Rutgers University
Juan Pablo Mejia-Ramos, Rutgers University

In this paper, we report the results of a qualitative study in which we interviewed nine mathematicians about how and why they read the published proofs of their colleagues. We then tested the hypotheses generated from these analyses with a quantitative study with 112 mathematicians. Key results from these studies are (a) mathematicians do not always read a proof to obtain conviction, but usually do so to find techniques that might be useful in their own research, (b) mathematicians use authoritative evidence to gain conviction in the proofs that they read, (c) some mathematicians do not always check every line in a proof, even when they referee, and (d) the consideration of examples was crucial in gaining understanding and conviction.

Session 14 – Preliminary Reports
Friday, 3:50 PM – 4:20 PM

ERIC HAUSER

Student Troubles with Simple Harmonic Motion Models

Gillian Galle, University of New Hampshire

Many studies exist on student difficulty transferring mathematical knowledge to physics, on student understanding of trigonometry, and student ability to create graphical representations of functions. However, there are no studies that exist in the intersection of these issues. This study sought to explore student understanding of simple harmonic motion by examining how their approach to graphing the sine and cosine functions impacted their ability to graph sine and cosine based models of simple harmonic motion. The findings of this study conclude that neither an object perspective or process perspective of the graphical representations of sine and cosine is sufficient for the ability to graph simple harmonic motion modeled based on cosine. There seems to be an element missing, a connection students must make between the changes in input type, that needs to be addressed in order for students to create a graphical representation of a cosine-based simple harmonic motion model.

JOHN STEINBACH

Children's Reasoning about Integers: Video Clips to Share with Preservice Teachers

Ian Whitacre, San Diego State University/University of California-San Diego
Jessica Pierson Bishop, San Diego State University
Mindy Lewis, San Diego State University
Lisa Lamb, San Diego State University
Randolph Philipp, San Diego State University
Bonnie Schappelle, San Diego State University

We have conducted interviews with children using integer-related tasks, and we have identified various ways of reasoning that children bring to bear on these tasks. One product of this work is a collection of compelling video clips. We will share examples of children's reasoning, and the audience will be engaged in discussions of children's reasoning and use of video in instruction. Attendees will receive a free DVD with video clips that can be used with preservice teachers.

MARSHALL JOFFRE

What do mathematicians do when they reach a proving impasse?

Milos Savic, New Mexico State University

I report how two mathematicians came to impasses while constructing proofs on an unfamiliar topic, from a set of notes, alone, and with unlimited time. By an impasse, I mean a period of time during the proving process when a prover feels or recognizes that his or her argument has not been progressing fruitfully and that he or she has no new ideas. What matters is not the length of time but its significance to the prover and his or her awareness thereof. I point out two kinds of actions these mathematicians took to recover from their impasses: one relates directly relates to the ongoing argument, while the other consists of doing something unrelated to the ongoing argument which can be mathematical or non-mathematical. Data were collected using a new technique being developed to capture individuals' autonomous proof constructions on tablet computers in real-time.

CHIEF POKER JIM

A Study of Abstract Algebra Textbooks

Mindy Capaldi, Valparaiso University

This study will use reader-oriented theory and the analysis of example spaces to understand abstract algebra textbooks. Textbooks can lay the foundation for a course, and greatly influence student understanding of the material. Multiple undergraduate abstract algebra texts were studied to investigate potential audiences of the books, the level of detail in explanations, examples, and proofs, and the overall material included in the book. Conclusions were drawn regarding some discrepancies between the intended reader and the actual reader and the appropriateness and differences among example spaces.

ROY YATES

The Use of Dynamic Visualizations Following Reinvention

Beth Cory, Sam Houston State University
Jason Martin, University of Central Arkansas
Michael Oehrtman, University of Northern Colorado
Craig Swinyard, University of Portland

This research is a part of a larger project to gain insights into how calculus students might come to understand formal limit definitions. For this study, a pair of students participated in an eight- day guided reinvention teaching experiment in which they created a formal definition for sequence convergence even though they had not previously received instruction on formal limit definitions. During the reinvention process they identified and coordinated relevant graphical attributes of sequences as they recognized and resolved problems with their emerging definition. For this paper, we detail the ninth day in which the students participated in an activity using The Geometer's Sketchpad where they had to interpret their understandings of sequence convergence on premade manipulate-able dynamic visualizations of sequences. We hypothesize that by using these dynamic visualizations, the definition and their resolutions to problems were reinforced by strengthening their connections between their definition and these visual representations.

SAM HILL

Mathematical Modeling and Engineering Majors

Jennifer Czocher, The Ohio State University

A first course in differential equations for engineers and scientists is intended to introduce the students to key principles and techniques involved in using mathematics as a modeling tool. However, a great many students emerge with only a limited number of analytic techniques that are applicable only to a narrow selection of equations, despite the inclusion of word problems in the curriculum. Previous research into mathematical modeling competencies indicates that the students' difficulties can be traced to coordinating mathematical with physical reasoning. The purpose of this research is to develop tasks for a data collection instrument that will allow for the development of a cognition based model of how such skills grow.

