RUME XVII CONFERENCE SCHEDULE

THURSDAY, FEBRUARY 27, 2014

1:00 – 1:15 pm
Grand Mesa DEF

OPENING SESSION

1:25 – 1:55 pm

SESSION 1 – PRELIMINARY REPORTS

Chasm Creek A

Characteristics Of Successful Programs In College Calculus At Bachelor's Granting Universities
Kathleen Melhuish, Sean Larsen, Erin Glover & Estrella Johnson

The CSPCC (Characteristics of Successful Programs in College Calculus) project is a large empirical study investigating mainstream Calculus 1 to identify the factors that contribute to success, and to understand how these factors are leveraged within highly successful programs. Phase 1 of CSPCC entailed large-scale surveys of a stratified random sample of college Calculus 1 classes across the United States. From these surveys, successful institutions were selected as case studies. At each case study institution, Calculus I instructors, students and relevant administration were interviewed. In this report, we will present preliminary analysis on the five bachelors granting institutions selected. We will discuss common themes and factors that have emerged from the five institutions.

Grand Mesa A

Students' Understanding Of Exponential Functions In The Context Of Financial Mathematics
Natalie E. Selinski

Exponential functions are one of the most critical mathematical topics used by students in financial mathematics. This presentation explores university finance students' notion of exponential function from two sets of data. First, I use data collected through surveys to examine students' understanding of exponential function in general and, more specifically, to identify the extent to which students conflate exponential functions with polynomials. I then draw on data collected in an inquiry-based instructional sequence aimed at improving financial mathematics students' understanding of exponential functions. Results include delineation of what ways of understanding exponential functions are critical to studying financial mathematics and insights into how best to guide students in developing these understandings within the context of their field of study.

Chasm Creek B

Diagrams In Advanced Mathematics: Affordances and Limitations
Kristen Lew, Tim Fukawa-Connelly, Juan Pablo Mejia-Ramos & Keith Weber

We report a case study aimed at researching the rationale of a university mathematics professor for using diagrams in his analysis lectures, what he hoped his students would learn from these diagrams, the ways students understand these diagrams, and what they learn from them. Preliminary analysis suggest that by

focusing on specific properties of the diagrams presented in mathematics lectures, or by attributing little importance to them, students fail to fully understand what professors hoped they would learn from these diagrams.

Grand Mesa B

Exploring Students' Questions From Online Video Lectures

Fabiana Cardetti, Konstantina Christodoulopoulou & Steven Pon

This study was designed to investigate the types of questions college students generate as they watch video lectures in a business calculus class. Thirty-six students taking an undergraduate calculus course participated in the study. In this paper we share the preliminary results of our qualitative analysis. We have found nine mutually exclusive categories that uncover the thoughts, struggles, and successes our students go through as they experience this new teaching modality of video-viewing. We also include three questions for the audience to help further our analysis and open up new research opportunities for the improvement of collegiate teaching through the study of students' questions.

Grand Mesa C

Transfer Of Learning: Examining Individuals In Social Settings

Jeffrey King, Stephenie Anderson & Gulden Karakok

In this preliminary report, we share the design and results of the first phase of our on-going research study. Our three-phase study is designed to investigate individual student's transfer of learning of linear algebra concepts along with social mathematical interactions in which such concepts developed in group-based courses. We first frame our study in relation to current literature, then discuss our initial analysis from the first phase. Finally, we give a description of upcoming phases along with questions we wish to discuss with the audience.

2:05 – 2:35 pm

SESSION 2 - CONTRIBUTED REPORTS

Chasm Creek B

Geometric Reasoning On The Complex Plane

Hortensia Soto-Johnson & Jonathan Troup

Using Bakker and Hoffman's (2005) framework on diagrammatic reasoning, we analyzed a video-taped interview to explore two undergraduates' ability to reason geometrically about tasks related to complex variables. Our findings indicate that in order to provide a geometric interpretation, our participants needed to first perform algebraic computations. These computations appeared to provide them with the pieces required to construct a diagram. Once these pieces were in place the participants used dynamic gesture to enact their geometric interpretations with the aid of their diagram. It appeared that their dynamic gestures assisted with embodying geometric interpretations and as such one particular task was influential throughout the interview. Furthermore, the participants integrated less dynamic gesture as they progressed with similar tasks.

Grand Mesa A

Preservice Elementary Teachers' Understanding Of Number Theory: Connecting Content Knowledge To Pck

Kristin Noblet

Little is known about the relationship between preservice elementary teachers' content knowledge and PCK (Shulman, 1986), especially in number theory. This was investigated as part of a larger case study of preservice elementary teachers' understanding of topics in number theory. An emergent perspective (Cobb & Yackel, 1996) as well as Ball and colleagues' conceptualization of Mathematical Knowledge for Teaching (e.g, Hill, Ball, & Schilling, 2008) were used to collect and analyze data in the form of field notes, student coursework, and responses to task-based one-on-one interviews. The study suggests that preservice elementary teachers' PCK can be strengthened and influenced by their specialized content knowledge (Ball, Thames, & Phelps, 2008) as well as their perspectives on how students learn.

Grand Mesa B

Students' Examples Usage In The Domain Of Functions

Muhammed Fatih Dogan

Mathematicians use examples strategically while working on mathematical conjectures, and this strategic usage helps them gain a lot of insight about mathematical phenomenon. However, students do not always have the same strategic example usage; instead, they tend to over rely on examples without understanding of example based reasoning. This study examines college algebra students' responses on a written assessment in the function domain and discusses students' example spaces. The results reveal that students have very limited example space in the function domain that affects their strategic example usage. Student example usage was very limited to conventional example spaces that they learned during instruction or from their textbook. This study suggests that having conventional example spaces does not guarantee that students can use examples strategically, which can help them better understand the mathematical conjectures.

Chasm Creek A

Perceptions In Abstract Algebra: Identifying Major Concepts and Concept Connections Within Abstract Algebra

Ashley Suominen

Abstract algebra is recognized as a highly problematic course for most undergraduate students. Despite these difficulties, most mathematicians and mathematics educators affirm its importance to undergraduate mathematical learning. Both the process of abstraction and constructing on past knowledge are essential to comprehending course material. The goal of this research was to establish a list of the important concepts in abstract algebra as perceived by graduate students in mathematics and understand how they believe these concepts are related. Through an interview study, the students' perceptions of Abstract Algebra were analyzed through the development of concept maps. The results revealed graduate students had great difficulty articulating what they learned and had differing views of major concepts and relationships within the course. Furthermore, their perceptions of concept importance equated to the amount of time their class spent discussing that concept.

2:35 – 3:05 pm
Atrium

COFFEE BREAK

3:05 – 3:35 pm

SESSION 3 – PRELIMINARY REPORTS

Grand Mesa A

Calculus Instructors' Resources, Orientations and Goals In Teaching Low Achieving Students

Misun Lee & Sepideh Stewart

Teaching and learning calculus has been the subject of mathematics education research for many years. Although the body of research is mainly concerned with students' difficulties with calculus, in this study we will be focusing our attention on the professors and instructors of calculus. In this research we used Schoenfeld's framework to examine four instructors' resources, orientations and goals in teaching calculus to low achieving students. So far, the preliminary results of the interviews show that although the professors thought differently about many aspects regarding teaching calculus, they all claimed that the first step to succeed in calculus courses is being prepared and having the right background.

Chasm Creek A

Conceptions Of Inverse Trigonometric Functions In Community College Lectures, Textbooks, and Student Interviews

Vilma Mesa & Bradley Goldstein

We present a textbook analysis of conceptions of key ideas associated with inverse trigonometric functions using Balacheff's model of conceptions (Balacheff & Gaudin, 2010). We found conflicting conceptions of angles, trigonometric functions, and inverse trigonometric functions that may help explain difficulties that community college trigonometry instructors and their students face when explaining tasks associated with this topic. We make suggestions for further research.

Chasm Creek B

An Analysis Of Transition-To-Proof Course Students' Proof Constructions With A View Towards Course Redesign

John Selden, Ahmed Benkhalti & Annie Selden

The purpose of the reported study was to gain knowledge about undergraduate transition-to-proof course students' proving difficulties. We analyzed the final examination papers of students in one such course. We have tentatively identified categories of difficulties such as nonstandard language/notation, insufficient warrants, and extraneous statements. The ultimate goal is to use these categories as pedagogical content knowledge with which to redesign an existing transition-to-proof course to alleviate the difficulties for future students.

Grand Mesa B

Graduate Students' Integrated Mathematics and Science Knowledge For Teaching

Shahram Shawn Firouzian

Previous studies have indicated that effective mathematics teaching relies on teachers' knowledge of both student thinking and mathematical content, however very little is known about the integration (combination) of teacher's mathematical knowledge and science knowledge for teaching important topics like derivative and applied derivative problems. The goal of this study was to examine the knowledge of mathematics and science that teachers draw on when teaching the concept of derivative and applied derivative problems. We conducted task-based interviews with nine graduate assistants (GTAs). Findings revealed that GTAs made use of their knowledge of science as well as of mathematics when discussing how to teach applied derivative problem. In this proposal, we only look at the results of two interviews and try to shed light into the nature of science and mathematics knowledge the teachers use for Teaching and how that can lead into opportunities in professional development for the novice teachers.

Wind Star (1st Floor)

Implementing Inquiry-Oriented Instructional Materials In Undergraduate Mathematics

Christine Larson, Megan Wawro, Michelle Zandieh, Chris Rasmussen, David Plaxco & Katherine Czeranko

Over the past years, research in the RUME community has driven the development of inquiry-oriented instructional materials in a number of undergraduate mathematics content areas including abstract algebra, differential equations, and linear algebra. Literature at the K-12 level has documented challenges inherent to scaling up the implementation of this kind of instruction. In this study, we explore how instructors make sense of and implement inquiry-oriented instructional materials in undergraduate mathematics, and the nature of supports these instructors report using and wanting when planning for instruction. We consider instructors' interpretations and desired supports as they relate to prior pedagogical experience and institutional setting. Data is taken from surveys, interviews, and video-taped instruction of three participating instructors at three different institutions as they work to implement two inquiry-oriented instructional units in undergraduate linear algebra.

Grand Mesa C

An Observation Instrument For Assessing The K-16 Mathematics Classroom

Jim Gleason

We describe the development of a new observation protocol instrument for classroom instruction that is mathematics-specific, spans K-16 mathematics, improves validity and reliability compared to existing instruments, and encompasses the Standards for Mathematical Practice. The instrument may be helpful for educators/researchers engaged in classroom evaluations of K-16 mathematics teaching.

3:45 – 4:15 pm

SESSION 4 – CONTRIBUTED REPORTS

Grand Mesa A

Generalizing Calculus Ideas from Two Dimensions to Three: How

Multivariable Calculus Students Think About Domain and Range

Allison Dorko & Eric Weber

We analyzed multivariable calculus students' meanings for domain and range and their generalization of that meaning as they reasoned about domain and range of multivariable functions. We found that students' thinking about domain and range fell into three broad categories: input/output, independent/dependent variables, and/or as attached to specific variables. We used Ellis' (2007) actor-oriented generalizations framework to characterize how students generalized their meanings for domain and range from single-variable to multivariable functions. This framework focuses on the process of generalization – what students see as similar between ideas in multiple contexts. We found that students generalized their meanings for domain and range by relating objects, extending their meanings, using general principles and rules, and using/modifying previous ideas. Our results about how students understand and generalize the concepts of domain and range imply that the domain and range of multivariable functions is a topic instructors should explicitly address.

