# The Effect of Mathematics Hybrid Course on Students' Mathematical Beliefs


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*Computer-based courses (e.g., online or hybrid) have significantly changed the design of pedagogy and curriculum in the past decade, which include online teaching and learning on mathematics education. As beliefs play an essential role on achievement, the impact of computer-based courses on mathematical beliefs is still underdeveloped. In particular, we are interested in whether mathematics hybrid class (blend of online and face-to-face) has different impact on students' mathematical beliefs compared to regular face-to-face class. A two-by-two design of instruction method (hybrid vs. regular) and mathematics performance (high vs. low) was employed. The results showed that both hybrid and regular class students believed understanding and memorization were equally important in mathematics learning. Hybrid class students showed more flexibility in selecting solution methods compared to regular class students on their beliefs about problem solving.*



*Key words:* Hybrid Course, Mathematical Beliefs, Quantitative Research, Developmental Mathematics


## Introduction

Computers have been used comprehensively in education in the past decade. The major computer-based course designs have been developed in the form of online (or internet) or hybrid (blend of online and face-to-face). Such course designs make it possible for students to learn any time anywhere (Lemone, 1999; Kadlubowski, 2001). The pedagogy has also been changed significantly (Czerniewicz, 2001; Macdonald et al., 2001) compared to the traditional one. For example, in mathematics, students can watch video lessons, follow step-by-step interactive tutorials, communicate through Internet, and do homework and tests online. Teachers are no longer troubled by pile of homework assignments and tests for grading (Engelbrecht & Harding, 2005; Juan et al., 2011)

A growing research about computer-based mathematics courses addressed a variety of issues including online curriculum/course design (Lee, 2014; Wenner, Burn, & Baer; 2011), factors related to online course achievement (Kim & Hodges, 2012; Kim, Park, & Cozart, 2014; Wadswarth et al., 2007), teaching (Cavanagh & Mitchelmore, 2011; Engelbrecht & Harding, 2005; Juan, Steegmann, & Huertas, 2011), and assessment (Engelbrecht & Harding, 2004; Groen, 2006). However, as mathematical reasoning and problem solving are the core of mathematics practice (Polya, 1954; Schoenfeld, 1992), a Google scholar and Eric Index search find little study about the effect of computer-based courses on students' mathematical beliefs about mathematical reasoning and problem solving methods and strategies. Mathematical beliefs play an essential role in the learning process of mathematics (Schoenfeld, 1983, 1985, 1989) or even academic performance (Carlson, 1999; Schommer-Aikins, 2002). Briefly speaking, one's mathematical beliefs affect his/her ways of thinking and mathematical practice, and ultimately affect his/her mathematics performance.

Therefore, the purpose of the study is to examine students' mathematical beliefs to explore the impact of mathematics hybrid course. In particular, we are interested in the differences of mathematical beliefs between hybrid (blend of online and face-to-face) and regular (face-to-face) class students. This study may contribute to the underdeveloped literature about the impact of online learning on students' mathematical beliefs.

## Literature Review

This section will review mathematical beliefs, particularly on mathematical reasoning and problem solving.

### Mathematical Reasoning

According to the literature in mathematics and mathematics education, mathematicians or experts emphasize on reasoning and understanding while novice mathematics learners emphasize on memorization and replication. Polya (1954) noted: "The result of mathematician's creative work is demonstrative reasoning, a proof; but the proof is discovered by plausible reasoning…" (p. vi). Ross (1998) noted: "It should be emphasized that the foundation of mathematics is reasoning. While science verifies through observation, mathematics verifies through logical reasoning" (p. 254). Mathematical reasoning includes the sense making of numbers and symbols, the derivation of rules, properties, and theorems, the emergence and utilization of mathematical methods, and the logical connection and analysis of mathematical statements. Rather than focusing on reasoning and understanding, novice mathematics leaners tend to memorize solution procedures and replicate them in problem solving. For example, a learner may be able to apply the method of isolating the variable for solving linear equations without understanding the equivalent relationship between the right and left side of an equation, or the equivalent equations that are transformed from the original equation. In other words, students may be able to apply the addition and multiplication properties by rote in solving equations without knowing the properties. Ross (1998) mentioned: "if reasoning ability is not developed in the student, then mathematics simply becomes a matter of following a set of procedures and mimicking examples without thought as to why they make sense." (p. 254)