Session 15 – Contributed Reports
Friday, 4:30 PM – 5:00 PM

ERIC HAUSER

Two-Variable Functions: Novice and Expert Shape Thinking

Eric Weber, Arizona State University

This study describes two first-semester calculus students' understandings of functions of two variables in a teaching experiment that focused on thinking about function as the simultaneous variation of quantities. The students' actions, responses, and construction of graphs revealed that one student thought about graphs as a malleable wire and another student considered graphs as the result of tracking the values of covarying quantities, which I characterize as novice and expert shape thinking. In this talk, I outline importance of understanding students' ways of thinking about functions of more than one variable, the methodology used to collect and analyze data. I conclude by discussing the implications of thinking about graphical representation of functions using novice and expert shape thinking.

CHIEF POKER JIM

To Reject or Not to Reject: One Student's Non-Normative Decision Procedure for Testing a Null Hypothesis

Michael McAllister, Arizona State University

The purpose of this study was to gain insight into how engagement with hands-on and computer resampling methods affected a statistically naive student's emergent understandings of statistical inference. In this study, simulation design activities provided a vehicle for engaging a student with the core ideas of hypothesis testing. The results highlight challenges the student experienced in coordinating the components of the logic into a coherent scheme of ideas and sheds light on aspects of engagement which need to be emphasized in order to resolve the inherent conceptual difficulties associated with reasoning that invokes a modes tollens-like argument. Moreover, I report on a heuristic the student used to make his inferential decisions—one that does not produce correct inferences. I've termed this the "similarity heuristic" because of a specific similarity relationship the student would look for and then use as a method for rejecting or not rejecting the hypothesis being tested.

JOHN STEINBACH

Prospective Elementary Teachers' Evolving Meanings for Generalizing, Doing Mathematics and Justifying

Jennifer Szydlik, University of Wisconsin-Oshkosh
Carol Seaman, University of North Carolina-Greensboro

We view the classroom as a culture of mathematizing (Bauersfeld, 1993), and document evolving meanings that prospective elementary teachers gave to their instructor's expectation that they "find general solutions," "do mathematics" and "justify solutions" during a semester-long inquiry-based course. Classroom observations and interviews with student informants suggest that almost three weeks passed before the students in the class gave normative meanings to their instructor's request for general solutions and to her expectation that they do mathematics, and it was not until the eleventh week that students understood that justifying a solution meant providing a mathematical argument that explained why the solution was valid. Furthermore, the data suggest that giving normative meaning to these mathematical activities is prerequisite to success, and that as students came to make sense of generalizing, doing mathematics and justifying, they improved in their abilities to do these activities and they began to see them as valuable.

Plenary Talk
Friday, 5:15 PM – 6:15 PM

COLONEL LINDBERG BALLROOM

The Engineers in Your Math Classes: What Are They Thinking?

Dr. Cynthia Atman
University of Washington

Mathematics is an essential component of engineering and as a result is a fundamental element of engineering education. Engineering students learn mathematics along with many other sets of knowledge and skills as they prepare to enter either the engineering profession or continue in graduate studies. The Academic Pathways Study (APS), part of the NSF-funded Center for the Advancement of Engineering Education, conducted in-depth research on the engineering education experience from the engineering student's perspective. Qualitative and quantitative data were gathered from multiple cohorts of undergraduate engineering students using a multi-method approach including surveys, structured and semi-structured interviews, and written design tasks. In addition, APS researchers used data from the National Survey of Student Engagement to compare with APS results and gain further insight into the engineering learning experience. This talk will present a sampling of findings from different components of the research, focusing on mathematics and design in undergraduate engineering education.

Saturday, February 25

Morning Session

7:30 AM – 8:30 AM Breakfast – Arcadian Garden
8:30 AM – 9:00 AM Session 16 – Contributed Reports
9:10 AM – 9:40 AM Session 17 – Preliminary Reports
9:50AM – 10:20 AM Session 18 – Contributed Reports
10:20 AM – 10:50 AM Coffee Break
10:50 AM – 11:20 AM Session 19 – Contributed Reports
11:30 AM – 12:00 PM Session 20 – Preliminary Reports
12:00 PM – 1:15 PM Lunch Colonel Lindberg Ballroom

Afternoon Session

1:30 PM – 2:30 PM Plenary Talk *Reading Processes and Proof Comprehension* *Lara Alcock* *Mathematics Education Center Loughborough University, UK* Colonel Lindberg Ballroom
2:40 PM – 3:10 PM Session 21 – Contributed Reports
3:10 PM – 3:40 PM Coffee Break
3:50 PM – 4:20 PM Session 22 – Theoretical Reports
4:30 PM – 5:00 PM Session 23 – Contributed Reports
5:10 PM – 5:40 PM Session 24 – Contributed Reports
5:40 PM – 6:30 PM Break Embassy Suites Complimentary Happy Hour – Arcadian Garden
6:30 PM – 9:00 PM Dinner and Awards & Plenary Talk *An Expanded Framing for Characterizing Mathematical Progression* *Dr. Chris Rasmussen* *San Diego State University & Center for Mathematics and Science Education* Colonel Lindberg Ballroom