Chasm Creek B

How To Make Time: The Relationships Between Concerns About Coverage, Material Covered, Instructional Practices, and Student Success In College Calculus

Estrella Johnson, Jessica Ellis & Chris Rasmussen

This report draws on data collected by the Characteristics of Successful Programs in College Calculus project in order to investigate issues around coverage and pacing. This includes identifying what topics are being taught in Calculus I, determining the extent to which instructors and departments feel pressure to cover a set amount of material, and investigating possible relationships between concerns over coverage, instructional practices, and the nature of the material covered at five institutions selected for having successful calculus programs.

Grand Mesa B

Do Experts and Novices Gesture Differently?

Brent Hancock, Marki Dittman & Hortensia Soto-Johnson

Previous gesture studies conjecture that as individuals develop expertise in a field of mathematics their gestures tend to become more metaphoric, iconic, and dynamic. In this mixed-methods study, we compared the gestures of six experts and four pairs of novices as they geometrically described the complex number arithmetic operations: z+w, zw, and 1/z. An ANOVA revealed that the factors Task and Gesture were statistically significant, but there was no statistically significant difference between the two groups' gesture use. A Hierarchical Cluster Analysis directed the qualitative analysis where we found that novices exposed to technology appeared to produce gestures that were innovative or similar to the experts' gestures. These findings suggest that bolstering students' awareness of their own and the instructors' gestures as well as exposing students to technology may help them develop more dynamic gestures and in turn possibly facilitate a more geometric perspective of the arithmetic of complex numbers.

Chasm Creek A

Why Lectures In Advanced Mathematics Often Fail

Kristen Lew, Tim Fukawa-Connelly, Juan Pablo Mejia-Ramos & Keith Weber

Research on mathematicians' pedagogical practice in advanced mathematics is sparse. The current paper contributes to this literature by reporting a case study on a proof that a professor presented in a real analysis course. By interviewing the professor, we focus on his learning goals in this proof and the actions that he took to achieve these goals. By interviewing six students, we investigate how they perceived the proof and what they learned from it. Our analysis provides insight into why students did not learn what the professor desired from this lecture.

4:25 – 4:55 pm

SESSION 5 – CONTRIBUTED REPORTS

Grand Mesa B

Working Together On Mathematics Homework: A Look At How University Students Spend Their Time Outside The Classroom

Gillian Galle

Despite the large amount of time university students are expected to spend studying material and learning on their own outside of the classroom, little is known about what specific student study habits look like. This study sought to start developing a description of what activities students engage in when studying together in self-formed groups outside of the classroom. By identifying a set of macrotasks, verbally-cued transactions that identify what activity the group is currently engaged in doing, this study provides a way to compare how different study groups allocate their time and distinguish between the enactment of social and sociomathematical norms outside of the classroom.

Chasm Creek B

A Comparison Of Four Pedagogical Strategies In Calculus

Spencer Bagley

The quality of education in introductory calculus classes is an issue of particular educational and economic importance. In work related to a national study of college calculus programs conducted by the MAA, I report on a study of four different pedagogical approaches to Calculus I at a single institution in the Fall 2012 semester. Using statistical methods, I analyze the effects of these four approaches on students' persistence in STEM major tracks, attitudes and beliefs about mathematics, and procedural and conceptual achievement in calculus. Using qualitative methods, I draw links from the statistical results to differences and commonalities in the four classroom strategies.

Chasm Creek A

Implied and Empirical Readers Of Newton's Method

Kristen Murphy, Celeste Glenn & Nicole Engelke Infante

The ability to translate a text into a mathematical process is a key goal of mathematics education. Knowing when students have the prerequisite knowledge to understand such a process is a perennial concern for instructors. Here we use Newton's method to evaluate reader oriented theory as a means to illuminate these issues. Through clinical interviews with twelve first semester calculus students, we

determined that knowledge of both tangent lines and roots is required for students to understand and apply Newton's method. Analysis was done from the perspective of the empirical, implied, and intended readers and was examined for the extent to which the empirical and implied readers aligned. It was found that although the alignment of the empirical and implied readers was helpful in determining the success of the students, it was not in itself a deciding factor.

Grand Mesa A

Differences In Expectations Between Explicit Statements and Actual Practices Using Vectors In A Trigonometry and Physics Course

Wendy James

Science and engineering instructors often observe that students have difficulty using or applying prerequisite mathematics knowledge in their courses. Historically, transfer theory is used to investigate students' issue applying their vector knowledge from a trigonometry course to a physics course, but this qualitative case-study is positioned differently epistemologically and theoretically from transfer theory to understand and describe the mathematical vector practices in the two courses. Saussure's (1959) concept of signifier and signified provided a lens for examining the data during analysis. Multiple recursions of within-case comparisons and across-case comparison were analyzed for differences in what the instructors and textbooks explicitly stated and later performed as their practices. While the trigonometry and physics instruction differed slightly, the two main differences occurred in the nature and use of vectors in the physics course.

5:00 – 6:10 pm
Centennial Room (12th Floor)

POSTER SESSION

Beyond Plug and Chug: The Nature Of Calculus Homework At Doctoral Institutions

Gina Nunez, Kady Hanson & Jessica Ellis

Prior research reflects a positive relationship between homework and student academic achievement in undergraduate mathematics courses. Additionally, recent research has indicated no significant difference in student learning based upon the medium of the assignment (on-line based versus paper-based). These findings led us to ask the question: How does the nature of Calculus I homework assignments at doctoral institutions with successful calculus programs compare to assignments at institutions with less successful calculus programs? Descriptive analyses of student and instructor responses from a large national survey given to mainstream Calculus I programs were conducted. Analysis revealed significant differences in the nature of homework between successful and less successful institutions, including differences in the content and frequency of assignments. The holistic approach to homework taken by successful institutions adds to the existing literature on homework at the undergraduate level and indicates an interesting relationship between homework and student success in Calculus I courses.

Students' and Experts' Ways Of Reasoning About Partial Derivatives Across Stem Contexts

Eric Weber, Tevian Dray, Corinne Manogue, Mary Bridget Kustusch & David Roundy

A common feature across STEM disciplines is the study of change, whether studying how changing a design parameter affects the operation of a prototype, or how pressure changes when we adiabatically compress a gas. Indeed, the nature of scientific measurement is to control some physical quantities while measuring others. Mathematically, we express the concept of changing one parameter while fixing others by using partial derivatives. However, how we use partial derivatives and how we talk about partial derivatives vary dramatically across STEM disciplines. The purpose of this poster is to share our preliminary results from student and expert problem-solving interviews about partial derivatives.

Formal Logic In Early Undergraduate Mathematics: A Cycle

Morgan Dominy

There are certain concepts in early undergraduate mathematics such as limits and linear independence which heavily rely on student understanding of formal logic. Since some undergraduates will eventually become pre-service teachers, any changes to early undergraduate courses will have a ripple effect throughout all levels of education. The purpose of this poster is to start a dialogue among math educators and gain insight on how to proceed in conducting research to measure the effect.

Raising Calculus To The Surface: Discovering Multivariable Calculus Concepts Using Physical Manipulatives

Aaron Wangberg, Brian Fisher, Jason Samuels & Eric Weber

Current research on algebraic and quantitative reasoning shows that many students experience mathematics as the manipulation of meaningless symbols (Smith & Thompson, 2007). In order to develop meaning in symbolic contexts, students must first conceive of relationships between the underlying quantities present in a particular context. Our project focuses on a quantitative reasoning approach to multivariable calculus, in particular the concepts of function, rate, area and volume by using physical surfaces. In this poster, we provide examples of identifying, measuring, and recording of essential quantities on physical surfaces.

Visualizing Mathematical Connections In Student Teaching Episodes

Danielle Champney

This poster aims to present a modified version of SPOT diagrams (Structure Perceived Over Time) (Yoon, 2012) – an aspect of analysis and data presentation used to present interactive student video data during which perceptual shifts may occur. Versions of this analytic tool are being explored with data from a study of students' teaching episodes during which they explained their understanding of infinite series convergence. Examining the teaching episode of one student (Molly) who had a literal “aha!” moment during her explanation, the aim of this poster will be to share the affordances of these modified diagrams, and discuss their benefits

for exposing some of the factors that contributed to Molly's developing understanding of infinite series convergence, as her teaching episode unfolded.

Developing Inquiry Oriented Instructional Materials For Linear Algebra (Dioimla): Overview Of The Research Project

Megan Wawro, Michelle Zandieh, Chris Rasmussen, Christine Larson, David Plaxco & Katherine Czeranko

The goals of the recently funded DIOIMLA research project are to produce: (a) student materials composed of challenging and coherent task sequences that facilitate an inquiry-oriented approach to the teaching and learning of linear algebra; (b) instructional support materials for implementing the student materials; and (c) a prototype assessment instrument to measure student understanding of key linear algebra concepts. Our poster will provide more detailed information about the DIOIMLA research project. Each of the three aspects of the project will be described in more detail and examples of each will be shared. The poster will also include an overview of the current status of the research project and a summary of the timeline for planned future work.

Measurement Definitions For Elementary School Teachers: Links To Graduate Level Mathematics

Visala Rani Satyam

Undergraduates planning to be teachers often encounter mathematics content textbooks written specifically for their population (preservice teachers). Elementary mathematics textbooks of this kind provide in-depth definitions of elementary school mathematics to foster deeper understanding of these basic concepts. I looked at measurement definitions (length, area, and volume) across 6 preservice textbooks and identified overarching themes, using an open coding method. These definitions tend to the precise and rigorous. For example, Parker & Baldridge (2008) define area as “a way of associating to each region R a quantity Area(R)” (p. 107). The following themes emerged across the set of definitions: discrete/continuous, unit, no overlaps/full cover, interior/exterior, function, measurement as an attribute, and space filling. I end with a discussion of the links to graduate level mathematics and what this means for preservice teachers and their future elementary students.

Using a Framing and Resources Framework For Analyzing Student Thinking About Matrix Multiplication

Warren Christensen

A student who has completed both Linear Algebra and Quantum Mechanics should have a wealth of conceptual and procedural knowledge that has been obtained from mathematics and physics classes. However in practice, students seem to struggle with this task. This investigation casts light on students’ thinking about matrix multiplication and how their thinking appears to be influenced by their framing of the problem as either a mathematics or physics question. Using Framing and Resources as a theoretical lens can provide insight into the ideas and

concepts that a student accesses from domains of mathematics and physics. Using lexicon analysis, it appears the student shifts from a “mathematical frame” to a “physics frame” and back again, but struggles to successfully transfer concepts between these two frames. I will highlight the markers for these frame shifts and demonstrate why framing and resources is the appropriate lens for this investigation.

Mathematical Thinking In Engineering and Mathematics Students

Jenna Tague

The past decades have brought a multitude of calls for improving the mathematical education of Science, Technology, Engineering, and Mathematics (STEM) students as well as increasing the number of STEM graduates (Ferrini-Mundy & Güçler, 2009). However, there is a need to examine what mathematics and mathematical thinking is needed for these STEM disciplines. This study examined the mathematical thinking of two purposefully selected students (one from mathematics and one from engineering) enrolled at a large Midwestern university as a starting place in addressing this gap. Interviews were analyzed through the socio cultural lens of zone theory (Valsiner, 1997) in order to investigate the resources the students drew upon while thinking mathematically. Additionally, a mathematical modeling cycle (Blum & Leiβ, 2007) allowed for cataloguing the particular phases involved in the participants’ mathematization processes.

