Results from a national survey of abstract algebra instructors: Math ed is solving problems they don't have

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There is significant interest from policy boards and funding agencies to change students' experiences in undergraduate mathematics classes. Abstract algebra specifically has been the subject of reform initiatives, including new curricula and pedagogies, since at least the 1960s; yet there is little evidence about whether these change initiatives have proven successful. Pursuant to answering this question, we conducted a survey of abstract algebra instructors to generally investigate typical practices, and more specifically, their knowledge, goals, and orientations towards teaching and learning. On average, moderate levels of satisfaction were reported with regard to the course itself or student outcomes; moreover, little interest in, or knowledge of, reform practices or curricula were identified. We found that 77% of respondents spend the majority of class time lecturing – not surprising when considering 82% reported the belief that lecture is the most effective way to teach.

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Teaching matters. It is the single most important factor in terms of what students might be able to learn from a class and what they can't learn from a class. Teaching matters because it affects how students understand their roles in the class, what it means to learn and understand the material, and the ways that students come to understand the content, and almost certainly what kind and how much students put into mastering the material. Students know this. In a time when retention of STEM majors could not be more critical for our nation, fewer than 40% of students entering college in pursuit of a STEM degree complete that degree (PCAST, p. i) citing ineffective teaching methods and uninspiring atmospheres in introductory-level STEM courses as the primary reason for attrition (PCAST, p. 5).

Mathematics, like other STEM majors, is not immune to the retention issue: even as the number of entering freshman declaring mathematics as a major in increasing, the number completing the major is constant (Kirkland, 2013); however, unlike other STEM majors, mathematics must be acutely aware of the effects of poor teaching in introductory-level courses because these courses are required for a myriad of disciplines and often act as a gateway to STEM careers. Mathematics courses, without the siren song of labs and experiments beckoning, historically have resorted to the use of lecture-style presentation in disproportionate numbers relative to other STEM majors despite mounting evidence contradicting its effectiveness.

Background and Literature

Lecture-based pedagogy has been labeled problematic for undergraduate learning, persistence, and success; instead, researchers recommend pedagogical reforms that are more reflective of how people learn and better reflect the nature of doing mathematics (Kyle, 1997; National Academy of Science, 2007; National Research Council, 1996; National Science

Foundation, 1992, 1996). Critics who do not wish to see the lecture vilified will argue that it is the students who are to blame, for they do not understand the pedagogical contract, they can't comprehend the intellectual difficulty of the work, and they have the inability to even pay attention to the correct things in a lecture (Burgan, 2006; Wu, 1999). Although these are valid concerns, the research community is fairly resolute in the position that diversifying teaching methods enhances critical thinking skills, long-term retention of information, and subsidiarily, retention of STEM majors (PCAST, p.9 – multiple resources cited).

There is mounting evidence to believe that mathematicians are not only aware of reform practices and goals, but that they do, or at least would consider, using them. There have been numerous articles published in the journals of the AMS (American Mathematical Society) about reforming teaching (c.f., Leron & Dubinsky, 1995; Halmos, Moise, & Piranian, 1975; Jones, 1977). Thanks to outreach efforts at the Joint Mathematics Meetings by proponents of the Moore Method (Copping, Mahavier, May, & Parker, 2009), there is reason to believe that its basic precepts are well-known. Calculus reform specifically has been very extensive with reform activities being supported by commercial publishers, discussed in the *American Mathematical Monthly* (c.f., Kaput, 1997; Ostebee & Zorn, 1997), and examined in session at the Joint Mathematics meetings. What is certainly true is that the National Science Foundation has spent a large amount of money, and mathematicians and mathematics education researchers have spent a large amount of time, designing new curricula. On a smaller scale, many instructors have developed their own materials, some via participation in Project NExT, the Academy of Inquiry-Based Learning, or Moore-Method conferences.

In terms of mathematicians, national professional organizations (e.g. the MAA), and mathematics education researchers, it is quite possible that no other upper-division course has gotten anywhere near a comparable amount of attention in terms of reform initiatives as undergraduate abstract algebra (e.g., Dubinsky & Leron, 1994; BLINDED; Hibbard & Maycock, 2002). Almost exclusively, these initiatives have concentrated their efforts into changing the undergraduate abstract algebra experience; namely, with more doing of mathematics during class. Yet, we believe that despite this single-mindedness, these efforts have had little to no effect on most students' experience of the abstract algebra course. This suggests that the field might have misplaced beliefs about what change is possible, or more importantly, that we are missing or misunderstanding something fundamental about the class, instructors, or instructors' beliefs about the class, students, and learning.