Session 16 – Contributed Reports
Saturday, 8:30 AM – 9:00 AM

JOHN STEINBACH

Student Learning of Key Concepts in Sequences and Series

Rebecca Schmitz, Michigan Technological University
Harvey Keynes, University of Minnesota

This study examines students' foundational understanding and misconceptions of infinite repeating decimals in a variety of calculus courses and the roles of instruction in improving knowledge. We discuss Tzur and Simon's (2004) Reflection on Activity-Effect Relationships Model for student understanding and its application to the survey used in the current study. We examine a variety of matched questions concerning .777… and .999… and use these results for both performance measures and indicators of higher levels of student cognition. We look at the influence of different instructional practices, which include conceptual discussions and related group works, and their impact on student understanding. Our results indicate that these instructional practices can significantly improve student performance and understanding, even among mathematically talented students.

CHIEF POKER JIM

Small Group Discussion and Student Evaluation of Presented Proofs

Jim Brandt, Southern Utah University
Gretchen Rimmasch, Southern Utah University

Research has demonstrated that proof validation, the process of reading and reflecting on a proof to determine its correctness, is difficult for many mathematics majors. Rather than reading written proofs, however, students' classroom experiences often involve presentations of proofs either by their instructor or by other students. In this study, we explore mathematics majors success in recognizing the validity of presented proofs. Participants watched videotaped proofs either at the beginning or at the end of a transition-to-proof course. After completing an initial evaluation of the proof, students discussed the proof in small groups and then evaluated the proof a second time. The impact of the course and the effect of the small group interaction will be discussed.

ERIC HAUSER

Increasing Student Success in Intermediate Algebra through Collaborative Learning at a Diverse Urban Community College

Claire Wladis, City University of New York
Alla Morgulis, City University of New York

There is evidence that cooperative learning can improve student outcomes, but much of the research has been focused on pre-college mathematics or college calculus-level mathematics and above. This project tests the hypothesis that a change in instructional format from a lecture-based class to one incorporating scripted collaborative discovery-based projects would increase student success in Intermediate Algebra and Trigonometry at a diverse urban community college.

Twelve pairs of experimental and control sections were chosen so that each pair had the same instructor and assignments. Surveys, pre/post-tests, and success rates were used to assess intervention effectiveness. Statistical analysis suggests that the intervention had a significant effect on student success that was contingent upon a suitable period of instructor training and revision of course assignments. Increases in student performance on exams of approximately two-thirds of a letter grade and a thirteen percentage point gain in successful course completion were obtained in experimental sections.

Session 17 – Preliminary Reports
Saturday, 9:10 AM – 9:40 AM

CHIEF POKER JIM

Undergraduate Proof in the Context of Inquiry-Based Learning

Todd Grundmeier, Cal Poly-San Luis Obispo
Alyssa Eubank, Cal Poly-San Luis Obispo
Shawn Garrity, Cal Poly-San Luis Obispo
Alyssa Hamlin, Cal Poly-San Luis Obispo
Dylan Retsek, Cal Poly-San Luis Obispo

This research project explores students' proof abilities in the context of an inquiry-based learning (IBL) approach to teaching an introductory proofs course. IBL is a teaching method that focuses on student discussion and exploration in contrast to lecture based instruction. Data was collected from three sections of an introductory proofs course, which included 70 students total. Data collection included a portfolio from each student, consisting of their work on every proof assigned throughout the course, as well as each student's final exam. Contrary to previously published research relating to courses taught in a more traditional lecture based setting, this data analysis suggests that students developed a strong grasp on how to correctly use definitions and assumptions within the context of their proofs. Results also suggest that within the IBL setting, students generally organized their proofs in an efficient, thoughtful, and logical manner.

ERIC HAUSER

Future Teachers' Views of Mathematics and Intentions for Gender Equity: Are These Carried Forward into Their Own Classrooms?

Jacqueline Dewar, Loyola Marymount University

Previous work indicated that an interdisciplinary mathematics and gender course about women mathematicians and their contributions to the field shifts students' views away from seeing mathematics as the study of numbers and toward a more expert view of what the subject entails. In addition, at the end of the course in the reflective writing portion of a portfolio, future teachers frequently volunteer their intentions to foster gender equity in their own classrooms. This preliminary research report will explore whether, and how, this enriched view of mathematics and the resolve for equity persist and influence the classroom teaching of four former students. It will also seek to determine the particular learning experiences that most contributed to any positive findings. Ethnographic methods, including interviews and classroom observations, will be employed.