Pencasts As Exemplars In Differential Equations

Jennifer Czocher, Jenna Tague, Amanda Roble & Gregory Baker

A substantial amount of students’ time in mathematics courses at the undergraduate level is spent working homework problems, but it is difficult to help students make sense of procedures or go beyond acquiring fluency. We introduced pencasts as an instructional medium to a differential equations course to create exemplars of challenging homework problems which cognitively modeled the professor’s mathematical thinking on each task. Using mixed-methods survey design, we assessed students’ use of and response to the pencasts. Students reported finding the pencasts helpful, that they enabled independent work on the homework, and they appreciated a detailed, verbal explanation of why particular steps were taken in solving each exemplar.

Mathematics Beyond Classroom: Students' ‘Value Creation’ Through Mathematical Modeling Within a Learning Community

Joo Young Park

This study examined how mathematical modeling activities within a collaborative group impact on students’ perceived ‘value’ of mathematics. With a unified framework of Makiguchi’s theory of ‘value’, mathematical disposition, and identity, the study identified the elements of the value-beauty, gains, and social good-with the observable evidences of mathematical disposition and identity. A total of 60 college students participated in ‘Lifestyle’ mathematical modeling project. Both qualitative and quantitative methods were used for data collection and analysis. The result from a paired-samples t-test showed the significant changes in students’ mathematical disposition. The results from analysis of students’ written responses and interview data described how the context of the modeling tasks and the

collaborative group interplay with students' perceived value. The poster will present the main findings and the examples of students' written tasks and responses.

A Proposal For a Theoretical Framework On Specialized Knowledge For Teaching Mathematics

Thorsten Scheiner

Building upon past and recent theoretical approaches and models in research on mathematics teacher knowledge, the presented work provides a theory-driven and research-based approach conceptualizing the construct of specialized knowledge for teaching mathematics. The crucial aspect of this approach is the underlying assumption that the transformation of knowledge from specific knowledge bases creates a new form of knowledge that possesses distinct characteristics that were not present in their original form. This new kind of knowledge is considered as being crucial for teaching mathematics, in particular, at an upper-secondary level.

6:15 – 9:00 pm
Grand Mesa DEF

DINNER & PLENARY SESSION

Plenary Speaker: Andrea diSessa

Knowledge In Pieces: How To Analyze The Process Of Learning At High Resolution

Abstract: I aim to give an overview of how the epistemological perspective of "Knowledge in Pieces" (KiP) has allowed the creation of high-resolution analyses of learning in process. "High resolution" entails analysis of real-time data, so that one can actually see learning steps at the grain-size at which learners experience it, and at which teachers and curriculum developers try to manage it. As such, this very rare kind of analysis might be extraordinarily helpful in designing instruction and learning materials.

I will first try to characterize the overall KiP program of studies and contrast it with other programs of studying learning. Then, I will use data from two recent studies to illustrate the principles in action. (1) I will show a case of a small class of students developing, on their own, some normative physics (Newton's law of thermal equilibration: Temperature difference drives rate of change of temperature). Here, we can see, element by element, what incoming knowledge was invoked, and how it changed and combined to result in the normative idea. (2) The other study involves micro-analysis of student learning from a well-studied instructional sequence (Brown and Clement's "bridging analogies"). In this, we track differences in incoming student knowledge well enough to see why some students succeeded and others failed to achieve the instructional goal.

FRIDAY FEBRUARY 28, 2014

8:45 – 9:15 am
Grand Mesa A

SESSION 6 – CONTRIBUTED REPORTS

Calculus Students' Early Concept Images Of Tangent Lines

Renee Larue, Brittany Vincent, Vicki Sealey & Nicole Engelke

This study began as an attempt to explore first-semester calculus students' understanding of Newton's method. Within that context, it was found that many students had difficulty sketching tangent lines. The research presented in this paper examined the language students use to describe tangent lines as well as their graphical illustrations of tangent lines. Task-based interviews were conducted with twelve first-semester calculus students who were asked to verbally describe a tangent line, sketch tangent lines for multiple curves, and use tangent lines within the context of Newton's method. We identified six prominent categories that described students' concept images of tangent lines and found that individual students often possessed multiple concept images. Furthermore, data shows that these concept images were often conflicting, and students were usually willing to modify their concept images in different contexts.

Chasm Creek A

Lessons Learned From Case Studies Of Successful Calculus Programs At Five Doctoral Degree Granting Institutions

Chris Rasmussen, Jessica Ellis & Dov Zazkis

In this report, we present initial findings from our case study analyses at five exemplary calculus programs at institutions that offer a doctoral degree in mathematics. Understanding the features that characterize exemplary calculus programs at doctoral degree granting institutions is particularly important because the vast majority of STEM graduates come from such institutions. Analysis of over 95 hours of interviews with faculty, administrators and students reveals seven different programmatic and structural features that are common across the five institutions. A community of practice and a social-academic integrations perspective are used to illuminate why and how these seven features contribute to successful calculus programs.

Grand Mesa B

Mathematics Teacher Models Of Quantitative Reasoning

David Glassmeyer, Michael Oehrtman & Jodie Novak

The purpose of this study was to document mathematics teachers' models of quantitative reasoning as they participated in a Model Eliciting Activity (MEA) grounded in their classroom practice. This MEA was designed and implemented in a master's course of 21 in-service mathematics teachers. The documents produced by the teachers were analyzed using a models and modeling perspective to determine how teacher models of quantitative reasoning developed through the MEA. Findings from this study detail how teachers' models of quantitative reasoning were not fully communicated in terms of defining quantitative reasoning in settings not connected to their classroom. As teachers went through the course and the MEA iterations, they began grappling with quantities and quantitative relationships as aspects of quantitative reasoning. Teachers' attention to these aspects better positioned these teachers to reason covariationally about the mathematical content in their documents, thus promoting deep conceptual understanding of functions and more advanced mathematical topics.

Chasm Creek B

The Selection and Use Of Examples By Algebraists: An Exploratory Study

John Paul Cook & Tim Fukawa-Connelly

This paper reports on an exploratory study of 10 algebraists designed to investigate the reasoning behind their selection of examples for their own teaching and research. Variation theory provided a lens with which to analyze the algebraists' goals for their collections of examples and to speculate about the resulting pedagogical implications. Though findings from this exploratory study should be regarded only as preliminary and in need of further justification, our results provide some initial evidence that mathematicians use a relatively small number of very well-chosen classes of examples in both their teaching and their research (suggesting that this might be a useful pedagogical strategy for students as well). We also report on the examples of groups and rings that the algebraists deemed to be the most important for students of introductory abstract algebra.

9:25 – 9:55 am

SESSION 7 – PRELIMINARY REPORTS

Wind Star (1st Floor)

Student Calculus Reasoning Contexts

Matthew Petersen, Sarah Enoch & Jennifer Noll

This paper analyzes how student discourse about Calculus is situated in a graphical representation of a physics problem. Students were asked to identify three unlabeled graphs as representing the position, velocity and acceleration of a car. Findings showed that the students reasoned in three distinct contexts - static-graphical, covariational, and physical. While the students were able to communicate effectively between the first two contexts, and leverage them to find a solution to the problem, the students' discourse in the physical context did not communicate well with their discourse in the other two contexts, nor was it very fruitful in finding a solution to the problem.

Grand Mesa B

Preservice Teachers' Uses Of The Internet To Investigate The Proof Of The Pythagorean Theorem and Its Converse

Aaron Brakoniecki

Learners of mathematics, including preservice teachers, often explore online resources when investigating mathematical problems. When asked to search online for resources that would help them be able to better explain a proof of the Pythagorean theorem and its converse, preservice teachers used a variety of different searching strategies to locate information. Further, the ways in which this information was incorporated into their understanding of mathematics became evident through concept maps. This proposal describes the study conducted and initial results from the data and asks the reader to consider possible ways this research might be extended and refined.

Grand Mesa C

Developing Pre-Service Secondary Math Teachers Capacity With Error Analysis Related To Middle-Grades Mathematics

Diana White

As part of a National Science Foundation Noyce Scholarship Grant, one university substantially revised its preservice secondary (grades 7-12) math teacher preparation program. As one component of this program, preservice teachers take three credit hours of middle level number and operation and geometry, with a focus on mathematical knowledge for teaching. As a research component, we investigated the impact of this course on preservice teachers capacity to identify, analyze, and respond to student errors. This paper provides additional background and results from the first two offerings of the course, as well as ideas for further study.

Chasm Creek A

Investigating Instructors' Concerns About Assessments In Inquiry-Based Learning Methods Courses

Inah Ko & Vilma Mesa

We present initial findings of ongoing research that investigates the nature of instructors' concerns as they design and use assessments for their students using inquiry-based learning (IBL) approaches. Using data collected from biweekly online-teaching logs written by 39 instructors, we categorized concerns into three major themes: Item Design/Assessment, Course/Resources, and Student difficulty. We compare two areas of concerns (designing assessment and using quizzes, tests, and exams) according to the type of concern and the instructor's experience with IBL, course level, and year by using the frequencies of each category cited for each log. Our work will contribute to IBL research by analyzing instructors' challenges as a preliminary study to enhancing IBL teaching and learning in college mathematics education.

Chasm Creek B

Teaching Inquiry-Based Mathematics To In-Service Teachers: Results From The Field

Karen Keene & Celethia McNeil

We present results from a classroom teaching data collection that involved practicing teachers as they participated in an inquiry-oriented differential equations course. Data was collected to investigate how the teachers' participation in this kind of course, different from any of their original mathematics courses, may influence their conceptions of teaching, mathematics, and student learning. Preliminary results indicate that teachers were changed by their experience in the class, at least as expressed in interviews. The teachers were likely to attempt to use more student-centered methods in their classroom and believe that student learning is better in the student-centered environment. Additionally, attitudes about non-lecture, although mixed, did indicate more positive attitudes towards the constructivist perspective on learning. Finally, the teacher's participation in argumentation increased during the course.

9:55 – 10:25 am
Atrium

COFFEE BREAK

10:25 – 10:55 am

SESSION 8 – CONTRIBUTED REPORTS

Chasm Creek B

Graduate Students Teaching Assistants' (GTAs') Beliefs, Instructional Practices, and Student Success

Jessica Ellis

In this report I present findings from a large, national study focused on Calculus I instruction. Graduate student Teaching Assistants (GTAs) contribute to Calculus I instruction in two ways: : as the primary teacher and as recitation leaders. As teachers, GTAs are completely in charge of the course just as a lecturer or tenured track/ tenured faculty would be, although they lack the experience, education, or time commitment of their faculty counterparts. In this study, I investigate how GTAs compare to tenure track/tenured faculty, and other full/part time faculty on their (a) beliefs about mathematics; (b) instructional practices; and (c) students' success in Calculus I. Findings from this report point clearly to a need to prepare GTAs adequately for the teaching of calculus but also for further examination of the nature and implications of the differences between GTA and other instructor types' beliefs about teaching and teaching practices.