### Problem Solving

Mathematicians or expert problem solvers try to solve difficult or challenging problems, while novice mathematics learners tend to master routine problems. Schoenfeld (1992) mentioned: "The unifying theme is that the work of mathematicians, on an ongoing basis, is solving problems - problems of the "perplexing or difficult" kind" (p. 15). He noted Halmos' argument that "students' mathematical experiences should prepare them for tackling such challenges. That is, students should engage in "real" problem solving, learning during their academic careers to work problems of significant difficulty and complexity." (p. 16). Indeed, solving difficult or perplexing problems enable one to think, overcome obstacles, and come up with some ways to apply mathematical methods to work out the answer. Novice mathematics learners tend to practice and master certain types of routine problems, which help focus on memorizing and replication. For example, mastering problem types and their corresponding procedures helps one identify similar problems and replicate solution procedures.

While novice mathematics learners try to memorize problems/problem types and their corresponding solution procedures, expert problem solvers try to see the way general rules can be used and worked out in solving challenging problems. As Carlson (1999) noted, an expert in mathematics is the one who "needs to concentrate more on the systematic use of general thought process rather than on memorizing isolated facts and algorithms" (p.247) Indeed, Polya emphasized general rules as “one must have them assimilated into one’s flesh and blood and ready for instant use” (Pólya and Szegö, 1925, preface, p. vii.)

The use of general rules does not mean to use only one way for solving a problem. Multiple ways of solving a problem means to try different plans or strategies, which may involve the same or different general rules, or different ways general rules or properties are applied. Star and Rittle-Johnson (2007) noted Dowker’s (1992) study that “expert mathematicians know and use more strategies than novices, even choosing to use different strategies when attempting identical problems on different occasions”. In other words, experts in mathematics may try different ways in exploring a problem instead of seeking an authoritative way for solving problems. As Carlson (1999) noted, the expert view in mathematics “examine situations in many ways…rather than follow a single approach from an authoritative source”. (p. 247)

## Theoretical Framework

Four pairs of contrasting mathematical beliefs are constructed based on the above literature review on mathematical reasoning and problem solving. The first pair of contrasting beliefs is about mathematical reasoning. The rest three pairs are about problem solving. The first pair of contrasting beliefs is “understanding versus memorization”. The second pair is “solving challenging problems versus solving routine problems”. The third pair is “using general methods versus using case-based methods”. The fourth pair is “using flexible methods versus using authoritative methods”.

The four pairs of contrasting beliefs, according the literature review above, can be characterized as the contrasting of expert beliefs versus novice beliefs. In the study, experts and novice learners were characterized as high and low performance students. Combined with the two types of Hybrid and Regular instruction methods, a two-by-two design of instruction method (hybrid vs. regular) and mathematics performance (high vs. low) was employed. The two-by-two table is illustrated below (see Table 1). The two independent variables are the instruction method (Hybrid, Regular) and student performance (High, Low). The dependent variable is student’ mathematical belief scores.

Table 1: The Two-by-Two Research Design

| | Hybrid | Regular |
|---|---|---|
| High | Four beliefs | Four beliefs |
| Low | Four beliefs | Four beliefs |

The Likert scale of 5 points has been widely used in the literature (Mason & Scrivani, 2004; Schommer-Aikins, Duell, & Hutter; 2005) for measuring mathematical beliefs. A Liker-type item contains the following features: (1) response levels are arranged horizontally; (2) response levels are anchored with consecutive integers; (3) response levels are also anchored with verbal labels which connote more-or-less evenly-spaced gradations and (4) verbal labels are bivalent and symmetrical about a neutral middle (Kislenko & Grevholm, 2008; Uebersax, 2006). A common Liker-type item for measuring mathematical belief ranges from 1 (= totally disagree) to 5 (= totally agree). However, to measure a pair of contrasting beliefs, the points will be arranged in the way that the two ends (1 and 5) mean strongly agreeing to each of the two contrasting beliefs. The two points (2 and 4) close to the middle point (3) mean somewhat agreeing. The middle point (3) remains neutral.