MARSHALL JOFFRE

Investigating Instructors' Concerns as They Learn to Teach with Inquiry-Based Learning Methods

Vilma Mesa, University of Michigan
Tim Whittemore, University of Michigan

We report initial findings of a study that seeks to investigate the changing nature of instructors' concerns as they learn to teach mathematics courses using inquiry-based learning approaches. Using year-long data from interviews with faculty and teaching logs, we seek to describe the concerns that instructors have as they teach with this method. Our analysis of pilot data with four instructors new to the method suggests that these concerns are organized into five major themes: Student preparation, motivation, and engagement; Coverage; Rigor; Difficulty of the material; and Student Learning. In addition, the nature of these concerns and their relative frequency seem to suggest that these faculty are preoccupied more with managerial aspects of teaching and less with student learning, which would be consistent with a proposed developmental model of professional expertise in teaching. We seek input on the instrument used to gather the data.

ROY YATES

Inverse, Composition, and Identity: The Case of Function and Linear Transformation

Spencer Bagley, San Diego State University/University of California-San Diego
Chris Rasmussen, San Diego State University
Michelle Zandieh, Arizona State University

In this report we examine linear algebra students' conceptions of inverse and invertibility. In the course of examining data from semi-structured clinical interviews with 10 undergraduate students in a linear algebra class, we noted a proclivity for students to identify 1 as the result of the composition of a function and its inverse. We propose that this may stem from the several meanings of the word "inverse" or the influence of notation from linear algebra. In addition, we examined how students attempted to reconcile their initial incorrect predictions with their later computational results, and found that students who succeeded in this reconciliation made heavy use of what we termed "do-nothing function" ideas. The implications of this work for classroom practice include a possible method to help students develop object conceptions of function, as well as the need to pay more explicit attention to often-backgrounded notational issues.

JOHN STEINBACH

Student Thinking of Function Composition and its Impact on their Ability to Set up the Difference Quotients of the Derivative

Gail Tang, University of Illinois-Chicago

While the concept of limit is troubling to many students, a difficulty that preceded this confusion was observed: students were not able to correctly set up the difference quotients as required in the limit definition of derivative. The purpose of this study is to investigate the cognitive processes involved in setting up the difference quotients and the associated errors. This explanatory case study seeks to explain why these particular errors occur from the perspective of student thinking of function composition. At the end of the study, a framework that aggregates criteria used (by past studies and this study) to assign student membership into a function conception category will be produced in an attempt to move towards a systematic classification of students' cognitive processes. Implications from this study can inform teaching practices by exposing students to expected errors. As observed from the data, this can lead to rich discussions on the concept of function itself.

SAM HILL

Definite Integral: A Starting Point for Reinventing Multiple Advanced Calculus Concepts

Craig Swinyard, University of Portland
Steve Strand, Portland State University

While recent Realistic Mathematics Education (RME) studies have shed light on students' abilities to formalize limit conceptions, it remains to be seen how students might make similar progress with other fundamental advanced calculus concepts like continuity, derivative, and integral at a depth required for success in upper-division courses. To address this gap in the literature, we conducted a fourteen session teaching experiment geared at students' reinvention of the formal definition of definite integral. Our presentation will address the following research questions: 1) Do students' efforts to formalize the concept of definite integral motivate a need (for them) to formalize the notion of convergence/limit?; and, 2) Once students reinvent a formal definition of definite integral, can they use this formalization as a tool for formalizing other advanced calculus concepts? If so, which concepts?

Session 18 – Contributed Reports
Saturday, 9:50 AM – 10:20 AM

JOHN STEINBACH

Challenging Convention: Mathematics Students' Resistance to the Unconventional

Ami Mamolo , York University
Rina Zazkis, Simon Fraser University

This article explores undergraduate mathematics students' responses to tasks that deal with areas, perimeters, volumes and derivatives, and their abilities to transfer appropriate knowledge to a novel and unconventional situation. Specifically we focus on an unconventional use of parameters in familiar formulas and investigate how students react to such a change and whether they proceed to implement a similar one. Our analysis attends to the specific mathematical ideas and connections transferred by our participants. We suggest that considering and accepting the unconventional is part and parcel to an appreciation of the overarching structure of mathematics.

ERIC HAUSER

Calculus beyond the Classroom: Application to a Real-Life Problem Simulated in a Virtual Environment

Olga Shipulina, Simon Fraser University
David Smith, McMaster University

This study concerns the correlation of mathematical knowledge with a corresponding real life object within the theoretical framework of Realistic Mathematics Education. It shows to what extent students, who had almost completed the AP calculus course, were able to apply their knowledge to a real-life situation. By simulating the interactive milieu in the Second Life Virtual Environment (VE), this study explores how students find a 'real-life' optimal path 'practically', and how they then re-invent the corresponding calculus task. The research revealed that one out of ten participants was able to mathematize the problem horizontally and vertically without any guidance. The study instructional design, based on simulation of a real-life situation in VE allowed students to explore mathematical solutions relative to their intuitive findings in VE. By mathematizing their own 'real-life' activities, students connected them with corresponding mathematics at an intuitive level.