Many theories have been posited about why new curricular practices have not been adopted. Coverage concerns seem to be paramount. There is evidence that faculty feel a significant tension between the breadth of required topics and the ability to focus on teaching and learning through problems, subsequently driving instructors to resort to more expeditious lecture approaches (e.g., Roth-McDuffie & Graeber, 2003, p.335). Other commonly cited barriers included: the demands of the position not allowing for innovation, lack of support from colleagues or supervisors, and a lack of common vision for reform (Roth-McDuffie & Graeber, 2003; Henderson & Dancy, 2007). While these studies do offer some reasons why mathematicians might not change their instructional practices, the results have limited applicability because the participants were neither mathematicians (Henderson & Dancy, 2007), nor instructors (Speer, 2008), or were not teaching abstract algebra (Roth-McDuffie & Graeber, 2003).

Theoretical Framework

The fact is that there is essentially no research that helps researchers and policy makers understand why some mathematicians adopt reform practices in their teaching and some do not (Speer et al., 2010). Maybe the goal of the funders and policy boards is inappropriate; alternatively, maybe the goal is good but there are no meaningful avenues for change. There has been little research attempting to explore these issues from the perspective of the instructors who are the ones being asked to change practice; consequently, we believe there is a considerable need for more investigation into university mathematicians' beliefs, knowledge, and goals about the teaching of abstract algebra. The present report is based upon a survey of abstract algebra instructors to examine typical practices in general, and more specifically, orientations towards teaching and learning. We investigate the following research questions: (1) What kinds of pedagogical practices do abstract algebra professors report using in their classrooms and why? (2) What affordances and constraints on their use of non-lecture practices do they perceive?

We designed our inquiry and analyzed our results through the lens of Schoenfeld's with Schoenfeld's (1999) framework of knowledge (resources), goals, and orientations. This framework, identified is useful for analyzing long-term decision making, supports the theory that mathematics instructors' "*thinking, judgments, and decision-making* as they prepare for and teach their class sessions" are important and shape their instruction (Speer, et al., 2010, p. 101).

Methods and Data Analysis

To create an instrument designed to measure the knowledge, goals, and teaching/learning orientations of mathematicians, we adapted questions from both Henderson and Dancy's physics-education survey (Henderson & Dancy, 2009) and Characteristics of Successful Programs in College Calculus survey (see surveys at www.maa.org/cspcc). In addition to basic demographic information, the survey questions asked the professors to rate the importance of various sources of information and to list factors that influenced their teaching decisions. In an attempt to elicit their beliefs about teaching and learning, we asked them to describe and characterize their classroom practices, including the motivation behind those choices. Finally, we asked questions to test claims from the literature about why undergraduate mathematics instructors were resistant to changing their pedagogical practices.

Requests for participation in our online survey were sent to departmental administrators at approximately 200 institutions, targeting instructors who teach undergraduate abstract algebra. We had 131 completed surveys (initial response rate of ~30%). In general, the respondents (92% tenure-stream faculty) had significant experience, both with teaching in general and abstract algebra specifically, and were most likely to be teaching an undergraduate groups-first course designed for a mixed audience. (See Figure 1.)