## Method

The two research questions of the study are: (1) What are the differences between high and low performance students about their mathematical beliefs? (2) What are the differences between hybrid and regular class students about their mathematical beliefs? In the study, the mathematical beliefs refer to mathematical reasoning beliefs (mathematical understanding vs. memorization) and problem solving beliefs (challenging problems vs. routine problems, general methods vs. case-based methods, and flexible methods vs. authoritative methods).

### Participants and Procedure

Students who took developmental mathematics courses (e.g., Foundations for Algebra, Introductory Algebra, or Intermediate Algebra) at a university in the west region of the U.S were invited to participate in this study. The students were given a questionnaire two weeks before the finals week in the Spring Semester 2013. There were 229 students involved in this study, where 204 student data were valid. Among the 204 students, 60 students were from 7 hybrid classes and 144 students were from 11 regular classes. Students' enrollment in either regular or hybrid class was based on their own will. They were not assigned to the classes.

### Instruments

The questionnaire contains 15 questions based on the four types of contrasting mathematical beliefs, as described in the framework. In particular, there were two questions (#5, #14) about mathematical understanding (understanding vs. memorization), two questions (#4, #12) about challenging problems (challenging vs. routine problems), four questions (#1, #6, #7, #13) about generality (general methods vs. case-based methods), and three questions (#2, #3, #11) about flexibility (multiple methods vs. authoritative methods).

A Likert-type item was used for each question in the questionnaire (see Table 2 for an example). The scale is from 1 to 5 where 4 and 5 mean the answer is toward (a) (e.g., somewhat (a) for 4 and far more (a) for 5), and 2 and 1 mean the answer is toward (b) (i.e., somewhat (b) for 2 and far more (b) for 1). The number 3 on the scale means equally (a) and (b).

Table 2: An Example of the Likert-type Question

My confidence in preparing for mathematics exams depends on
(a) how many problems I attempted
(b) how many challenging problems I attempted

| Far More (a) | Somewhat More (a) | Equally (a) & (b) | Somewhat More (b) | Far More (b) |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |

The following Table 3 shows one example question for each of the four mathematical belief categories.

Table 3: Categories and Example Questions of the Mathematical Belief Questionnaire

| Category | Example Question |
|---|---|
| Mathematical understanding | When studying mathematics in a textbook or in course materials:<br>(a) I find the important information and memorize it the way it is presented.<br>(b) I organize the material in my own way so that I can understand it. |
| Challenging | My confidence in preparing for mathematics exams depends on: |

| | |
|---|---|
| problems | (a) how many problems I attempted.<br>(b) how many challenging problems I attempted. |
| Generality | To me, it is important to:<br>(a) find one method that can be used to solve many problems.<br>(b) memorize different methods for solving different problems. |
| Flexibility | For learning to solve problems, it is important to:<br>(a) follow the way my teacher teaches or the textbook suggests.<br>(b) find the way I feel like and/or comfortable with. |

The questionnaire items were designed in the way that some items had higher score (e.g., 4 or 5) for expert beliefs, and some items had lower score (e.g., 1 or 2) for expert beliefs. This design was to prevent students from seeing any pattern in answering the questionnaire.

## Analysis

Students' performance levels (High and Low) were characterized by their final letter grades. Grades A, A– or B+ were grouped as high performance. Grades B or below (including no pass) were grouped as low performance. Students received B+ if their final number grades were 87 or above.

The scores of some Liker-items of the questionnaire were transformed to match the score distribution of 1 to 5 from novice beliefs to expert beliefs.

Four 2-by-2 two-way ANOVA tests were conducted for the average mean of each of the four belief categories – mathematical understanding, challenging problems, generality, and flexibility. Each test contained two independent variables (instruction method, student performance) and one dependent variable (mean score of mathematical beliefs).

