Classroom Culture, Technology, & Modeling: A Case Study of Students' Engagement with Statistical Ideas

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Advances in technologies have changed the way statisticians do their work, as well as how people receive and process information. The case study presented here follows two groups of two students who participated in a reform-oriented curriculum that utilized technology to engage students with modeling and simulation activities to develop their statistical literacy, thinking, and reasoning. Our analysis applies a social theory of learning and a framework for student engagement as a means for studying students' development of statistical reasoning. In addition, we investigate the impact of a curriculum focused on modeling and simulation on the development of students' statistical reasoning skills.

Key words: Statistics, Engagement, Statistical Reasoning, Technology, TinkerPlots™

Introduction

Today there appears to be a consensus among the statistics education community that approaches to teaching introductory statistics should utilize technology and place emphasis on data and the core concepts of inference rather than on the dissemination of statistical theory (ASA, 2005; Cobb, 1992; 2007, Garfield, delMas, & Zeiffler, 2012). Statistics educators argue that many of the components of our introductory statistics courses (e.g., using a z-score to calculate a 95% confidence interval; computing a standard deviation) are relics dating back to the 1900's historical roots of statistics and need to be reconceived in light of our data-driven, technologically based world (Cobb, 2007; Gould, 2010; Nolan & Lang, 2010). In the hope of aligning curriculum more with the practice of statistics, educators are developing new curriculum (e.g. Garfield et al., 2012, Lock et al., 2013; Tintle et al., 2011) that utilize technology and engage students with modeling and simulation activities to develop students' statistical literacy, thinking, and reasoning. We argue that a better understanding of the ways in which technology and curriculum work together to impact students' development of key statistical ideas is an important next step in statistics education research.

Theoretical Perspective

The authors take the perspective that learning is a result of participation in a classroom community (see for example, Bowers, Cobb & MacClain, 1999; Gresalfi, 2013). Our theory of learning leads us to the view that the curriculum materials, the classroom culture, and the technology all work together to compel students to engage at a critical level. Specifically we focus on a framework of *affordances* for students to *engage* with statistical ideas (see Greeno & Gresalfi, 2008; Gresalfi, 2013). Gresalfi defines affordances as "the set of actions that are made possible by a particular object"; effectivities as "an individual's ability to realize those affordances"; and, "the extent to which an affordance is realized depends on the dynamic *intention* that emerges among elements of the system" (p. 17). She suggests that this framework (affordances, effectivities, and intention) provides a way to document learning (where learning is seen as tied to context and situation). Gresalfi and Barab (2011) use four types of engagement in their work: procedural, conceptual, consequential, and critical. They define procedural engagement as "using procedures accurately" and conceptual engagement as "understanding why an equation works the way it does" (p. 302). Consequential engagement "involves recognizing the usefulness and impact of disciplinary content" and critical engagement "involves questioning the appropriateness of using particular disciplinary procedures for attaining desired ends" (p. 302). They argue that the goal of curricular design and implementation of curricula is to foster consequential and critical engagement so that students use procedures and concepts as tools for investigating problems in meaningful ways.

The research presented here investigates students' reasoning as they engage in modeling and simulation activities while using the Change Agents for Teaching and Learning Statistics (CATALST; see Garfield et al., 2012) curriculum, coupled with TinkerPlots™ technology (Konold & Miller, 2015). In particular, in this report our intention is to study *if* and *how* the classroom culture impacts students' reasoning as small groups are presented with a statistical problem, asked to reason about the context to make conjectures, and then model and simulate the research question using TinkerPlots™ technology. Our overarching research questions are: 1) How do students who receive the CATALST curriculum and use TinkerPlots™ software develop and reason about the viability of their conjectures while engaging in the modeling process? and 2) What aspects of the classroom culture impact students' reasoning about the viability of their initial conjectures?

Methods

Data was collected in an introductory statistics course at a large urban university in the Northwest region of the United States. Students enrolled in this course as a prerequisite for the traditional introductory sequence (descriptive statistics, probability, inferential statistics) or to satisfy the required math elective needed to graduate. A total of 21 students enrolled in the course and all students consented to be participants in the study. Data collection consisted of all student work on in-class activities, video, audio, and screen capture recordings (with a subset of students from the class), and student assessment items.

The third author implemented the CATALST curriculum (Garfield et al., 2012) and TinkerPlots™ technology during the 10-week course. The philosophical stances underlying the development of the CATALST curriculum harmonize with Gresalfi's (2013) affordances for engagement framework. Like Gresalfi's framework, modeling activities are fundamentally designed to provide *strong affordances* for students to *critically engage* with statistical ideas by providing opportunities for modeling, generalizing, and reflecting. To illustrate the alignment of the course materials with Gresalfi's framework we present excerpts from both the video data collected as two groups of students (containing 2 students in each group) reason through the Cereal Box Activity as well as the group work turned in upon completion of the activity.