CHIEF POKER JIM

Outcomes of Inquiry-Based Learning for Pre-Service Teachers: A Multi-site Study

Sandra Laursen, University of Colorado-Boulder
Marja-Liisa Hassi, University of Colorado-Boulder

Inquiry-based learning (IBL) is argued to be particularly important for pre-service teachers, both to deepen their mathematical knowledge and provide powerful first-hand experience with student-centered teaching and learning. Yet relatively few published studies document outcomes from such courses. We have studied student outcomes—cognitive, affective, and social--and learning processes in three two-term mathematics courses targeted to pre-service elementary and secondary teachers, taught at two universities. The study draws on tests of future teachers' mathematical knowledge for teaching (MKT), pre/post-course surveys of their learning gains, beliefs and attitudes, and interviews with both students and instructors. Of particular interest are test results from the Learning Mathematics for Teaching instrument, developed by Hill, Ball and coworkers. This nationally validated test was developed for assessing growth in-service teachers' MKT but, in this study, successfully documents growth in MKT among future teachers.

Session 19 – Contributed Reports
Saturday, 10:50 AM – 11:20 AM

JOHN STEINBACH

On Mathematicians' Different Standards When Evaluating Elementary Proofs

Matthew Inglis, Loughborough University
Juan Pablo Mejia-Ramos, Rutgers University
Keith Weber, Rutgers University
Lara Alcock, Loughborough University

Many mathematics educators and philosophers of mathematics believe there is an unusually high level of agreement in their evaluations of the validity of a mathematical proof. The data in this paper challenge this assumption. 108 research-active mathematicians were shown an adaptation of a published proof from elementary calculus and were asked to evaluate its validity. 24% of the participants judged the proof to be valid, while 76% judged it invalid. Applied mathematicians were more likely to judge the proof to be valid than pure mathematicians (43% vs. 17%). Participants who judged the proof to be invalid were more confident in their judgments. These findings suggest that different groups of mathematicians may use different standards in evaluating proofs.

ERIC HAUSER

Assessment of College Students' Understanding of the Equals Relation

Gregory Wheeler, Utah State University

Incorrect usages of the equal sign by undergraduate students indicate a tendency for students to comprehend the equal sign as an operator symbol or to ignore the equal sign altogether. This article focuses on college students' understanding of the equals sign and the equals relation, and how that understanding is influenced by the context in which the equals sign is presented. This study indicates that college students often fail to correctly interpret the equals relation and suggests two explanations for these misinterpretations: 1) students fail to recognize the extent of the sameness suggested by an equation, 2) when students focus on solving, evaluating, or coming up with "the answer" they fail to recognize the contribution of the equals sign or other indications of an equals relation in a given context.

CHIEF POKER JIM

The Evolution of Classroom Mathematical Practices in a Mathematics Content Course for Prospective Elementary Teachers

Ian Whitacre, San Diego State University/University of California-San Diego

We report on the classroom mathematical practices that developed in a mathematics content course for prospective elementary teachers. The course focused on number and operations and was intended to promote number sense development. Instruction was guided by a local instruction theory for number sense development, which we have described previously. The present report focuses on the classroom mathematical practices that emerged in the class involved in a recent teaching experiment. The actual learning trajectories identified inform elaboration and refinement of the local instruction theory and shed light on prospective teachers' number sense development.

Session 20 – Preliminary Reports
Saturday, 11:30 AM – 12:00 PM

CHIEF POKER JIM

Do Generic Proofs Improve Proof Comprehension?

Keith Weber, Rutgers University
Evan Fuller, Montclair State University
Juan Pablo Mejia-Ramos, Rutgers University
Kristen Lew, Rutgers University
Philip Benjamin, Rutgers University
Aron Samkoff, Rutgers University

In undergraduate mathematics courses, proofs are regularly employed to convey mathematics to students. However, research has shown that students find proofs to be difficult to comprehend. Some mathematicians and mathematics educators attribute this confusion to the formal and linear style in which proofs are generally written. To address this difficulty, some researchers have suggested that students be exposed to generic proofs. We report preliminary results of a study that employs a recent model of proof comprehension to assess the extent to which reading a generic proof improves student understanding over reading a traditional proof.

JOHN STEINBACH

Student Understanding in the Concept of Limit in Calculus: How Student Responses Vary Depending on Question Format and Type of Representation

Rob Blaisdell, University of Maine

Research indicates that calculus students have difficulties with limit. However, underlying reasons for those difficulties and possible influences of question format have not been examined in detail. Since limit is foundational to calculus it would help the mathematics education community to know not only the difficulties students have, but also how questions used to assess knowledge affect responses. Data for this study came from surveys administered to 111 first semester calculus students. Survey questions focused on limit in multiple representations including graphs, mathematical notation and definitions. Questions were multiple choice and free response. Student difficulties documented in previous research were evident in this population. Findings also indicated that difficulties students exhibited in one question were sometimes different then the difficulties those same students exhibited when asked about the same idea in a different representation. Students in general had less difficulty with graphical representations than mathematical notation or definition questions.