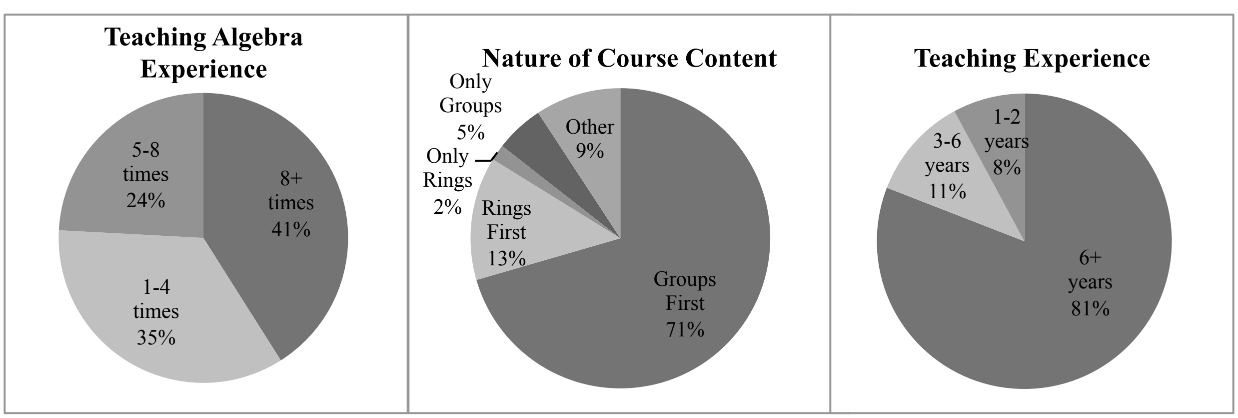

Figure 1. Information about Survey Respondents

To analyze the data, we first calculated basic descriptive statistics appropriate for each item. After compiling the demographic information, we focused our attention on instructor satisfaction in order to determine if any impetus for change existed. To address the first research question, we examined the self-reported teaching practices of the respondents and compared that to both level of satisfaction and extent of agreement with the Likert-scale belief statements designed to measure teaching/learning orientations. In our discussion, we highlight areas where the respondents appear to hold beliefs that should lead to certain pedagogical actions but who do not report engaging in those actions. To address the second research question, we categorized instructor reports on constraints and affordances to implementation of non-lecture reform practices, and we compared these with those cited in the literature. In each case, we have attempted to align these with Schoenfeld's (1999) framework of knowledge (resources), goals, and orientations.

Results

Satisfaction

When measuring satisfaction, several dimensions were considered. For this report, we choose to discuss two in particular: textbook and student learning outcomes. Of all the factors contributing to abstract algebra professor's overall levels of satisfaction, the aspect with the greatest percentage (87.6%) of satisfied or very satisfied respondents was the textbook. Instructor comments indicated that the satisfactory rating stemmed from the breadth, depth, and sequencing of content. It is important to note however, that even amongst the satisfied, complaints about pricing and frequency of new editions was rampant.

When reporting on satisfaction with student learning outcomes, approximately half of the classified responses (a number gave responses that we could not reliably categorize) reported being satisfied (44), with the remainder being evenly split between very satisfied (23) and dissatisfied (22). The responses were organized by domain and level of satisfaction, allowing us to look for common themes. Figure 2 shows a matrix illustrating typical comments.

	Very Satisfied	Moderately Satisfied	Dissatisfied
Student Engagement	▪ My students work hard. ▪ My students ask a lot of questions. ▪ My students put time in outside of class ▪ My students are excited to see how this course fits with past/future coursework	▪ The students who want to learn put in the time and do well ▪ My students generally work hard enough to get through the course but I wish they were more motivated to learn ▪ My students demonstrate infrequent or inconsistent participation in class	▪ My students don't appreciate the material ▪ My students don't do work outside of class ▪ My students are not interested in math ▪ My students view the course as irrelevant to their careers ▪ My students don't participate in class
Student Preparation	▪ My students are very well-prepared ▪ My students have a working understanding of prerequisite material and understand how to construct proofs ▪ My students' preparation is sufficient to be successful in my class	▪ My students' preparation varies by background and major ▪ Most of my students have weak proof backgrounds but develop this over the course ▪ Most of my students have insufficient prior knowledge relative to what I would like, but with the right work ethic can be successful in my class	▪ My students are unprepared to take this course ▪ My students lack proof skills ▪ My students have poor general math skills ▪ My students' insufficient preparation and ability hinders their ability to be successful in my class