## Results

### Beliefs about Mathematical Understanding

A two-way ANOVA test of instruction method (Hybrid, Regular) and student performance (High, Low) on beliefs about mathematical understanding showed a significant main effect for student performance. The high performance students significantly recognized the importance of understanding (M=3.21) in mathematics learning compared to the low performance students (M=2.96). However, there was no main effect for instruction method. Both hybrid (M=3.00) and regular (M=3.08) class students' belief about mathematical understanding are close to neutral (M=3.00). All hybrid and regular class students believed understanding and memorization were equally important. The statistics of the ANOVA test are shown in Table 4.

Table 4: The Statistics of ANOVA Test on Beliefs about Mathematical Understanding

| | M | SD | *F*-score | Interaction | *p*-value |
|---|---|---|---|---|---|
| Instruction Method | | | $F(1, 191)=0.05$ | No | 0.819 |
| Hybrid | 3.00 | 1.05 | | | |
| Regular | 3.08 | 0.95 | | | |
| Student Performance | | | $F(1, 191)=4.40$ | No | 0.037* |
| High | 3.21 | 0.93 | | | |
| Low | 2.96 | 1.00 | | | |

*Note.* $*p<.05$. $**p<.01$

**Beliefs about Challenging Problems in Problem Solving**

A two-way ANOVA test of instruction method (Hybrid, Regular) and student performance (High, Low) on beliefs about challenging problems showed a significant main effect for students' performance. High performance students significantly recognized the value of challenging problems (M=3.39) in learning to solve problems compared to low performance students (M=3.00). Low performance students took both of challenging problems and the amount of problems equally important in preparing for a test. There was no significant main effect for instruction method. Both hybrid (M=3.14) and regular (M=3.15) class students slightly favored doing challenging problems in learning to solve problems. The statistics of the ANOVA test are shown in Table 5.

Table 5: The Statistics of ANOVA Test on Beliefs about Challenging Problems

| | M | SD | *F*-score | Interaction | *p*-value |
|---|---|---|---|---|---|
| Instruction Method | | | *F*(1, 190)=0.18 | No | 0.673 |
| Hybrid | 3.14 | 1.00 | | | |
| Regular | 3.15 | 0.86 | | | |
| Student Performance | | | *F*(1, 190)=7.72 | No | 0.006** |
| High | 3.39 | 0.86 | | | |
| Low | 3.00 | 0.96 | | | |

*Note*. *$p$<.05. **$p$<.01

**Beliefs about Generality in Problem Solving**

A two-way ANOVA test of instruction method (Hybrid, Regular) and student performance (High, Low) on beliefs about generality (regarding problem-solving methods) showed that there were no significant main effects for both of instruction method ($M_{hybrid}$=2.92, $M_{regular}$=3.13, $p$=0.136) and student performance ($M_{high}$=3.15, $M_{low}$=3.01, $p$=0.207).

**Beliefs about Flexibility in Problem Solving**

A two-way ANOVA test of instruction method (Hybrid and Regular) and student performance (High, Low) on beliefs about flexibility showed significant main effects. Hybrid class students were significantly more flexible in choosing problem-solving methods (M=3.33) compared to regular class students (M=3.11). High performance students were significantly more flexible (M=3.31) compared to low performance students (M=3.09) in choosing problem-solving methods. The statistics are showed below in Table 6.

Table 6: The Statistics of ANOVA Test on Beliefs about Flexibility

| | M | SD | *F*-score | Interaction | *p*-value |
|---|---|---|---|---|---|
| Instruction Method | | | *F*(1, 191)=7.10 | No | 0.008** |
| Hybrid | 3.33 | 0.71 | | | |
| Regular | 3.11 | 0.66 | | | |
| Student Performance | | | *F*(1, 191)=7.19 | No | 0.008** |
| High | 3.31 | 0.66 | | | |
| Low | 3.09 | 0.68 | | | |