SAM HILL

The Status of Capstone Courses in the Preparation of Secondary Mathematics Teachers

Mary Beisiegel, Harvard University
Joshua Chesler, California State University-Long Beach
Dana Cox, Miami University
Rachael Kenney, Purdue University
Jill Newton, Purdue University
Jami Stone, Black Hills State University

Capstone courses have been recommended as a way to connect the mathematics pre-service secondary mathematics teachers learn in college to the school mathematics they will teach in their own classrooms. Yet little is known about the status of these courses across the U.S, in whether they are offered, the topics that are covered, the curriculum used, and the pedagogical approach, among other aspects of the course. We will present findings from a 2011 survey of U.S. colleges and universities that investigated the status of such capstone courses at these institutions. Discussion will be centered around the importance and future of such courses in teacher preparation programs.

ROY YATES

Doing Mathematics: Perspectives from Mathematicians and Mathematics Educators

Jim Brandt, Southern Utah University
Jana Lunt , Southern Utah University
Gretchen Rimmasch, Southern Utah University

Learner-centered teaching strategies such as inquiry-based learning ask students to actively engage in the material they are learning, to do mathematics in order to learn mathematics. A teacher's interpretation of the meaning of "doing mathematics" is related to his or her beliefs about mathematics and about mathematics teaching. In this exploratory study, we report the results of interviews with sixteen university level mathematics and mathematics education faculty regarding their perspectives on the meaning of doing mathematics within the context of a calculus course, a proof-oriented course, and their own mathematical experiences.

MARSHALL JOFFRE

Teaching Methods Comparison in a Large Introductory Calculus Class

Warren Code, University of British Columbia
David Kohler, University of British Columbia
Costanza Piccolo, University of British Columbia
Mark Maclean, University of British Columbia

We have implemented a classroom experiment similar to a recent study in Physics (Deslauriers, Schelew, & Wieman, 2011): each of two sections of the same Calculus 1 course at a research-focused university were subject to an "intervention" week where a less-experienced instructor encouraged a much higher level of student engagement by design; we employed a modified pseudoexperiment structure for our methods comparison with a Calculus 1 student population and with further steps to improve validity. Our instructional choices encouraged active learning (answering "clicker" questions, small-group discussions, worksheets) during a significant amount of class time, building on assigned pre-class tasks. The lesson content and analysis of the assessments were informed by existing research on student learning of mathematics, in particular the APOS framework.

ERIC HAUSER

Summing Up Students' Understandings of Sigma Notation

Steve Strand, Portland State University
Sonya Redmond, Portland State University
Dov Zazkis, San Diego State University/University of California-San Diego

Within the context of an advanced calculus instructional design teaching experiment, four students encountered interesting difficulties with sigma notation. This report tells the story of those students' progress; it describes the nature of the difficulties encountered and the ways these difficulties were resolved. Specifically, we wish to answer the questions:
1) How do post-calculus students talk about and use sigma notation?
2) How do they handle the transition from discrete to continuous cases in their use of sigma notation? In particular,
a) What challenges do students encounter when transitioning from sums involving the terms of a sequence to sums involving approximate area under a function?
b) What skills or tools do students use to meet these challenges?

Plenary Talk
Saturday, 1:30 PM – 2:30 PM

COLONEL LINDBERG BALLROOM

Reading Processes and Proof Comprehension

Dr. Lara Alcock
Mathematics Education Centre Loughborough University, UK

In this talk I will present a series of studies designed to investigate the processes associated with reading mathematical proofs. First, I will establish that the format in which a proof is presented causes differences in students' comprehension levels. Second, I will use eye-movement data to demonstrate that experts and novices exhibit different behaviours while reading proofs. I will discuss these results in relation to the opportunities offered by multimedia learning technologies and in relation to prior theoretical claims about proof comprehension and about expert mathematical behaviour.

Session 21 – Contributed Reports
Saturday, 2:40 PM – 3:10 PM

JOHN STEINBACH

The Effect of Structure-based Instruction on the Transfer of Learning to Solve Algebra Word Problems

Kuo-Liang Chang, Utah Valley University
Robert Floden, Michigan State University

A problem in learning to solve mathematics word problems students have been facing is to transfer the learned problem-solving knowledge from one story context to another story context. Some studies have provided evidence that structure facilitates transfer of learning to solve word problems. In this study we examine the effect of teaching structures (structure-based instruction) on the transfer of learning to solve algebra word problems. Sixty-one college students participated in a 2-hour controlled experiment. The results showed that students who received structure-based instruction had better performance in some types of transfer of solving algebra word problems.