Grand Mesa A

Undergraduate Students' Stochastic Understanding Of Probability Distribution

Darcy Conant

Stochastic conceptions undergird development of conceptual connections between probability and statistics and support development of a principled understanding (Greeno, 1978) of probability distribution. This study employed mixed research methods to investigate the impact of an instructional course intervention designed to support development of stochastic understanding of probability distribution. Instructional supports consisted of supplemental lab assignments comprised of anticipatory tasks designed to engage students in coordinating thinking about complementary probabilistic and statistical notions along a hypothetical learning trajectory aimed at development of stochastic understanding of probability distribution. Participants were 184 undergraduate students enrolled in a lecture/recitation, calculus-based, introductory probability and statistics course. Results of quantitative analyses showed completion of stochastic lab assignments had a statistically significant impact on students' stochastic understanding of probability distribution. Student interviews revealed those who held stochastic conceptions also indicted integrated reasoning related to probability, variability, and distribution and presented images supporting principled understanding of probability distribution.

Grand Mesa B

Technology and Algebra In Secondary Mathematics Teacher Preparation Programs

Eryn Stehr & Lynette Guzman

Most recently, the Conference Board of the Mathematical Sciences has advocated for incorporating technology in secondary mathematics classrooms. Colleges and universities across the United States are incorporating technology to varying degrees into their mathematics teacher preparation programs. This study examines preservice secondary mathematics teachers' opportunities to expand their

knowledge of algebra through the use of technology and to learn how to incorporate technology when teaching algebra in mathematics classrooms. We explore the research question: What opportunities do secondary mathematics teacher preparation programs provide for PSTs to encounter technologies in learning algebra and learning to teach algebra? We examine data collected from a pilot study of three Midwestern teacher education programs conducted by a larger project investigating algebra. Our data suggest that not all secondary mathematics teacher preparation programs integrate experiences with technology across mathematics courses, and that mathematics courses may provide few experiences with technology to PSTs beyond strictly computational.

Chasm Creek A

Understanding Students' Conceptualizations Of Logical Tools

Casey Hawthorne

While a significant amount of research has been devoted to exploring why university students struggle applying logic, limited work can be found on how students actually make sense of formal logic itself and the logical mechanisms used to communicate logical equivalence. This project borrows the theoretical framework of unitizing and reification, that have been effectively used to explain the types of integrated understanding required to make sense of symbols involved in numerical computation and algebraic manipulation, to investigate students' conceptualization of truth tables and the implication statements. By using a continuum as a framework to analyze the degree to which students' thinking of each is compartmentalized versus unified, results indicate that students tend to favor one logical mechanism over another, without establishing a holistic view of both or an integrated view of the two together.

11:05 – 11:35 am

SESSION 9 – CONTRIBUTED REPORTS

Chasm Creek B

Comparing Calculus Students' Representation Use Across Different Settings

Dov Zazkis

The distinction between analytic (notation-based) and visual (diagram-based) representations within students' mathematical problem-solving has been part of the mathematics education literature for more than 40 years. However, in spite of this long history there are many unanswered questions regarding how and why particular students choose particular representations, and what influences their social surroundings have on their individual representation use. This study coordinates analyses of undergraduate calculus students' analytic and visual reasoning across both one-on-one interview and group-work settings. These analyses help clarify differences between individual representation use and representation use in group settings.

Chasm Creek A

An Investigation Of College Students' Statistical Literacy

Erin Glover & Sean Larsen

Many statistics educators consider statistical literacy a vital skill because it

supports students in thinking critically about the way data is used every day in social, political, and medical contexts. An important component of statistical literacy is the ability to read, interpret, and contextualize graphical information. As part of a classroom teaching experiment in an introductory college statistics course, students were given a set of graphs and asked to interpret the graphs, compare them and to describe real life contexts that might explain differences in the graphs. This research presentation will share an analysis of student responses to this task, with a particular focus on students' use of statistical language and their abilities to contextualize situations that would produce the given data. Implications for future research as well as pedagogical implications will be discussed.

Grand Mesa A

What Is Simplifying?: Using Word Clouds As A Research Tool

Benjamin Wescoatt

This paper describes the utilization of word clouds within a research methodology. To explore student notions of the concept of "simplify" in a trigonometry course, students responded to the prompt "In your own words, what does it mean to simplify?" The researcher created a word cloud derived from the student responses to explore and identify themes. These themes formed an initial framework for an in-depth analysis of the responses. During the textual analysis, the word cloud was consulted to confirm findings. Using the word cloud in preliminary and confirmatory roles adhered to the framework put forth by McNaught and Lam (2010). From the analysis, students appeared to view the act of simplifying as a process of taking an expression to its most basic state in order to reduce the perceived size (physical or cognitive) of the expression. Moreover, word clouds played a valuable role, providing visual representations of data.

Grand Mesa B

Prospective Secondary Teachers' Conceptions Of Proof and Interpretations Of Arguments

Annamarie Conner, Richard Francisco, Carlos Nicolas Gomez, Ashley Suominen & Hyejin Park

We analyzed the interviews of three prospective secondary mathematics teachers to examine their conceptions of proof and how they validated arguments in the context of students' answers. Our participants had differing views of the definition of proof and its role in mathematics. Their work when validating arguments in large part aligned with their professed views of proof, with some deviations on the part of one participant. Further research must examine whether this consistency is prevalent across prospective teachers and how this relates to teachers' work with proof in classrooms.

11:40 – 12:40 pm
Grand Mesa DEF

LUNCH

12:45 – 1:15 pm

SESSION 10 – PRELIMINARY REPORTS

Grand Mesa C

Student Understanding Of The Fundamental Theorem Of Calculus At The Mathematics-Physics Interface

Rabindra Bajracharya & John Thompson

We studied students' understanding of the Fundamental Theorem of Calculus (FTC) in graphical representations that are relevant in physics contexts. Two versions of written surveys, one in mathematics and one in physics, were administered in multivariable calculus and introductory calculus-based physics classes, respectively. Individual interviews were conducted with students from the survey population. A series of FTC-based physics questions were asked during the interviews. The written and interview data have yielded evidence of several student difficulties in interpreting or applying the FTC to the problems given, including attempting to evaluate the antiderivative at individual points and using the slope rather than the area to determine the integral. The interview results further suggest that students often fail to make meaningful connections between individual elements of the FTC.

Grand Mesa B

Transforming Remedial Mathematics Instruction With High-Quality Peer Teaching

Kristen Bieda, Raven Mccrory & Steven Wolf

This project investigated the potential of a hybrid remedial mathematics course (RMC), taught by a corps of undergraduate peers in a secondary mathematics teacher preparation program, to provide remedial mathematics students with opportunities to develop robust mathematical proficiency. We collected data from four semester exams as well as a final exam for students in both intervention and control sections of the RMC. We also conducted interviews with students in the intervention section of the RMC as well as prospective secondary mathematics teachers (PSMTs) who served as instructors for the course. Our findings show that the instructional model appears to positively impact the learning of students in a RMC, while also providing PSMTs with meaningful opportunities to learn to teach. We will share revisions to our instructional model and seek audience input about approaches to scaling this model to remedial mathematics courses at other institutions.

Wind Star (1st floor)

Mathematicians' Views On Transition-To-Proof and Advanced Mathematics Courses

Milos Savic, Melissa Mills, & Robert Moore

This study explores mathematicians' views on 1) knowledge and skills students need in order to succeed in subsequent mathematics courses, 2) content courses as transition-to-proof courses, and 3) differences in the proving process across mathematical content areas. Seven mathematicians from three different universities (varying in geographic location and department size), were interviewed. Precision, sense-making, flexibility, definition use, reading and validating proofs, and proof techniques are skills that the mathematicians stated were necessary to be successful in advanced mathematics courses. The participants agreed unanimously that a content course could be used as a transition-to-proof course under certain conditions. They also noted differences in the proving processes between abstract algebra and real analysis. Results from this study will be used to frame a larger study investigating students' proof

processes in their subsequent mathematics content courses and investigating how these skills can be incorporated into a transition-to-proof course.

Chasm Creek B

Current and Future Faculty Members' Mathematical Knowledge For Teaching Calculus

Natasha Speer & Shahram Shawn Firouzian

Findings from research into "mathematical knowledge for teaching" have informed the design of preparation and professional development programs for K-12 teachers. At the college level there has been limited research into mathematical knowledge for teaching. We lack findings that demonstrate that expert teachers of college mathematics know and make use of knowledge beyond solely mathematical content.. The goal of this study is to examine the knowledge of student thinking possessed by mathematicians who teach calculus. Data come from interviews on student thinking about core calculus concepts.. Interviewees were research mathematicians who have been recognized for their teaching excellence and mathematics graduate students.. Findings demonstrate that the mathematicians were more able to identify known student difficulties as well as to describe common strategies students use to successfully solve the problems. Implications for research and professional development for novice college mathematics instructors are discussed.

Chasm Creek A

Assessment In Undergraduate Inquiry-Based Learning Mathematics Courses

Timothy Whittemore & Vilma Mesa

We report initial findings of a study that seeks to investigate the methods instructors' use to assess their students' learning and how these assessments affect the instruction in their classrooms. Using data collected from 13 instructors using inquiry-based learning methods, we seek to discuss the instructors' goals for the students, the ways they measure the students' progress towards these goals, the feedback they give students, and how these assessments affected their instruction. Our analysis of the data uses open coding of the transcripts and of the documents (e.g., syllabi, exams, homework assignments) that the instructors gave to the students. Instructors cite using informal assessments and focusing on presentations when asked about "knowing" that students are learning. They cite formal assessments and examinations when asked about "measuring" that students are learning. We seek input on the analysis of the materials as current results may depend on the coding system used.

Grand Mesa A

Professional Development and Student Achievement On Standardized State Exams

Melissa Goss, Rebecca Anne Dibbs & Robert Powers

Although teacher quality is positively correlated with student achievement, easily quantified measures of teacher quality are not accurate measures of quality; teacher pedagogical content knowledge and skills are better predictors, but difficult to measure. Professional development may be a cost-effective vehicle for

developing new skills in in-service teachers, but there is conflicting research on whether professional development measurably raises student achievement on high stakes standardized tests. The purpose of this causal-comparative study was to examine Andrew, an in-service, high school teacher participant in the master's program. State mathematics assessment and student demographic data were collected from school districts for 4 academic years spanning from pre-program through program completion. One-way ANOVA analysis on student scale scores factoring by year showed a significant decrease in student mathematics scale scores potentially attributable to differences in population. Independent-samples t tests on the final two years showed a statistically insignificant increase in student growth percentiles.

1:25 – 1:55 pm

SESSION 11 – THEORETICAL REPORTS

Chasm Creek B

The Duality Principle and Learning Trajectories In Mathematics Education

Eric Weber & Elise Lockwood

The purpose of this paper is to argue that attention to students' ways of thinking should complement a focus on students' understanding of specific mathematical content, and that attention to these issues can be leveraged to model the development of mathematical knowledge over time using learning trajectories. To illustrate the importance of ways of thinking, we draw on Harel's (2008a, 2008b) description of mathematical knowledge as comprised of ways of thinking and ways of understanding. We use data to illustrate the explanatory and descriptive power that attention to the duality of ways of understanding and ways of thinking provides, and we propose suggestions for constructing learning trajectories in mathematics education research.