In the Cereal Box Activity students are asked to model and investigate the number of boxes of Munchy Crunch cereal a person would expect to buy in order to collect all six possible prizes, assuming that the manufacturer placed one of the six possible prizes in each box at random during manufacturing. Once the students are presented with the problem, students are then asked to work in groups and make conjectures about the number of boxes of Munchy Crunch cereal they think would need to be purchased (on average) in order to collect all six prizes. Students were then asked to create a model (both a conceptual model and in Tinkerplots™) and generate data that would help them answer the statistical research question. Other then this initial information, students were not given any additional directions. The context and open-ended nature of the activity afforded students opportunities for engaging both consequentially and critically with statistical ideas.

Initial Results

Initial analysis of the transcripts of the video data from the two groups identified four primary instances during the activity where students reasoned about their conjectures: 1) when groups formulated their initial conjectures; 2) when the instructor asked them to discuss and explain their original conjectures; 3) after examining the results of a single trial; and 4) after constructing (a plot of) their empirical sampling distribution. To gain a better understanding of how students reasoned when making and evaluating their conjectures and to illustrate the alignment of the course materials with the engagement framework, we present excerpts from two groups' written and recorded video data. The remainder of the results section will be organized according to the instances presented above.

Formulating Initial Conjectures

During their initial conjectures, Group 1 and Group 2 reasoned that a person would need to buy 36 boxes of Munchy Crunch cereal in order to collect all six prizes. An excerpt from Group 1 is presented below to illustrate how they generated and reasoned about their initial conjecture.

Student A: MmHm.
Student B: Not, I think - well obviously you have to buy more than six...
Student A: Yeah.
Student B: To get all six of them. So at least six.
Student A: Maybe it'd be like, since there are six prizes and you'd probably have to get six boxes at least for each prize to rule out one of them. No.
Student B: So thirty-six boxes.
Student A: So like thirty-six. Yeah. That's a lot of cereal. Let's say thirty.
Student B: Thirty?
Student A: Yeah. Or...we can come back to it.
Student B: Okay.
Student A: Let's just say thirty-six question mark.
Student B: Okay.

In the above transcript, we see that the students engaged with one another to begin to reason through and formulate a initial conjecture of 36 boxes, however it is clear that both students are unsure of the initial conjecture and are struggling to articulate their reasoning behind it. We would characterize this response to the task as procedural engagement.

Reasoning About Conjectures While Interacting with the Instructor

After making initial conjectures, both groups had the opportunity to discuss them with the instructor. We believe that interaction with the instructor assisted students in articulating their reasoning behind the original conjecture and even resulted in Group 1 evaluating the validity of their conjecture. When the instructor first joins the students she inquires about their initial conjecture (36 boxes). When prompted to explain their reasoning behind the conjecture Student A offers the following reasoning, "Well I think it's because you have six possible prizes and let's say you have a one out of six chance of getting each prize. So kind of multiplying it on itself makes sense because if you get six boxes of cereal you have the chance of getting at least one different one then the rest of them". Student A's response demonstrates conceptual engagement as she is offering justification to support their original conjecture.

While the students in Group 1 are able to explain that one of the sixes (in their multiplication of six and six to obtain 36 boxes) comes from the number of possible prizes they admit that they are confused as to why they chose to multiply the number of possible prizes by another six. As the instructor and the students continue to discuss the multiplication by six, the students realize that rather than considering how many boxes of Munchy Crunch cereal a *single* person would need to buy to collect all six prizes they were considering how many boxes total *six different* people would need to buy for each of them to collect one prize that was unique from the other five people. In response to this realization, Student B states, "But this is only one person though, right? It's not six people…So it shouldn't have been the six". The discussion between the instructor and Group 1 and the assertion from Student B that the multiplication by six was wrong prompted student to reevaluate their initial conjecture. This provides evidence that the interaction between the instructor and group members assisted students in increasing their level of engagement (from conceptual to critical), as the students are now questioning the appropriateness of the reasoning behind their original conjecture (and therefore the appropriateness of their original conjecture all together).

Reasoning About Initial Conjectures After Examining the Results of a Single Trial

Group 1 showed further evidence of reasoning about their initial conjecture after running and examining the results of a single trial using their sampler in TinkerPlots™. Their single trial produced all six prizes in 11 cereal boxes. Given the results of the single trial, the students revisited their initial conjecture.

Student B: I would say twelve. Between six and twelve.
Student A: Yeah
Student B: Because...
Student A: I mean you need at least six, right?
Student B: Yeah.
Student A: And then, I feel like if you got more...
Student B: Cause the chances of you getting it on the first try are not...
Student A: I don't know. Yeah. The chances of getting like all six different prizes of your first six boxes doesn't make sense. But…
Student B: But two. I feel like if you get two and two and two and two. And then the last one.
Student A: Is like a fifty fifty chance.
Student B: Yeah.
…
Student B: So twelve is a more reasonable number.

