We summarize the preliminary results of a study of conceptual understanding of mathematics by pre-service secondary school math teachers. Our research involves the statistical analysis of data from an actual mathematics Praxis II licensure exam, which was administered nationwide. Through a quantitative, item by item analysis, using a classification of these test items by conceptual difficulty, we obtain insight into the conceptual issues that pre-service teachers have great difficulty with. Our preliminary results show a significant gap between computational and abstract mathematical processes. This in turn, affects the ability of pre-service teachers to be fluent in the domains of both subject and pedagogical content knowledge.

Key words: [Pre-service Secondary Teachers, Mathematics Praxis, Quantitative Analysis, Content Knowledge]

Introduction
The knowledge required for effectively teaching mathematics has been studied by many researchers in mathematics education. Ball, Thames, and Phelps (2008) have investigated this topic thoroughly for pre-service teachers for K-8. Also, Krauss et al. (2008) investigated the topic for secondary school mathematics teachers.

In light of these investigations into the knowledge base for teaching mathematics, math teacher educators at the postsecondary level need to examine how well versed pre-service secondary math teachers are in their ability to move through different concepts and ideas in the mathematics they will teach. What are some ways we can obtain insight into students’ mathematical understanding and flexibility? This is not an easy question to answer since the mathematical training of secondary teachers in the United States varies widely. However, the need for mathematical understanding and flexibility will be the same, regardless of where mathematics is being taught. In order to gain insight into pre-service teachers’ abilities to conceptualize, we examine scores from an administration of a Praxis II Mathematics Content Knowledge Exam. Since virtually all states use the Praxis II as a licensure test, it is practically the only standard measure that cuts across all math teacher preparation programs in the country.

Literature Review
Thames and Ball (2010) indicate that solving every day “teaching problems” demands “mathematical understanding and flexibility”. They have formulated a need for pedagogical content knowledge (PCK) as well as subject matter knowledge. Pedagogical content knowledge encompasses knowledge of content and students, knowledge of content and teaching, and knowledge of curriculum.

Krauss et al. (2008) investigated the topic of pedagogical content knowledge for secondary school mathematics teachers. In their work, they examine tasks that would assess pedagogical content knowledge in mathematics. These tasks involve moving beyond standard explanations for basic mathematical ideas.

Conceptually, the more difficult questions in the Praxis II exam also involve moving beyond the standard routines and procedures in mathematics. Thus, analyzing these questions
can provide a window into the types of associations that pre-service teachers may have difficulty making. This can, in turn, increase awareness of concepts and ideas which need to be taught or emphasized in greater depth in the undergraduate training of math teachers. These concepts and ideas are necessary for both subject matter knowledge and pedagogical content knowledge. A teacher with a poor grasp of conceptual underpinnings of the mathematics they teach will be less able to facilitate students’ understanding of those concepts.

**Research Question and Methodology**

Using the data on the Praxis II exams, we would like to ask the question, “what types of mathematical thinking do future teachers have the most difficulty with?” To help answer this question, we analyzed Praxis II exam results with a close examination of items that require connections across multiple mathematical domains and involve unpacking of the underlying knowledge. Our analysis is based on test records of 2299 examinees across the United States, who took the sixty-minute version of the Praxis II Mathematics: Content Knowledge test in November 2008. The data was collected by the Educational Testing Service (2008).

To answer our question, we follow the method of test item classification as proposed by Wainer, Sheehan, and Wang (2000). The results follow.

**Results and Discussion**

The Praxis II exam in mathematics consists of 50 questions, broken down into categories as follows. These categories are provided by the Educational Testing Service (2008).

<table>
<thead>
<tr>
<th>Category</th>
<th>Topics covered</th>
<th>Number of questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Algebra and Number Theory</td>
<td>8</td>
</tr>
<tr>
<td>II</td>
<td>Measurement, Geometry and Trigonometry</td>
<td>12</td>
</tr>
<tr>
<td>III</td>
<td>Functions and Calculus</td>
<td>14</td>
</tr>
<tr>
<td>IV</td>
<td>Data Analysis and Statistics and Probability</td>
<td>8</td>
</tr>
<tr>
<td>V</td>
<td>Matrix Algebra and Discrete Mathematics</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 1

The percentage of the 2299 respondents who answered each item correctly was given by the Educational Testing Service (2008). The side-by-side boxplots in Figure 1 show the percentage of respondents choosing the correct answers for questions in the various categories.

![Figure 1](image-url)
Test items in categories II and IV had a maximum of 90% correct, while Categories I and V had the lowest maximum, at 65% and 66% correct, respectively. Categories I, III, and V represent the more abstract and conceptual portions of the test. Categories II and IV involve more numerical computations.

Wainer, Sheehan, and Wang (2000) also propose a different classification scheme for a more meaningful analysis of test results. In their work, they analyze Praxis I test scores for Education in the Elementary School Assessment according to hierarchically ordered skill levels. Gitomer (2010) analyzes results of Praxis II mathematics exams in terms of major trends using the total scores. We analyze some of the Praxis II mathematics exam results with a close examination of items that require connections across multiple mathematical domains and involve unpacking of the underlying knowledge. To do so, we modified the classification system proposed by Wainer et al. (2000) to reflect the mathematical complexity and processes involved in the Praxis II questions. The classification is as follows, with each of the fifty questions assigned only to a single category.

<table>
<thead>
<tr>
<th>Category</th>
<th>Processes involved</th>
<th>Number of questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>NC</td>
<td>primarily involves straightforward numerical computation</td>
<td>15</td>
</tr>
<tr>
<td>GR</td>
<td>primarily graphical or geometric reasoning</td>
<td>16</td>
</tr>
<tr>
<td>AC</td>
<td>involves computation, but uses abstract reasoning, and perhaps links multiple concepts</td>
<td>10</td>
</tr>
<tr>
<td>AR</td>
<td>involves abstract reasoning, but no numerical answer is produced</td>
<td>9</td>
</tr>
</tbody>
</table>

Using descriptive data analysis, we found that the students fared significantly worse in the categories of AC (abstract reasoning with computation) and AR (abstract reasoning without numerical answer). Side-by-side boxplots for the percentage of respondents choosing the correct answers for questions in the different categories are shown in Figure 2. The data clearly indicates a need for explicit emphasis on connections of concepts in mathematics.

Our work thus far suggests large gaps between computational and abstract understanding of mathematics by pre-service teachers. We expect to further analyze the data with more
refined methodologies in testing and measurement. To this end, we plan to modify the research methodology discussed in Wainer et al. (2000) and Sheehan and Mislevy (1990).

We also plan to use this data to investigate implications for the undergraduate teaching of mathematics, and how it can better serve the needs of pre-service mathematics teachers.

**Questions**

1. Is the proposed classification robust for the type of questions that appear on the Praxis II exam?
2. What are the implications of these results for the teaching of undergraduate mathematics courses, both at the lower and upper level?
3. Should further avenues for exploration expand on the measurement details or on the implications for pedagogy in undergraduate mathematics?

**References**