Chasm Creek A

What Is A Proof? A Linguistic Answer To A Pedagogical Question

Keith Weber

Proof is a central concept in mathematics education, yet mathematics educators have failed to reach a consensus on how proof should be conceptualized. I advocate defining proof as a clustered concept, in the sense of Lakoff (1987). I contend that this offers a better account of mathematicians' practice with respect to proof than previous accounts that attempted to define a proof as an argument possessing an essential property, such as being convincing or deductive. I also argue that it leads to useful pedagogical consequences.

Grand Mesa A

The Construction Of Cohomology As Objectified Action

Anderson Norton

The purpose of this paper is to investigate a theory about the nature of mathematical development, in which mathematics is characterized as the objectification of action. Informed by existing research on how students construct new mathematical objects, we consider as an example, the psychological construction of cohomology and related objects of algebraic topology. This

example is used to test theories of mathematical development through extension and through the author's lived experience. Findings from the self-study are used to integrate existing research on students' learning of abstract algebra, particularly from a neo-Piagetian perspective.

2:05 – 2:35 pm

SESSION 12 – THEORETICAL REPORTS

Grand Mesa A

An Origin Of Prescriptions For Our Mathematical Reasoning

Yusuke Uegatani

To build a supplementary theory from which we can derive a practical way of fostering inquiring minds in mathematics, this paper proposes a theoretical perspective that is compatible with existing ideas in mathematics education (radical constructivism, social constructivism, APOS theory, David Tall's framework, the framework of embodied cognition, new materialist ontologies). We focus on the fact that descriptive and prescriptive statements can be treated simultaneously, and consider both descriptive and exemplary models in our minds. This indicates that descriptive statements in mathematics come from our descriptions of models, and prescriptive statements come from the exemplarity of exemplary models. As a practical suggestion from the proposed perspective, we point out that careful communication is needed so that inquiring minds do not recognize the refutation of their arguments as a denial of their way of mathematical thinking.

Chasm Creek B

Disambiguating Research On Logic As It Pertains To Advanced Mathematical Practice

Paul Dawkins

Many consider logic a hallmark of mathematical practice and an integral part of proof-oriented mathematical instruction. This is true of the term "logic" whether it refers to a domain of mathematical study or to aspects of reasoning, but I claim that these formalized and psychological senses of the term must be carefully distinguished in mathematics education research. In the course of identifying how the abstraction criterion has been misapplied across various types of logic in psychological and mathematics education research, I outline a framework for the disambiguation of the range of research constructs referred to as "logic". By distinguishing the types of logic pertinent to mathematics education instruction, I hope to provide a language by which future research can better specify the constructs they investigate. Clearer research constructs should help the community to understand the role various logics play in students' apprenticeship into the practices of advanced mathematics.

Chasm Creek A

Two Metaphors For Realistic Mathematics Education Design Heuristics: Implications For Documenting Student Learning

Estrella Johnson

The primary goal of this work is to articulate a theoretical foundation based on Realistic Mathematics Education (RME) that can support the analysis of student learning. To do so, I will first frame the guided reinvention and emergent models

design heuristics separately in terms of both increasingly general student activity and in terms of concept development. Then, I will consider how the RME design heuristics could inform how one conceptualizes student learning. To do so, I will draw on two metaphors for learning and, by drawing on these two perspectives, propose ways in which the RME design heuristics can inform the analysis of student learning.

2:35 – 3:05 pm
Atrium

COFFEE BREAK

3:05 – 3:35 pm

SESSION 13 – PRELIMINARY REPORTS

Chasm Creek A

Characterizing Mathematical Complexity Of Tasks In Calculus I

Nina White, Vilma Mesa & Cameron Blum

We present findings from a revised framework created to analyze tasks that calculus teachers assign their students. In the presentation we will highlight the features of the analytical framework and the steps taken to ensure high inter-coder reliability. The framework has been used to analyze all tasks (N=2,996) present in homework, quizzes, and exams from six faculty teaching Calculus I in two two-year colleges. We highlight some insights we have gained in creating this framework and possible uses by other researchers and other contexts.

Chasm Creek B

The Value Of Systematic Listing In Correctly Solving Counting Problems

Elise Lockwood & Bryan Gibson

Although counting problems are easy to state and provide rich, accessible problem solving situations, there is much evidence that students struggle with solving counting problems correctly. With combinatorics (and the study of counting problems) becoming increasingly prevalent in K-12 and undergraduate curricula, there is a need for researchers to identify potentially effective instructional interventions that might give students greater success as they solve counting problems. In this study, we tested one such intervention – having students engage in systematic listing of what they were trying to count. We found that even creating partial lists of the set of outcomes was a significant factor in students' success on problems. Our findings suggest that more needs to be done to refine instructional interventions that will facilitate listing. We discuss these findings, suggest follow-up studies, and request feedback from the audience.

Grand Mesa A

Student Conceptions Of Trigonometric Identities Through Apos Theory

Benjamin Wescoatt

This preliminary study attempts to describe an initial genetic decomposition of a trigonometric identity for college students. Scant research exists into the concepts found in trigonometry. Thus, little is known about how students actually understand a trigonometric identity. Following the guidelines of APOS theory, an initial genetic decomposition for a trigonometric identity was proposed. According to this

decomposition, students with action conceptions can verify identities explicitly using step-by-step manipulations while students holding a process conception are able to visualize steps to demonstrate that the identity is true. Having an object conception means students recognize the truth of the equality without verification and are able to then use the identity to verify other identities. After observing students in task-based interviews, needed modifications to the genetic decomposition became apparent. For example, students' conceptions of the function argument appeared to influence the verification process.

Grand Mesa B

What Constitutes A Well-Written Proof?

Robert Moore

The purpose of this study was to identify some of the characteristics mathematicians value in good proof writing. Four mathematicians were interviewed. First, they evaluated and scored six proofs of elementary theorems written by students in a discrete mathematics or geometry course, and second, they responded to questions about the characteristics they value in a well-written proof and how they communicate these characteristics to students. Preliminary results indicate that these mathematicians agreed that the most important characteristics of a well-written proof are (a) correct logic and (b) clarity. Although these mathematicians differed in the attention they give to layout, grammar, punctuation, and mathematical notation, they agreed in giving these characteristics relatively little weight in the overall score. The results also showed that, in addition to demonstrating good proof writing in class, writing comments on students' papers is an important way they teach their students to write good proofs.

Wind Star (1st Floor)

Characteristics Of Successful Programs In College Calculus: How Calculus Instructors Talk About Their Students

Sean Larsen, Estrella Johnson & Dov Zazkis

The CSPCC (Characteristics of Successful Programs in College Calculus) project is a large empirical study, investigating mainstream Calculus 1, that aims to identify the factors that contribute to successful programs. The CSPCC project consists of two phases. Phase 1 entailed large-scale surveys of a stratified random sample of college Calculus 1 classes across the United States. Phase 2 involves explanatory case study research into programs that were identified as successful based in part on the results of the Phase 1 survey. During our case study site visits, we interviewed calculus instructors and asked a number of questions that prompted them to discuss their students. The purpose of the analyses we will present here is to characterize the ways that calculus instructors talk about their students. To do so, we will examine instructor survey responses and analyze instructor interviews conducted at the case-study institutions (PhD and Bachelors granting levels).

Grand Mesa C

Student Views About Truth In Axiomatic Mathematics

Brian Katz

An undergraduate mathematics major should come to hold appropriate views about the conclusions reached by our disciplinary methods. This project explores

the views about truth in axiomatic mathematics of a group of students who are (mostly) in their final proof-based course, Modern Geometry. Do these students hold expert-like views about truth in mathematics, and do those views change during a course that emphasizes epistemological themes? I find preliminarily that many of these experienced students do not distinguish the truth-value of theorems from that of definitions or axioms at the start of the term, but they develop more expert-like perspectives on truth during the course.

3:45 – 4:15 pm **SESSION 14 – CONTRIBUTED REPORTS**

Grand Mesa B

Model-Of To Model-For In The Context Of Riemann Sum

Kritika Chhetri & Jason Martin

This research focuses on mental challenges that students face and how they resolve these challenges while transitioning from intuitive reasoning to constructing a more formal mathematical structure of Riemann sum while modeling “real life” contexts. A pair of Calculus I students who had just received instruction on definite integral defined using Riemann sums and illustrated as area participated in ten interviews. They were given three contextual problems related to Riemann sums but were not informed of this relationship. While modeling these problem situations, the intent was to observe students’ transitioning from “model-of” to “model-for” reasoning based on Gravemeijer and Stephan (2010). Results indicate that it was not the end results but records of their ways of acting and reasoning about their contextual problem through multiple representations along with real life intervention that served as tools for supporting their transition from “model-of” informal activities to “model-for” more formal mathematical reasoning.

Chasm Creek B

Student Understanding Of Mean, Distribution and Standard Deviation

Samuel Cook & Tim Fukawa-Connelly

This study investigates the understandings of mean, median, distribution and standard deviation that undergraduate students have at the end of an introductory statistics course. The goal was to explore their understandings as a follow-up to previous studies documenting incoming student difficulties with the concepts and determine whether a course would help them achieve a more statistically appropriate understanding. They overwhelmingly think about the mean as the “average” and via the calculating formula, meaning they understand it as a process. Similarly, they understand the median in terms of the process for determining it, or via the location-based term, “middle.” As a result, students do not generally understand the two measures to be describing a similar concept. Students do, reliably connect the shape of a distribution to standard deviation, but that connection varies by type of display and is not based on a reliable rule.

Grand Mesa A

Preservice Secondary Teachers' Understanding Of The Cartesian Connection and Equivalence

Kyunghee Moon

Both prior research and national standards emphasize the importance of critical

ideas, such as the Cartesian Connection and equivalence, in algebra problem solving. The mathematics education community, however, has yet to determine whether the secondary teachers who teach such ideas fully grasp these ideas themselves. To investigate this, I interviewed a cohort of nine preservice secondary teachers in a teacher education program with two algebra problems that embed these ideas. The results showed that many of the teachers failed to understand equivalence as a relation between geometric objects, and thus could not solve an algebra problem by relating algebraic equations to their corresponding graphs. Many also misinterpreted the meaning of the term "solution," and thus could not use the Cartesian Connection to find a solution of an equation. It is advisable that secondary teacher education programs focus more on these critical ideas so that secondary teachers can impart such ideas on their students.

Chasm Creek A

Living It Up In The Formal World: An Abstract Algebraist's Teaching Journey

John Paul Cook, Ameya Pitale, Ralf Schmidt & Sepideh Stewart

Abstract algebra is a fascinating field of study among mathematics topics. Despite its importance, very little research has focused on the teaching of abstract algebra. In response to this deficiency, in this study we present an abstract algebra professor's daily activities and thought processes as shared through his teaching diaries with a team of two mathematics educators and another abstract algebraist over the period of two semesters. We examined how he was able to live in the formal world of mathematical thinking while also dealing with the many pedagogical challenges that were set before him during the lectures.

4:25 – 4:55 pm

SESSION 15 – CONTRIBUTED REPORTS

Chasm Creek A

Are Students Better At Validation After A Transition-To-Proof Course?

Annie Selden & John Selden

This paper presents the results of an empirical study of the proof validation behaviors of sixteen undergraduates after taking a transition-to-proof course that emphasized proof construction. Students were interviewed individually towards the end of the course using the same protocol used by Selden and Selden (2003) at the beginning of a similar course. Results include a description of the students' observed validation behaviors, a description of their proffered evaluative comments, and the suggestion that taking a transition-to-proof course does not seem to enhance students' validation abilities. We also discuss distinctions between proof validation, proof comprehension, proof construction and proof evaluation and point out the need for future research on how these concepts are related.

