Impact of Advanced Mathematical Knowledge on the Teaching and Learning of Secondary Mathematics

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There has been a longstanding debate in the mathematics and mathematics education communities concerning the knowledge secondary mathematics teachers need to provide effective instruction. Central to this debate is what content knowledge secondary teachers should have in order to communicate mathematics to their students, assess student thinking, and make curricular and instructional decisions. Many educators believe that mathematics teachers should have a strong mathematical foundation along with the knowledge of how advanced mathematics is connected to secondary mathematics (Papick, 2011). But according to others, more mathematics preparation does not necessarily improve instruction (Darling-Hammond, 2000; Monk, 1994). Therefore, it is important that, as a field, we investigate the nature of the present mathematics content courses offered (and required) of prospective secondary mathematics teachers to gain a better understanding of which concepts and topics positively impact teachers' instructional practice.

This exploratory study aims to advance our understanding of the nature of mathematics offered to prospective mathematics teachers by looking at mathematical connections. We investigate how in-service and pre-service teachers make connections between tertiary and secondary mathematics as well as if and how the understanding of connections influences teachers' thoughts about teaching and learning mathematics. The research questions for this study are as follows:

1) How does exposure to and instruction in tertiary mathematics impact the way teachers understand secondary mathematics?
2) How does exposure to and instruction in tertiary mathematics impact the way teachers approach secondary classroom instruction?

Conceptual Framework

We consider connections between tertiary and secondary mathematics to be ones that encompass both mathematical content and ways of thinking about and engaging with that content. To better understand these connections, we draw on three areas of research: mathematical knowledge for teaching (e.g., Ball, Thames, & Phelps, 2008), mathematical practices (e.g., Council of Chief State School Officers [CCSSO], 2010; RAND, 2003) and habits of mind (e.g., Cuoco, Goldenberg, & Mark, 1996). Mathematical knowledge for teaching (MKT) (Ball et al., 2008) incorporates both subject-matter knowledge and pedagogical content knowledge. One component in the larger domain of subject-matter knowledge is called *horizon content knowledge*. We believe this particular aspect of MKT is a potentially useful idea for thinking about what advanced content knowledge prospective mathematics teachers at the secondary level need for their teaching.

To expand the notion of subject-matter knowledge, it is also useful for us to consider what secondary teachers need to know beyond content and concepts and to encompass mathematical habits of mind (e.g., Cuoco et al., 1996) and engagement in mathematical practices (e.g., CCSS, 2010). These include looking for patterns, making conjectures, attending to precision, and connecting representations. Such habits and practices in mathematical thinking and learning

extend across content areas and levels of mathematical study. Therefore, as we consider how advanced mathematical content impacts teachers' knowledge and understanding of the teaching and learning of secondary mathematics, it is important for us to consider habits and practices that may also influence how tertiary ideas are learned and interpreted for teaching.

We drew on these ideas to develop a unit for practicing middle school mathematics teachers that highlights a particular connection between advanced mathematics and secondary mathematics. This unit, described below, seeks to not only show how connections can be purely mathematical in nature and relate directly to subject-matter knowledge, but to also illustrate how connections can go beyond knowledge *of* mathematics and encompass engagement *in* mathematics through the lens of mathematical habits of mind and mathematical practices.

Research Methodology

Participants are 14 students in a master's level mathematics education course. Of this group, one is a special education teacher, two are pre-service teachers, and eleven are in-service teachers with one to fifteen years teaching experience.

There will be three data sources: a mathematics questionnaire, an instructional unit, and interviews. The researchers will first give participants a mathematical content knowledge questionnaire; the purpose of which is to gain insight into the level of mathematical content knowledge that the participants possess as well as their thoughts on the impact of tertiary mathematics on instruction. Responses to this questionnaire will also be considered when analyzing the video and audio data collected during the instructional unit. After completing the questionnaire, one researcher will teach one two-and-half hour lesson. The lesson will be filmed and all written artifacts will be collected. During the lesson, participants will engage in an instructional unit on solving equations; the purpose of which is to challenge teachers' understanding of procedures used for solving equations and to consider how attention to the algebraic structures and their properties may inform procedures and solutions. Following the classroom lesson, approximately four volunteers will be asked to participate in a follow-up interview; the purpose of which is to clarify ideas discussed in class and to probe students' thinking on the impact of tertiary knowledge on the understanding of secondary mathematics and instruction.

To analyze the data, the researchers will use initial coding (Saldana, 2009) of the video data transcripts to split the data individually coded segments. The researchers will then use theoretical coding as a way to "constantly compare, reorganize, or "focus" the codes into categories" (Saldana, 2009, p. 42). The goal is to code the data based on thematic or conceptual similarities with respect to how the participants make connections between tertiary and secondary mathematics as well as if and how the understanding of connections influences their thoughts about teaching and learning mathematics.

Applications and Implications

In this exploratory study, we investigate questions regarding mathematics teachers' content knowledge and preparation. In particular, we would like to better understand which concepts might positively impact teachers' instructional practice. We do this by considering how in-service and pre-service teachers make connections between tertiary and secondary mathematics as well as if and how the understanding of connections influences teachers' thoughts about teaching and learning mathematics. While many researchers of mathematics and mathematics education may intuitively understand how secondary mathematics teachers' deep knowledge of mathematics is related to the ability to be an effective mathematics instructors in secondary schools, the field still needs to understand how secondary teachers use their tertiary mathematics

instruction in teaching secondary mathematics, which can lead to a better sense of the kinds of mathematics courses that can provide teachers with the content knowledge they need to make best use of these connections.

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