CHIEF POKER JIM

A Model of Students' Combinatorial Thinking

Elise Lockwood, University of Wisconsin-Madison

Combinatorial topics are prevalent in undergraduate curricula, and research indicates that students face difficulties when solving counting problems. The literature has not sufficiently addressed students' ways of thinking about combinatorial concepts at a level that enables researchers to understand how students conceptualize counting problems. In this talk, a model of students' combinatorial thinking is presented that emphasizes relationships between formulas/expressions, counting processes, and sets of outcomes. The model serves as a conceptual analysis of students' thinking and activity related to counting; it sheds light on relevant aspects of students' combinatorial thinking, and it provides language to describe and explain aspects of students' counting activity. In this way, the model has practical implications, both for researchers (providing a lens through which to examine data on combinatorics education) and for teachers (providing an aid to instructional design based on student thinking).

ERIC HAUSER

Musings on infinite sample space

Rina Zazkis, Simon Fraser University
Ami Mamolo, York University

We examine the responses of secondary school teachers to a probability task with an infinite sample space. Specifically, the participants were asked to comment on a potential disagreement between two students when evaluating the probability of picking a particular real number from a given interval of real numbers. Their responses were analyzed via the theoretical lens of reducing abstraction. The results show a strong dependence on a contextualized interpretation of the task, even when formal mathematical knowledge is evidenced in the responses.

ROY YATES

Two parallel calculuses: The one taught and the one used

Leann Ferguson, Indiana University

Calculus is an important tool for building mathematical models of the world around us and is thus used in a variety of disciplines, such as physics and engineering. These disciplines rely on calculus courses to provide the mathematical foundation needed for success in their courses. Unfortunately, due to the parallel nature of the calculus taught, many students leave their calculus course(s) with an understanding misaligned with what is needed in the discipline courses and are thus ill-prepared. By working with presumed experts (undergraduate mathematics and other discipline faculty members), this study developed a small number of prototype tasks that elicit, document, and measure students' understanding of a few calculus concepts the faculty participants believe to be essential to successful academic pursuits within their respective disciplines. This presentation details the data and analysis from the concluding rounds of research. Implications of this research for calculus instruction and curriculum are mentioned.

Session 22 – Theoretical Reports
Saturday, 3:50 PM – 4:20 PM

ERIC HAUSER

1+1= A Window: On the Polysemy of Symbols

Ami Mamolo, York University

This paper illustrates how mathematical symbols can have different, but related, meanings depending on the context in which they are used. In other words, it illustrates how mathematical symbols are polysemous. In particular, it explores how even basic symbols, such as '+' and '1', may carry with them meaning in 'new' contexts that is inconsistent with their use in 'familiar' contexts. This article illustrates that knowledge of mathematics includes learning a meaning of a symbol, learning more than one meaning, and learning how to choose the contextually supported meaning of that symbol.

JOHN STEINBACH

Promoting students' object-based reasoning with infinite sets

Robert Ely, University of Idaho
Iuliana Radu, Rutgers University

Recent research about the thorny Tennis Ball Problem has revealed that students respond in different ways depending on the type of properties they generalize from the finite steps to the envisioned final state of the infinite process. These generalizations, in turn, depend on the features of the finite steps their attention is directed toward. Undergraduate students who attend to the labeling of objects, rather than simply counting the objects, are using object-based reasoning, which is crucial to their ability to understand Cantorian set theory. We propose a sequence of tasks centered around the Tennis Ball Problem that our research has shown to help students build object-based reasoning.

CHIEF POKER JIM

Towards a description of symbol sense for statistics

Samuel Cook, University of New Hampshire
Tim Fukawa-Connelly, University of New Hampshire

This theoretical report aligns itself with Arcavi's (1994) work and the tradition of onto-semiotic research in mathematics education (Font, Godino, & D'Amore, 2007) and is situated in the context of statistics education. This report will:
• articulate a notion of symbol sense in statistics
• explain the importance to student understanding of the development of symbol sense.
The goal of this work is to guide both research and curriculum design efforts for introductory undergraduate statistics courses. The paper begins by describing statistical analogs of Arcavi's algebraic symbol sense, then furthers this by noting the importance of reading symbols generally, reading symbols through the context of the question, and the reading of symbols related to the visualization or selection of the display. Finally, the paper briefly explores how the understanding of symbols becomes more difficult and important in the use of the Central Limit Theorem and estimation of parameters.

Session 23 – Contributed Reports
Saturday, 4:30 PM – 5:00 PM

ERIC HAUSER

Embodiment of Struggle in Research Mathematicians: The Case of Proximal Inhibition

Michael Smith, University of California-San Diego

It's often considered desirable for students to develop ways of engaging with mathematics that mimic the thinking styles of mathematicians. However, there have been very few ethnographic studies of mathematicians to explore how they actually practice mathematical research. This study involves an embodied phenomenological analysis of videos of pairs of mathematicians working together on a current problem in their field. I outline one resulting embodiment of their struggles with the material, which I term "proximal inhibition." The value and implications of this contribution are discussed briefly at the end.