Grand Mesa A

Considering Mathematical Practices In Engineering Contexts Focusing On Signal Analysis

Reinhard Hochmuth, Rolf Biehler & Stephan Schreiber

In the light of a rough description of the different contexts in which mathematics is

learned and used in engineering studies, this report addresses epistemic relations between mathematics in higher mathematics lectures and mathematics in advanced engineering courses. In particular it elaborates on how different meanings of symbols, as subjectively relevant aspects of mathematical objects, are related to different institutional contexts and their dominant discourses. It is argued that modeling cycles are not an adequate tool in this context. Instead, we suggest using concepts from Anthropological Theory of Didactics (ATD). Inspired by (Castela & Romo Vázquez, 2011), exemplarily concepts from ATD are applied to topics and data from signal analysis. Finally, we claim this research could serve as a step towards investigating empirical questions relevant to students' learning and competences and, in particular, optimizing curricula and teaching in undergraduate mathematics.

Chasm Creek B

Evaluating Professional Development Workshops Quickly and Effectively

Charles Hayward & Sandra Laursen

Many funding agencies require evaluation of the impact of professional development projects they support. However, improved student outcomes, the ultimate goal, may take longer to be realized than the project time frame allows. Instructors need time to implement and refine new skills before positive student outcomes are realized, a delay that may be exacerbated in classes that are not taught frequently. We report on an efficient and cost-effective self-report measure designed to detect the initial changes in teaching practices that lead to improved student outcomes over time. We discuss the ability for timely and accurate measures through this instrument. Results support the interpretation that instructors' reported teaching practices show changes consistent with methods taught at professional development workshops on Inquiry-Based Learning in mathematics. Additionally, correlations with self-reported level of implementation suggest that instructors are reporting honestly, and not just socially desirable changes consistent with their concept of "real Inquiry-Based Learning."

Grand Mesa B

An Eye To The Horizon: The Case Of Delia's Hexagon

Ami Mamolo

This paper explores pre-service secondary school mathematics teachers' preferences when advising a student on how to determine the area of an irregular hexagon. The research attends to participants' personal mathematical knowledge, as interpreted through the lens of Knowledge at the Mathematical Horizon. Philosophical notions of inner and outer horizons of conceptual objects are adapted to provide a refined analysis of participants' personal strategies and preferences as evoked by an unconventional problem. The interplay amongst participants' understanding of mathematical structure, their focus of attention when interpreting a problem, and the advice they offer to a student are of interest. Implications for teacher education and further avenues of research are suggested.

5:10 – 6:10 pm
Grand Mesa DEF

PLENARY SESSION

Plenary Speaker: Anna Sfard

Mathematics Learning: Does Language Make A Difference?

Abstract: Mathematics and its learning are generally believed to be relatively independent of the language in which they are practiced. This assumption tacitly underlies the nowadays popular idea of international comparisons such as TIMSS or PISA, in which young people from all over the world are being tested with the help of a single mathematical questionnaire. The fact that the questions appear in different languages does not diminish examiners' conviction that, wherever they go, they are testing "the same mathematics," thus assessing fully comparable types of learning.

And yet, in the view of recent theoretical developments and some new empirical findings, the assumption about the language-proof nature of mathematics and its learning may be questioned. This issue is of particular importance to those who teach mathematics in schools and universities. Indeed, if it turns out that the way people learn is shaped by their main language, there may be significant differences in the needs of learners gathered in the same multilingual classroom.

The question of the impact of language on mathematics learning is the focus of this talk. I will begin with a brief historical survey of research guided by the famous Sapir-Whorf Hypothesis, according to which all human thinking is shaped by language. I will follow with a theoretical reflection on the relation between thinking and communication, undertaken in an attempt to reconceptualize the topic. I will then use the resulting conceptual apparatus while summarizing and interpreting results of two studies, one on learning limits and infinity and the other on learning fractions and probability, both of them launched in the quest for dissimilarities in mathematical discourses of learners coming from different linguistic backgrounds.

DINNER ON YOUR OWN

SATURDAY, MARCH 1, 2014

9:00 – 9:30 am

Chasm Creek B

SESSION 16 – CONTRIBUTED REPORTS

Students' Struggle With The Temporal Order Of Delta and Epsilon Within The Formal Definition Of A Limit

Aditya Adiredja & Kendrice James

Studies about students' understanding of the formal definition of a limit, or the epsilon delta definition suggest that the temporal order of delta and epsilon is an obstacle in learning the formal definition. While such difficulty has been widely documented, patterns of students' reasoning are largely unknown. This study investigates the degree of difficulty students have with the temporal order, along with justifications that students provide to support their claim. diSessa's Knowledge in Pieces provides a suitable framework to explore the context specificity of students' knowledge as well as the potential productivity of their prior knowledge in learning.

Chasm Creek A

The Construction Of A Video Coding Protocol To Analyze Interactive Instruction In Calculus and Connections With Conceptual Gains

Matthew Thomas

Instruments called concept inventories are being used to investigate students' conceptual knowledge of topics in STEM fields, including calculus. One interactive instructional style called Interactive-Engagement has been shown to improve students' gains on such instruments in physics. In this paper, we discuss the development of a video coding protocol which was used to analyze the level of Interactive-Engagement in calculus classes and investigate the correlation with gains on the Calculus Concept Inventory.

Grand Mesa A

Academic and Social Integration Revealed In Characteristics Of Successful Programs In College Calculus Project: The Two-Year College Context

Vilma Mesa, Nina White & Helen Burn

We present an analysis of features common across four Calculus I programs at two-year colleges identified as successful in the Characteristics of Successful Programs in College Calculus (CSPCC) study. In this paper we discuss how these features emerged in the analysis of the four cases and their connection to theories of student academic and social integration. Student academic and social integration have been identified as closely related to student persistence in college. We used a constant comparative analysis to identify themes within and across institutions, using transcripts of 22 interviews with faculty, staff, and administrators, and student focus groups. We discuss three of the seven major themes that arose, High quality instructors, Faculty autonomy and trust in the teaching of calculus, Supporting students academically and socially, and Attention to placement, which support a model of student academic and social integration. We present further research steps and some implications for practice.

Grand Mesa B

Teaching The Concept Of Mathematical Definition Using Student Construction and Self-Assessment

Susanna Molitoris Miller

Definitions are an important part of the study of mathematics, yet many students

struggle with successfully understanding and using this construct. It has been suggested that students may improve their understanding of mathematical definitions by engaging in the act of writing definitions (de Villiers, Govender, & Patterson, 2009). Through a mixture of survey and teaching experiment methodology this study explores pre-service elementary teachers' understanding of mathematical definitions before and after engaging in a teaching experiment which provided many opportunities for the participants to write their own mathematical definitions for familiar and novel classes of quadrilaterals. Definitions were assessed as having necessary, sufficient and minimal conditions. It was found that while many students initially struggled to write definitions that meet these qualifications, the process of trying to construct their own definitions did improve students' understanding of these characteristics of mathematical definitions.

9:40 – 10:10 am **SESSION 17 – PRELIMINARY REPORTS**

Grand Mesa B

Differential Participation In Formative Assessment and Achievement In Undergraduate Calculus

Rebecca Dibbs & Michael Oehrtman

Prior formative assessment research has shown positive achievement gains when classes using formative assessment are compared to classes that do not. However, little is known about what, if any, benefits students that are not participating regularly in formative assessment gain from these assignments. The purpose of this study was to investigate the achievement of the students in two introductory calculus courses using formative assessment at the three different participation levels observed in class. Although there was no significant difference on any demographic variable other than gender and no significant difference in any achievement predictive variables between the groups of students at the different participation levels, there were significant differences in achievement on all but the first activity write-up and the final exam.

Grand Mesa A

Cognitive Processes and Knowledge In Activities In Community College Trigonometry Lessons

Linda Leckrone & Vilma Mesa

Over 50,000 students take trigonometry at two-year colleges in the U.S., yet little is known about their instruction. We report an analysis of activities in trigonometry classes taught at a community college attending to two dimensions, the type of knowledge used (Factual, Procedural, Conceptual, and Metacognitive) and the cognitive processes (Remember, Understand, Apply, Analyze, Evaluate, Create) intended in the activity as enacted by teachers in their lessons. Most of the 163 activities were classified as applying procedural knowledge; over one-fifth of the activities were coded as remembering factual knowledge or understanding conceptual knowledge. We discuss these findings in light of the community college setting and offer some questions for further research.

Chasm Creek B

Using The Flipped Model To Address Cognitive Obstacles In

Differential Equations

Jenna Tague, Jennifer Czocher, Amanda Roble & Gregory Baker

Recent work has shown that there is a lack of coherence from calculus to differential equations: the knowledge calculus knowledge students are expected to gain by the end of the calculus sequence is different from how that knowledge is expected to be used in differential equations (Authors). In this report, we describe how we have begun to address some of these issues with coherence through utilizing the flipped classroom model. We share our theoretical perspective, how it was enacted using the classroom model and technology, and also a preliminary evaluation of students' expectations of and perceptions of the coherence of the course and its content.

Chasm Creek A

An Exploration Of Mathematics Graduate Teaching Assistants' Teaching Philosophies

Kedar Nepal

This is an investigation of the teaching philosophies of beginning mathematics graduate teaching assistants. Three teaching philosophy statements from each of four participants were collected at different stages of a semester-long teaching assistant preparation program and analyzed. Principal elements found in these statements before they underwent training and how their philosophies changed over time during training will be discussed.

Wind Star (1st Floor)

Approximation: A Connecting Construct Of The First-Year Calculus?

Kimberly Sofronas, Thomas Defranco, Hariharan Swaminathan, Charles Vinsonhaler, Nicholas Gorgievski & Brianna Wiseman

This report will present preliminary findings from a research study designed to investigate calculus instructors' perceptions of approximation as a central concept and possible unifying theme of the first-year calculus. The study will also examine the role approximation plays in participants' self-reported instructional practices. A survey was administered through Qualtrics to a stratified random sample of 3930 mathematicians at higher education institutions throughout the United States with a desired N = 300. Quantitative and qualitative methods were used to analyze the data gathered. Findings from this research will contribute to what is known about the perceptions and teaching practices of calculus instructors regarding the role of approximation in first-year calculus courses. Research-based findings related to the role of the approximation concept in the first-year calculus could have implications for first-year calculus curricula.

Grand Mesa C

Noticing The Math In Issues Of Social Justice

Ami Mamolo

This preliminary report examines pre-service secondary mathematics teachers' engagement with problems which contextualized mathematics in issues of social justice. A framework for Teaching Mathematics for Social Justice was employed and participant responses were analysed with respect to what mathematics they

noticed and attended to in and after the problem solving. Results suggest participants had difficulty “seeing” the math in non-math contexts, and that their ability to notice the embedded mathematics was influenced by the specific social context as well as their orientation towards mathematics (both in general and regarding specific content). Implications for research and teacher education are described.