Student Performance	▪ My students get good grades on exams ▪ My students produce high quality projects ▪ My students submit carefully considered homework assignments ▪ Very few of my students fail the course	▪ My students get decent grades on exams, but not as good as I would like ▪ My students produce mediocre projects ▪ My students submit homework that is often inadequate, incomplete, or rely on help to finish it satisfactorily ▪ I often have as many D/F/W grades as I do A/B/C	▪ My students do poorly on exams, without a curve, the majority would not pass ▪ My students produce poor projects or are incapable of completing them altogether ▪ My students don't/can't do homework or need extensive help to do so ▪ A large portion of my students fail or withdraw
Student Understanding	▪ My students are capable of coauthoring journal articles with faculty ▪ My students leave my class prepared for future advanced coursework and often get accepted to reputable grad school programs ▪ My students demonstrate algebraic reasoning and mathematical maturity	▪ My students leave my class adequately prepared for future coursework, but not necessarily grad school ready ▪ My students don't grasp all the subtleties, but come away with a level of understanding suitable for their backgrounds, abilities, and future plans ▪ My students have a working understanding of fundamental concepts and can usually make definitions, sort conjectures, and build useful examples	▪ My students master only a small fraction of the topics covered ▪ My students don't come away with a real understanding of the material ▪ My students leave without really getting the point ▪ My students are generally unprepared for future coursework
Curriculum Issues	▪ My curriculum covers lots of presently relevant examples from applications in diverse fields (physics, chemistry, math, etc) ▪ My curriculum requires that students work on finding proofs for themselves and this approach has been successful in generating student growth. ▪ Having the students work in small groups instead of traditional lectures has proven successful ▪ My curriculum gives the students the right taste of modern math and supplies them with the right language to be successful ▪ My curriculum has struck a successful balance between abstraction and computational topics to keep all students engaged	▪ My curriculum is ok but could benefit from extended motivation for topics and guided self-discovery ▪ My curriculum is ok for math majors but does not adequately serve the pre-service teacher population ▪ I am satisfied that they get a good introduction to group theory but would like to go deeper into the subject and have the students formulate and explore conjectures on their own ▪ I consider my course 'algebra appreciation' rather than a careful, complete introduction for those who should master the material	▪ My curriculum is out of date ▪ My curriculum is divorced from the true motivations and applications of algebra ▪ My curriculum materials are lacking and I often have to supplement with worksheets/handouts ▪ I spend too much time teaching how to write proofs and not enough time on algebra topics

Figure 2. Satisfaction Matrix

In summary, instructors who were moderately satisfied indicated (unsurprisingly) that students learned most of the important content and worked reasonably hard. The courses might be in need of a little reorganization or supplemental materials, but major pedagogical overhauls were not considered warranted or desired. The comments of the instructors who were dissatisfied were complaints about the unsatisfactory work ethic, motivation, and ability of the students. Instructors who reported high levels of satisfaction were the most likely to comment on the format and curriculum of their courses, with approximately half of them indicating belief that their course was different than most traditional abstract algebra courses due to the use of some form of inquiry-based learning (increased use of examples, student research, Modified Moore Method, etc.).

While the groups did vary widely in typical responses, it was interesting to note that there were two common themes that emerged across all levels of satisfaction. The first observation was a general frustration with students' lack of prerequisite proof skills and poor proof-writing ability. The other common opinion was that it was both difficult and inappropriate to design and teach a course for different constituencies (most often cited was the comingling of Math and Math Education majors). Due to different backgrounds, abilities, and occupational goals, the consensus was that neither population was being adequately served by teaching them simultaneously. However, even with this mixed sense of satisfaction with student learning outcome, we were surprised to find that, for the instructors completing the survey, the passing rates were quite high with the average grade break down being: A 33.37%, B 33.85%, C 20.55%, and D/F/W 12.18%.

Teaching methods

Lecture was the most common pedagogical practice with 77% of respondents claiming that they currently lecture to teach abstract algebra, 15% of respondents currently teach in some other way, and 8% used to do something different in the past but now lecture. Of the 23% who either now, or in the past, used non-lecture pedagogy and curricular materials, most (15 respondents) created it themselves without formal support (typically drawing on a mixture of texts and problem-sets). There were only two respondents who cited use of a particular established curriculum (Teaching Abstract Algebra for Understanding, Larsen, 2013; Learning Abstract Algebra with ISETL, Dubinsky & Leron, 1994). The others used their own experiences with Moore Method classes, collaboration with other Moore Method instructors, or participation in the Academy of Inquiry-Based Learning as a guide to develop their materials and shape their practice.