CHIEF POKER JIM

A Hypothetical Learning Trajectory for Conceptualizing Matrices as Linear Transformations

Megan Wawro, Virginia Tech
Christine Larson, Vanderbilt University
Michelle Zandieh, Arizona State University
Chris Rasmussen, San Diego State University

In this presentation we articulate a hypothetical learning trajectory (HLT) designed to support students' development and elaboration of a transformation view of matrix multiplication. The major learning goals of this HLT are (a) interpreting a matrix as a mathematical object that transforms input vectors to output vectors, (b) interpreting matrix multiplication as the composition of linear transformations, (c) developing the imagery of an inverse as "undoing" the original transformation, and (d) reasoning about matrices as objects that geometrically transform a space. Within this HLT, we extend students' conceptualization of the "matrix acting on a vector" view to a more global view of a matrix transforming an entire space, as opposed to the localized view wherein matrices are interpreted as transforming one vector at a time.

JOHN STEINBACH

An Analysis of Calculus Instructor Grading Inconsistencies

Jana Talley, Jackson State University

Despite the consensus among mathematics educators that prior knowledge is essential to student success, calculus instructors vary widely in their assessment of prior knowledge errors found on student assignments and exams. This phenomenological study of five calculus instructors at a large research institution investigated the influence that instructor belief systems have on the consistency of grading across instructors. The results showed that the intricacies of instructor sensible systems, as outlined by Leatham's (2006) study of mathematics teacher beliefs and practices, play a vital role in the assessment of student errors.

Session 24 – Contributed Reports
Saturday, 5:10 PM – 5:40 PM

CHIEF POKER JIM

Students' Ways of Thinking about Enumerative Combinatorics Solution Sets: The Odometer Category

Aviva Halani, Arizona State University

This presentation aims to address students' ways of thinking about the set of elements being counted in enumerative combinatorics problems. Fourteen undergraduates with no formal experience with combinatorics participated in individual task-based interviews in spring 2011. Open coding was used to identify students' ways of thinking about the set of elements being counted, called the solution set. One category of ways of thinking which emerged from the data analysis involves holding an item constant and cycling through possible items for the remaining spots in order to generate all elements of the solution set. This category is known as Odometer thinking and two ways of thinking from this category, Standard Odometer and Wacky Odometer, are presented here. The conjectured Generalized Odometer way of thinking, which involves holding an array of items constant, is introduced as an extension of Wacky Odometer thinking. Its potential to coordinate set- and process-oriented thinking is discussed.

ERIC HAUSER

Preparing Mathematics Teachers to Teach Definitions

Joshua Chesler, California State University-Long Beach

Mathematics teachers, at all levels, must help their students become thoughtful users of mathematical definitions. This paper examines pre-service secondary mathematics teachers, at the end of their undergraduate training, interacting with mathematical definitions. They were tasked with choosing and using definitions; evaluating the equivalence of definitions; and interpreting a definition from a high school mathematics textbook. Their performances indicated that many of these future mathematics teachers have difficulty reasoning with and about mathematical definitions. These deficiencies have implications for undergraduate teacher preparation.

JOHN STEINBACH

The Effectiveness of Local Linearity as a Cognitive Root for the Derivative in a Redesigned First- Semester Calculus Course

Jason Samuels, City University of New York

In this report we investigate an innovative, reorganized curriculum for first-semester calculus which emphasizes local linearity and uses it as the fundamental principle on which the rest of the curriculum is based. Technology and visualization are used as tools for guided discovery of local linearity and other aspects of calculus. How students used local linearity as a cognitive root for the derivative will be discussed. Student learning outcomes will also be presented, with some examples of student work demonstrating the results of the approach.

Dinner, Awards, and Plenary Talk
Saturday, 6:30 PM – 9:00 PM

COLONEL LINDBERG BALLROOM

An Expanded Framing for Characterizing Mathematical Progression

Dr. Chris Rasmussen
San Diego State University & Center for Mathematics and Science Education

Learning Progressions (LPs) in mathematics and science education specify levels of sophistication marked by conceptual waypoints. While LPs offer a tool of a certain grain size for instructional design and assessments, they neglect some of the complexities and realities of teaching and learning. In particular, LPs ignore mathematical progression in terms of mathematical practices and they only consider progression in terms of individuals. In this talk I argue that mathematical progress is more completely seen as a progression in an ecology of ideas and ways of reasoning for individuals and their classroom community. In service of this goal, I develop and illustrate an expanded version of Cobb and Yackel's (1996) interpretive framework that coordinates sociological and psychological perspectives. This expansion focuses on the following four analytic framings for characterizing mathematical progression: collective disciplinary practices, classroom mathematical practices, individual participation, and individual, acquisition.