10:10 – 10:40 am
Atrium

COFFEE BREAK

10:40 – 11:10 am

SESSION 18 – PRELIMINARY REPORTS

Chasm Creek A

Presentation Of Matrix Multiplication In Introductory Linear Algebra Textbooks

John Paul Cook & Sepideh Stewart

We conducted an analysis of 17 modern, introductory linear algebra textbooks to investigate presentations of matrix multiplication. Using Harel’s (1987) textbook analysis framework, we examined the sequencing of matrix multiplication and its accompanying rationale. We found two principal sequences: one which first defines the operation as a linear combination of column vectors before introducing the dot product method (LC to DP), and another which invokes the dot product method before linear combinations (DP to LC). The rationale for these two trajectories varied in interesting ways. LC to DP demonstrates that solving a system of linear equations is equivalent to solving its corresponding matrix equation Ax=b. The rationale for DP to LC was less focused, opting in several cases to postpone the explanation until linear transformations are covered. We hope to initiate a discussion about the effectiveness of and pedagogical implications for these two contrasting approaches.

Grand Mesa A

Differentiated Student Thinking While Solving A Distance Vs. Time Graph Problem

Eric Pandiscio

This study probes the thinking of students at different stages: a) secondary students taking calculus, b) college students taking calculus, and c) college students pursuing teacher certification taking a mathematics course other than calculus. The study asks: 1) what is the nature of student thinking when solving a graph problem, and 2) do students with different levels of mathematical experience solve a graph problem differently? A pilot investigation reveals many students estimate answers, even if they had studied calculus. For the current study, data will be collected during Fall, 2013. Oral interviews will be conducted with a subset of the participants and coded via Grounded Theory (Strauss & Corbin, 1990; Dick, 2005). This work follows physics education (McDermott, Rosenquist & van Zee, 1987; Thornton & Sokoloff, 1990; Kim & Kim, 2005), and mathematics education (Chiu, Kessel, Moschkovich & Munch-Nunez, 2001; Moschkovich, 1996) that describe difficulties students have with graph interpretation.

Grand Mesa B

Undergraduate Students' Use Of Intuitive, Informal, and Formal Reasoning To Decide On The Truth Value Of A Mathematical Statement

Kelly Bubp

Although deciding on the truth value of mathematical statements is an important part of the proving process, students are rarely engaged in making such decisions. Thus, little is known about the ways in which students use intuitive, informal, and formal reasoning to evaluate conjectures. In this study, task-based interviews will be conducted with undergraduate students in which they will be asked to determine the truth value of five mathematical statements on functions and relations. Students' reasoning on these tasks will be classified as intuitive, informal, or formal, and then further categorized according to the findings of current research, with new categories added as needed. This study should contribute to our understanding of the ways in which students reason when dealing with uncertainty in the proving process. Additionally, this study may suggest ways in which educators can assist students in navigating the often difficult process of proving and refuting mathematical statements.

Grand Mesa C

A Framework and a Study To Characterize a Teacher's Goals For Student Learning

Frank Marfai

In this study, a secondary school teacher's goals for student learning were characterized using a framework that emerged from prior work. Observed lessons spanning the use of both conceptually rich and conceptually poor curricula were analyzed and lead to unexpected findings, suggesting that both challenges and opportunities for professional development endeavors exist that center around perturbing a teacher's goals.

Chasm Creek B

Instructors' Beliefs On The Role Of Calculus

Kathleen Melhuish & Estrella Johnson

In this report we will draw on the Characteristics of Successful Programs in College Calculus data set in order to investigate instructor beliefs about the role calculus plays. Specifically, in this preliminary report, we have analyzed instructor interview transcripts in order to address the question: How do instructors perceive the role of calculus at successful four-year universities? Our preliminary analysis has uncovered six emerging themes. Each will be presented and illustrated with an instructor's quote.

Wind Star

Mathematical Perceptions and Problem Solving Of First Year Developmental Mathematics Students In A Four-Year Institution

Anne Cawley

I report initial findings of a study that seeks to investigate the change in developmental (remedial) mathematics students' mathematical problem solving skills. I report on the analysis of one-on-one interviews with six students before a

four-week Intermediate Algebra course. The ultimate goal is to see the extent to which their skills changed after the course. Using a framework of reasoning developed by Lithner (2000), I describe events in which one particular student shows plausible reasoning and also reasoning based on established experience. I seek input with regard to alternative frameworks or analysis of the data that may help me interpret the findings.

11:20 – 11:50 am

SESSION 19 – CONTRIBUTED REPORTS

Grand Mesa B

Teaching Methods and Student Performance In Calculus I

Barbara Trigalet, Lisa Mantini & R. Evan Davis

Classroom teaching in multiple sections of Calculus I at a large comprehensive research university was observed and coded using the Teaching Dimensions Observation Protocol (TDOP). Multiple teaching styles were identified ranging from low engagement to moderate engagement to high engagement sometimes including student group work. Student performance on two course-wide uniform exams and on the Calculus Concept Inventory was analyzed for any correlations with teaching methods. Significant correlations were found with high engagement teaching styles on both the first exam and the final exam. However, no significant correlations were found on the Calculus Concepts Inventory, indicating that students may not have exerted much effort on this assessment.

Chasm Creek A

Exploring Students' Ways Of Thinking About Sampling Distributions

Aaron Weinberg

The concept of a sampling distribution plays a central role in the process of making statistical inferences. However, students typically struggle to understand and reason about sampling distributions. This study seeks to characterize the ways undergraduate students think about sampling distributions in scenarios involving repeated sampling and making statistical inferences. Eight students in an introductory statistics class worked on problems involving sampling distributions during a semi-structured interview. A framework was developed based on their responses to describe the ways they discussed and coordinated various aspects of the population and sampling distributions by focusing on the processes of sampling and repeated sampling; these descriptions suggest that explicitly coordinating particular aspects of these processes may correspond to the robustness of students' conceptions of sampling distributions.

Grand Mesa A

Supporting Students To Construct Proofs: An Argument Assessment Tool

Martha Byrne & Justin Boyle

Engaging students in the construction of proofs often does not include conversations about what does and does not count as proof, to the detriment of the students. The critiques of student-generated arguments should be communicated in a language common to instructor and student; such a language can be developed via an assessment tool that is accessible to both parties. This

paper describes the development of an argument assessment tool that will be useful for instructors and researchers both to assess students' and participants' ability to construct proofs and to communicate those assessments. The tool is introduced and two assessed student arguments are to illustrate the tool's application. Future work with the argument assessment tool will include its use in a classroom as an instructional tool for establishing a common language for instructor and students and providing the foundation for discussions about proof production.

Chasm Creek B

Students' Use Of Parameters and Variables To Reason About Multivariable Functions

Eric Weber

The purpose of this paper is to characterize students' ways of thinking about parameters and variables to reason about the behavior of multivariable functions. I focus on two single variable calculus students, Lisa and Carl, as they participated in a sequence of semi-structured exploratory teaching interviews intended to gain insight into 1) their approaches to reasoning about the behavior of single variable functions, and 2) what role those approaches played in their initial thinking about the behavior of functions of two, three and four variables. The interviews suggest that the students' ability to move flexibly between thinking about a function's variables as parameters allowed them to generalize their reasoning patterns about functions of n variables and extend that to functions of n+1 variables. I argue that their ability to parameterize functions allowed them to reason about functions for which they could not initially visualize representations.

11:50am–1:50 pm
Grand Mesa DEF

LUNCH

1:50 – 2:20 pm

SESSION 20 – PRELIMINARY REPORTS

Chasm Creek A

Slope and Derivative: Calculus Students' Understanding Of Rates Of Change

Jen Tyne

Studies have shown that students have difficulty with the concepts of slope and derivative, especially in the case of real-life contexts. I used a written survey to collect data from 75 differential calculus students. Students answered questions about linear and nonlinear relationships and interpretations of slope and derivative. My analysis focused on students' understanding of slope as a constant rate of change and derivative as an instantaneous rate of change, and what these meant in the context of the problems. Preliminary results indicate that students have more success with slope questions than derivative questions (McNemar's test, $p<0.03$), and that while students correctly use the slope of a linear relationship to make predictions, they do not demonstrate an understanding of the derivative as an instantaneous rate of change and an estimate of the marginal change. Plans for a modified survey and interviews are in place for fall 2013.

Chasm Creek B

Student Understanding Of Linear Independence Of Functions

David Plaxco, Megan Wawro & Lizette Zietsman

In this study, we present preliminary findings regarding student understanding of linear independence of vector-valued functions. Students were given a series of homework questionnaires and participated in individual and paired interviews. The researchers used grounded theory to categorize student approaches for determining linear (in)dependence of functions. In order to gain insight into students' intuitive notions, data were collected before any formal instruction about the definition of linear independence of functions. The researchers describe initial analyses of student approaches, conjecturing their treatment of vector-valued functions at specific t-values or for varying t as a potentially beneficial lens of analysis. Students who evaluated specific t-values determined the linear independence of a set of vectors in R2 rather than the linear independence of the set of functions, themselves elements of a function space. The analytical construct of process/object pairs (Sfard, 1991) could be a useful lens to explore this distinction.

Grand Mesa B

Proof Conceptions Of College Calculus Students

Jon Janelle

This study investigated 52 college Calculus students' views about the nature and purpose of mathematical proof and which forms of empirical argument they perceived as valid and convincing proofs. Past studies of student proof conceptions have primarily focused on three groups: students in secondary geometry courses, pre-service and in-service teachers, and advanced undergraduate and graduate students who have received formal instruction in the creation of deductive proofs. This study fills a gap in the literature by examining students' conceptions after the completion of a high school geometry course, but before enrollment in a course focused on the creation and evaluation of mathematical proofs. Survey questions and coding systems were adapted from previous studies. Preliminary findings suggest that a majority of college Calculus believe that the inspection of a few cases and the testing of an extreme case are valid methods for proving mathematical conjectures.

Wind Star (1st Floor)

An Investigation Into Students' Use Of Given Hypotheses When Proving

Kathleen Melhuish

The mathematical practice of strengthening or weakening a theorem requires careful attention to hypothesis and conclusion. Selden and Selden (1987) reported that students often unintentionally weaken theorems raising concerns of undergraduates' attention to hypothesis. In this paper, I consider both the prevalence of this error and what the practice of strengthening/weakening a theorem may look like. A survey of prove/disprove prompts was piloted with five graduate students. A subset of these prompts was then given to undergraduates in an introductory group theory course. Preliminary results indicate that the error of weakening the theorem is prevalent amongst both populations. The graduate students participated in follow-up interviews where they were prompted to

strengthen/weaken conjectures to further examine their attention to the hypotheses. In this preliminary report, I will present the survey results and one graduate case to illustrate what the practice of strengthening/weakening a theorem may look like.

Grand Mesa A

The Effect Of 5 Minute Preview Video Lectures Using Smart Board, Camtasia Studio, and Podcasting On Mathematical Achievement and Mathematics Self-Efficacy

Minsu Kim

The purpose of this study is to examine the effectiveness of 5 minute preview video lectures for each lecture using podcasting in terms of mathematical achievement and mathematics self-efficacy in intermediate algebra and college algebra courses at a university. Data from 128 students in six sections collected for two semesters through first and final exams, questionnaires, classroom observation checklist, and the Mathematics Self-Efficacy Scale. The preliminary findings indicate no significant difference on the mathematical achievement and mathematics self-efficacy between the control group who did not watch the preview lectures and the treatment group who watched the preview lectures while the treatment group slightly developed their mathematics self-efficacy and abilities for mobile technology. In addition, the treatment group was significantly satisfied with the preview lectures. When the treatment group was divided into intermediate low and high subgroups based on the first exam, the intermediate low subgroup significantly improved their mathematical achievement.