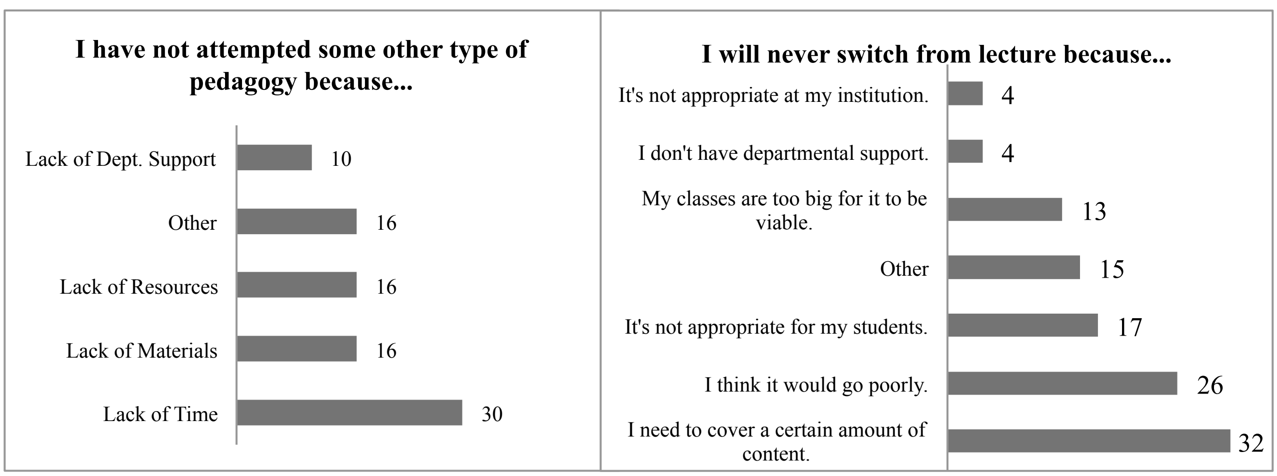

Figure 3. Perceived constraints on the use of non-lecture practices

Of the 85% who are currently teaching with lecture, 56% of them say that they would consider teaching with non-lecture practices (the remaining 44% say they would never do so). The reasons instructors provided for not yet attempting other pedagogy and the concerns mentioned explaining why they would never change their habits can be seen in Figure 3. In short, the two main themes in the comments related to the effort and support needed to revise and teach such a class and concerns about covering the appropriate amount of material. Of the 32 instructors who stated coverage as a reason to not adopt a non-lecture format, 23 of them answered “no” when asked “Do you feel pressure from your department to cover a fixed set of material in your abstract algebra course?” It appears therefore, that concerns about coverage may be more tied to an internalized goal or orientation, as opposed to an external pressure.

One of the most interesting findings was the apparent contradiction that emerged when comparing the responses to the following prompts. 82% of respondents agreed with the statement: *Lecture is the best way to teach.*; however, 56% agreed (and 26% more slightly agreed) with the statement: *I think students learn better when they do mathematical work (in addition to taking notes and attending to the lecture) in class*. This result was promising for the prospect of non-lecture class activities; yet when asked what students do in class besides take notes (given a list of options), the only things that instructors claimed that students did in class, even at a rate of once per month, was doing calculations, working with examples, or working with applications. Moreover, 63% reported that students never spent time working on mathematics problems in class. It appears that what instructors think is best for student learning (students doing mathematical work in class) is not happening with any frequency; thus, we argue that there exists a mismatch between beliefs about student learning and actual teaching practice.

Findings and Implications for Future Research

There are three primary findings that we highlight. First, that lecture is the predominant mode of instruction, and that even those who have tried other pedagogies appear to switch back to lecturing at very high rates. Moreover, given the significant amount of time, money, and energy spent developing, testing, promoting, and training mathematicians to use new curricula and pedagogies, there is almost no uptake. Those using non-traditional materials are far more likely to have developed their own materials than to have adopted NSF-supported curricula.

The second primary finding relates to the factors that influence pedagogical decisions. In decreasing order of significance, the participants reported that their experiences as a teacher and student were far and away the most significant (more than 90% agreement) influence; followed by talking to colleagues about how to teach specific content, and looking at other texts (70-90% agreement that it is a significant influence). Little importance was assigned to the normal means of learning about new teaching ideas; e.g., Project NExT, MathFest, MAA mini-courses or other workshops, or reading publications about teaching such as the MAA Notices series or PRIMUS (ranging from the single digits to about 15% indicating that it was significant). If mathematicians essentially give no weight to the traditional means of dissemination of new pedagogical ideas and techniques (and evidence of their effectiveness), reformers have little means of promoting change other than individual conversation. This alone suggests why reforming undergraduate abstract algebra instruction is difficult, especially with the currents modes of dissemination.

Finally, while faculty claim they have the ability to change their courses, the reported satisfaction levels indicate they do not have the desire to do so; furthermore, the majority of dissatisfaction stems from the students and not the course materials. Given the strong content focus and high belief in the efficacy of (and preference for) lecture, it appears that as a collective, the abstract algebra teaching faculty have little interest in adopting new pedagogical approaches at this time. Thus, we propose two concurrent research directions: first, we need to better explore the reasons that mathematicians appear to strongly believe in their current practice, the types of evidence that they hold as dispositive, and what means of dissemination of new approaches achieve meaningful penetration. Second, we argue that we need to further explore the types of changes to the practice of lecture that mathematicians would adopt. In other words, how can the RUME researchers meet the perceived needs of the abstract algebra community while taking into account what is understood as practical and feasible in the eyes of the faculty?

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