The students reason together about the results in a way that leads them to conclude that a more reasonable conjecture would be "between six to twelve" boxes (an interval) because "you need at least six" and "twelve is a more reasonable number" than 36 based on the results they obtained and the reasoning that there is a 50/50 chance of getting a unique prize for each subsequent draw. While this reasoning in not entirely sound the students seemed to believe that the results of the single trial supported their conjecture. Therefore, we assert these students are demonstrating critical engagement because they recognize the need to re-evaluate their initial conjecture in light of the new information obtained and they question the appropriateness of using a single number (rather than an interval) to capture their conjecture.

Reasoning About Conjectures After Constructing an Empirical Sampling Distribution

After creating an empirical sampling distribution of the number of boxes of Munchy Crunch cereal needed to obtain all six prizes, both groups of students showed evidence of evaluating the validity of their conjectures. In the below excerpt from Group 2, we see Student C discussing with the instructor the likelihood of his group's original conjecture.

Instructor: What was your conjecture?
Student C: Um. Thirty six.
Instructor: Would your conjecture be shocking to you?
Student C: Yes, it would actually.

In light of the new information gleaned by constructing and examining the empirical sampling distribution (See Figure 1), Student C recognized that the group's original conjecture would be unusual. While he does not provide explicit reasoning in the above excerpt, further evidence of his reasoning can be seen in their written work. Student C said, "Based on the results of my simulation, I would give a point of estimate of 15. I arrived at this number by using the average function in tinkerplots. This gave me an average of 14.61, which I then rounded up to 15."

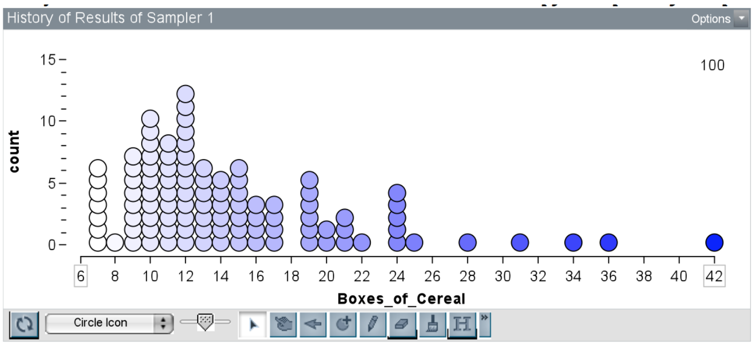

Figure 1. Student C's empirical sampling distribution

In the excerpt presented above and in Student C's written work we see evidence of Student C engaging with the technology, task, and his instructor to determine their original conjecture was unusual and a better estimate would be 15 boxes of cereal, which he based on the mean of the empirical sampling distribution. While selecting the mean may not be most desirable point estimate (since the sampling distribution is right-skewed) we believe that Student C's work demonstrates consequential engagement. Had Student C reasoned about the appropriateness of using the mean (versus another statistical measure) to determine the point estimate then he would have demonstrated critical engagement.

Conclusion

The question posed in the Cereal Box Activity is not a simple one. To arrive at the theoretical answer (14.7 boxes) students would need to have knowledge of formal statistical concepts such as expected value as well as knowledge of the geometric distribution. Although this problem is very complex, the curriculum, the technology, and the classroom culture afforded students the opportunity to generate conjectures about the number of boxes one would need to buy as well as opportunities to evaluate their conjectures and their reasoning. Our analysis provides insights into how the classroom culture impacted these students' reasoning while participating in modeling and simulating the Cereal Box Activity using TinkerPlots™. In particular, analysis of both groups' transcripts provided evidence that while each group was able to initially decide that 36 boxes seemed like a reasonable estimate, neither group was able to fully explain their reasoning behind the selection initially. This result is not entirely surprising given the difficult nature of the task. However, interactions between the group members and instructor, and using technology to explore single-trial results and empirical sampling distributions afforded students the opportunity to reconsider their initial conjectures and engage at a higher level with the statistical concepts. While our analysis is still in the preliminary phases we believe that the above work suggests that the curricular approach focused on modeling, the technology, and the classroom culture appeared to work together in a way that supported these students' engagement with statistical ideas at a consequential and critical level. We assert that this deeper level of engagement resulted in gains in students' statistical reasoning skills.

Questions for the Audience

1) Was the engagement framework useful in analyzing the impact of the classroom culture on the development of students' statistical reasoning skills?
2) Do you foresee any limitations in utilizing this framework as we continue to investigate our research questions?

3) Are there other frameworks that may be more useful in teasing apart the impact of the classroom culture on the development of students' statistical reasoning skills?

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