Grand Mesa C

Characteristics Of Successful Programs In College Calculus: Instructors' Perceptions Of The Usefulness and Role Of Instructional Technology

Erin Glover & Sean Larsen

The CSPCC (Characteristics of Successful Programs in College Calculus) project is a large empirical study, investigating mainstream Calculus 1, that aims to identify the factors that contribute to successful programs. The CSPCC project consists of two phases. Phase 1 entailed large-scale surveys of a stratified random sample of college Calculus 1 classes across the United States. Phase 2 involves explanatory case study research into programs that were identified as successful based in part on the results of the Phase 1 survey. This second phase will lead to the development of a theoretical framework for understanding how to build a successful program in calculus and in illustrative case studies for widespread dissemination. Technology was one of the topics we explored with students, instructors, administrators, and other individuals that we interviewed during our case study site visits. In this preliminary report, we will focus on calculus instructors' views on instructional technology.

2:30 – 3:00 pm

SESSION 21 – CONTRIBUTED REPORTS

Grand Mesa A

Reinventing Permutations and Combinations

Elise Lockwood, Craig Swinyard & John Caughman

Counting problems provide an accessible context for rich mathematical thinking, yet they can be surprisingly difficult for students. While some researchers have addressed these difficulties, more work is needed to uncover ways to help students count effectively. In an effort to foster conceptual understanding that is grounded in students' thinking, we had two undergraduate students engage in guided reinvention in a ten-session teaching experiment. In this experiment, the students successfully reinvented four basic counting formulas. In follow-up problems, combinations proved to be the most problematic for them, however, suggesting that the learning of combinations may require special attention. In this presentation, we describe the students' successful reinvention, and we discuss potential reasons for the students' issues with combinations. We additionally present potential implications and directions for further research.

Chasm Creek B

Calculus Students' Understanding Of Units

Allison Dorko & Natasha Speer

Units of measure are critical in many scientific fields. Instructors often note that students struggle with units, yet little research has been conducted about the nature of these difficulties or why they exist. Area and volume play important roles in calculus topics such as optimization, volumes of revolution, and related rates, yet we do not know what understandings of area and volume students bring with them to their study of these topics. We used written surveys and interview data to investigate calculus students' use of units in area and volume computational problems. Only 26.6% of students gave correct units for all tasks. These students understood arrays, dimensionality, and rules of exponents. In contrast, students who struggled with units did not. Common errors included misappropriation of length units and difficulty identifying the units of computations that involved π. Our findings are similar to findings about elementary school students' difficulties with units.

Chasm Creek A

Proof Scripts As a Lens For Exploring Proof Comprehension

Rina Zazkis & Dov Zazkis

We examine perspective secondary teachers' conceptions of what constitutes comprehension of a given proof and their ideas of how students' comprehension can be evaluated. These are explored using a relatively novel approach, scripted dialogues. The analysis utilizes and refines Mejia-Ramos, Fuller, Weber, Rhoads and Samkoff's (2012) proof comprehension framework. We suggest that this refinement is applicable to other studies on proof comprehension.

Grand Mesa B

A Typology Of Validating Activity In Mathematical Modeling

Jennifer Czocher

Mathematical modeling tasks are used to help students learn mathematics and also to improve their modeling skills. Validating has been identified as the process by which students check and revise their models, but little is known about when or how students choose to do so. This study examined engineering students' validating activity and identified a typology of different kinds of validating activity

satisfying different roles in ensuring accuracy of the model.

3:00 – 3:30 pm
Atrium

COFFEE BREAK

3:30 – 4:00 pm

SESSION 22 – CONTRIBUTED REPORTS

Grand Mesa A

On The Sensitivity Of Problem Phrasing - Exploring The Reliance Of Student Responses On Particular Representations Of Infinite Series

Danielle Champney

This study will demonstrate the ways in which students' ideas about convergence of infinite series are deeply connected to the particular representation of the mathematical content, in ways that are often conflicting and self-contradictory. Specifically, this study explores the different limiting processes that students attend to when presented with five different phrasings of a particular mathematical task - ∑(1/2)^n - and the ways in which each phrasing of the task brings to light different ideas that were not evident or salient in the other phrasings of the same task. This research suggests that when attempting to gain a more robust understanding of the ways that students extend the ideas of calculus – in this case, limit – one must take care to attend to not only students' reasoning and explanation, but also the implications of the representations chosen to probe students' conceptions, as these representations may mask or alter student responses.

Grand Mesa B

How Does Undergraduates' Understanding of the Function Concept Evolve During The Course of a Semester?

Eyob Demeke, Vincent Mateescu & Anek Janjaroon

Functions are a crucial topic in the study of mathematics. Research has found that a lack of deep understanding of functions is one of the main reasons why students struggle in calculus (Eisenberg, 1991; Ferrini-Mundy & Graham, 1991; Lauten, Graham, Ferrini-Mundy, 1994; McDonald, Mathews, & Strobel, 2000; Monk, 1994). In light of these studies, we investigate – using traditional paper-and-pencil assessments, concept maps, and an interview – what pre-calculus students' understanding of functions is, to what extent students have a repertoire of functions at their disposal, how students' understanding evolves over a semester, and what non-traditional assessments can tell us about this understanding. We found that (1) As Williams (1998) suggested, concept map assessments do reveal something that traditional assessments do not; (2) participants have trouble giving non-examples of functions, and (3) there does not seem to be a major change in participants' understanding of functions over time.

Chasm Creek B

Naive Brouwerian Visions: A Study Of Students' Interpretations Of Non-Constructive Existence Proofs

Stacy A. Brown

This paper shares findings from a three-phase study exploring students' conceptions of non- constructive existence proofs. Data are used to illustrate

students' tendency to apply a naïve Brouwerian lens to non-constructive proofs; that is, a perspective in which learner's proof conceptions are governed by a potentially subconscious anticipation of construction, which enables the learner to construe proofs of existence (be they constructive or non-constructive) as providing actual instances of (or algorithms for producing) mathematical phenomena. Questions concerning researchers proof scheme inferences are raised.

Chasm Creek A

Three Conceptualizations Of The Definite Integral In Mathematics and Physics Contexts

Steven Jones

Student understanding of the integral is a topic of recent interest in undergraduate education. We are just beginning to learn how different interpretations of the definite integral influence student thinking in both mathematics and science classrooms. This paper examines the relative "productivity" of three conceptualizations of the definite integral in mathematics and physics tasks. It appeared that the notion of the integral as an "addition over many pieces" was especially useful for understanding applied problems.

4:10 – 4:40 pm

SESSION 23 – CONTRIBUTED REPORTS

Chasm Creek B

The Ability To Reject Invalid Logical Inferences Predicts Proof Comprehension and Mathematics Performance

Lara Alcock, Toby Bailey, Matthew Inglis & Pamela Docherty

In this paper we report a study designed to investigate the impact of logical reasoning ability on proof comprehension. Undergraduates beginning their study of proof-based mathematics were asked to complete a conditional reasoning task that involved deciding whether a stated conclusion follows necessarily from a statement of the form "if p then q"; they were then asked to read a previously unseen proof and to complete an associated comprehension test. To investigate the broader impact of their conditional reasoning skills, we also constructed a composite measure of the participants' performance in their mathematics courses. Analyses revealed that the ability to reject invalid denial-of-the-antecedent and affirmation-of-the-consequent inferences predicted both proof comprehension and course performance, but the ability to endorse valid modus tollens inferences did not. This result adds to a growing body of research indicating that success in advanced mathematics does not require a normatively correct material interpretation of conditional statements.

Grand Mesa C

Factors Associated With The Success Of Female Mathematics Doctoral Students

Emily Miller

Although the gender gap in participation in undergraduate mathematics has narrowed, disparities still exist at the doctoral level. Only 30 percent of recent doctoral recipients in mathematics were women (Hill, Corbett, & St. Rose, 2010). To increase retention of women in mathematics doctoral programs, it is critical to

study the factors that are associated with their success. A survey was distributed to 142 female mathematics professors asking them to assess the impact of factors that could have contributed to their success. Results point to changeable factors that can be implemented to narrow the gender gap. Salient factors include persistence and dedication, strong undergraduate preparation and quality doctoral courses, and support from the doctoral advisor. Results show that gender still has an impact on the experiences of the participants, but there may be reason for optimism. Respondents who received their doctorates more recently reported less gender discrimination.

Grand Mesa B

Deploying Problems Assessing Mathematical Knowledge For Teaching As Tasks For Professional *Preparation*

Yvonne Lai & Heather Howell

Mathematical knowledge for teaching (MKT) has been shown to be a measurable construct impacting instructional quality and student outcomes. The primary examples that educators have for MKT tasks are those that were designed and validated for assessment purposes. It is not known to what extent features of a task that support its use as assessment may support or hinder its use in instruction. We examine this tension by studying the use of two such MKT tasks in a course for prospective teachers. Key considerations for using MKT tasks in professional preparation tasks were how the MKT task represents teaching practice and the possible purposes of using that representation in teacher education.

Chasm Creek A

Exploring Differences In Teaching Practice When Two Mathematics Instructors Enact The Same Lesson

Joseph Wagner & Karen Keene

Investigating teacher practice at all educational levels has become an important research arena. We analyze teacher practice by comparing two implementations of the same fragment of a student-centered curriculum by two mathematics professors. We highlight differences in their practices and the consequent classroom results by analyzing their participation in class discussions, and we show how Schoenfeld's (2011) resources, goals, and orientations framework may be used to explain these differences. Using classroom and interview data, we identify resources that each instructor believed he lacked, we highlight prominent mathematical and social goals that each instructor held, and we infer orientations toward teaching and learning mathematics that guided each instructor's practices. All of these in combination suggest explanations for the observed differences in the implementations and class outcomes. We believe that this analysis provides an important technique to understand and improve teaching and learning at the undergraduate level in mathematics.

4:45 – 5:15 pm
Atrium

BREAK (CASH BAR)

5:15 – 8:30 pm
Grand Mesa DEF

AWARDS BANQUET & PLENARY SESSION

Plenary Speaker: Ron Tzur

Promoting Teachers' and Students' Learning To Reason Multiplicatively: A Units-and-Operations Developmental Approach

Abstract: This paper presents an approach to the learning and teaching of multiplicative reasoning that focuses on units and operations students may construct, and use, when solving and posing mathematical problems. To explain learning, this approach includes a *6-scheme developmental framework* rooted in studies on children's construction of multiplicative and divisional schemes. This framework (a) distinguishes between two types of units—singletons (1s) and composite—each possibly comprised of concrete, figural, or abstract items, and (b) articulates advances in students ways of coordinating operations on either or both unit types. To promote students' learning and teacher development, this approach foregrounds a *student adaptive mathematical pedagogy* (STAMP; shorthand – adaptive teaching). Adaptive teaching stresses the need to tailor goals for student learning ("what should we teach next?") and activities for accomplishing these goals ("how should we teach this?") to students' available ways of operating on various units. Specifically, teachers learn to design and implement tasks for reactivating, and transforming, schemes that are both available to the students and instigate conceptual pathways to the intended mathematics. Data collected and analyzed in studies that employed this approach will be presented to elucidate and substantiate how it can contribute to teacher change and to student learning and outcomes (both those with learning disabilities and their normal-achieving peers). Implications of this approach to teaching undergraduate students entering university mathematics courses as well as prospective elementary teachers will be discussed.