*Note.* *$p$<.05. **$p$<.01

## Discussions

The ANOVA tests show that there were no differences between hybrid and regular class students on the mathematical beliefs of understanding, challenging problems, and generality. For mathematical understanding, both of the hybrid and regular class students believed understanding and memorization were equally important in learning mathematics (i.e., the belief mean scores of the two groups are 3.00 and 3.08). A possible explanation could be that teachers in the face-to-face developmental mathematics lectures might not emphasize enough on mathematical reasoning (e.g., teaching why), but more on mathematical facts and procedural skills. For challenging problems, both of the hybrid and regular students slightly preferred doing challenging problems in problem solving (i.e., the belief mean scores of the two groups are 3.14 and 3.15). A possible explanation for no difference between the two groups could be the problem solving opportunities (e.g., solving difficult problems) for both hybrid and regular students were similar. Homework problems teachers assigned to the students might have similar level of difficulty for both hybrid and regular classes. This may be due to that all developmental mathematics courses had the same departmental final exam. For generality, a possible explanation for no difference between the two groups could be that the teachers focused mainly on (general) standard algorithms without enough introductions to multiple ways of solving problems. Both of the regular and hybrid students received help from their teachers about the benefit of general methods in solving problems (i.e., hybrid courses have face-to-face sections). For the students' beliefs about flexibility, the hybrid class students were significantly more flexible in selecting solution methods ($M=3.33$) compared to the regular class students ($M=3.11$). It is possible that the hybrid students received less authority or emphasis from their teachers (i.e., less face-to-face time) about the selection of solution methods in problem solving.

The ANOVA tests show that there were significant differences between high and low performance students on the mathematical beliefs of understanding, challenging problems, and flexibility. The results are consistent to the literature. High performance students show more appreciation on mathematical reasoning (Carlson, 1999). They are more willing to take challenge in problem solving (Schoenfeld, 1983, 1985). High performance students are also more flexible in the selection of problem-solving methods (Dowker, 1992). However, there was no significant difference between high or low performance students in the use of general or case-based solution methods. A possible explanation could be it is difficult to differentiate general methods from case-based methods in developmental mathematics. Another possible explanation could be that the idea of general methods might not be emphasized in teachers' teaching.

## Implications

High performance students may hold more factors of success in computer-based learning context. They may also hold more potential of being enriched and promoted by the extended materials computer-based course can offer. The reasons are: first, hybrid or online classes significantly require self-efficacy and self-regulated learning ability (Hung et al., 2010; Smith, Murphy, & Mahoney, 2003). High performance students, according to this study, hold significantly more positive beliefs in mathematical reasoning and understanding compared to low performance students. The beliefs may strengthen students' confidence in self-regulated learning in understanding mathematics in computer-based learning context. Second, hybrid or online courses have more flexibility in offering students challenging problems or extended materials to challenge students compared to traditional face-to-face classes (Lin & Hsieh, 2001; del Valle & Duffy, 2009). High performance students, according to this study, are more likely to do challenging problems in learning to solve problems. Traditional face-to-face classes generally have more constraints on lecture time, lecture content, and homework assignments due to the different performance levels of students in a class. Hybrid or online mathematics courses may allow high performance students to move faster in watching lecture videos and doing homework, and therefore, to invest more time on self-regulated exploration as well as doing more challenging problems for extra credit. Third, hybrid or online mathematics courses provide more freedom and less authority in problem solving activities (Rosa & Lerman, 2011; van de Sande, 2011). According to the study results, the hybrid and high performance students were significantly more flexible in choosing solution methods. High performance students, therefore, may have more space to develop their own methods or knowledge in problem solving.

Finally, we hope this study sheds light on students' mathematical beliefs under computer-based learning context, and contribute to the effort of enriching online mathematics education.

## Limitations

This study has two unfortunate limitations. The first limitation is that this study was not a pre-post research design. The initial design was to attain the gain scores of the pre-post belief items. This study was initiated to help the developmental mathematics department make decision about retaining or dropping hybrid courses. Due to the timeline of decision making as well the IRB (Institutional Review Board) process, the experiment was finally conducted as a one-shot experiment (i.e., questionnaires at the end of the semester for hybrid and regular students). The second limitation is the categorization of high/low performance students. Due to the need of quantitative study, the classification of mathematics performance based on grades was a practical way for this study, but it could result in debates due to different definitions on high/low performance (e.g., the cutting grade could be C for high/low performance). Sometimes, it is even acceptable to differentiate grade A students just because there are different types of grade A students. We understand the limitation, but it seems inevitable.

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