What do students attend to when first graphing in R^3?

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This poster considers what students attend to as they first encounter R^3 coordinate axes and are asked to graph functions with free variables. Graphs are critical representations, yet students struggle with graphing functions of more than one variable. Because prior work has revealed that students' conceptions of multivariable graph are often related to their conceptions about single variable functions, I used an actor-oriented transfer perspective to identify what students see as similar between graphing functions with free variables in R^2 and R^3. I considered what students attended to mathematically, and found that they focused on equidistance, parallelism, and coordinate points.

Key words: generalisation, multivariable calculus, multivariable functions, graphing

Including multivariate topics in K-12 mathematics is one way to increase mathematical competence for all students (Ganter & Haver, 2011; Shaughnessy, 2011). Because multivariable topics share many similarities with their univariate counterparts, many researchers studying student learning of multivariable topics focus on how students generalise from the single- to multivariable context (e.g., Dorko & Weber, 2013; Kabael, 2011; Yerushalmy, 1997). This poster exhibits some initial findings from a longitudinal study that seeks to explore how calculus students generalise function and limit from the single- to multivariable context. Specifically, it considers what students attend to as they first encounter R^3 coordinate axes and are asked to graph functions with free variables.

Graphs are critical representations in calculus, yet students struggle with creating graphs of multivariable functions (Kabael, 2011; Martinez-Planell & Trigueros, 2012). Students' correct understandings about the shapes of graphs in R^2, for instance, may interfere with their learning about graphs in R^3. Some students graph $f(x,y) = x^2$ as a parabola rather than as a parabolic surface. Students may also draw $f(x,y) = x^2 + y^2$ as a cylinder or a sphere because they are accustomed to $x^2 + y^2$ representing a circle in R^2. These examples illustrate that part of students' thinking about multivariable functions' graphs comes from generalising the ways they think about graphs in R^2. I sought to further explore this, with the hypothesis that learning more about what students attend to when graphing can help instructors emphasize the productive connections students see across situations and target students' misconceptions. Toward that end, this poster focuses on the following research question: *what do students attend to as they first think about graphing multivariable functions with free variables?*

Theoretical Framework

I use an actor-oriented transfer lens to study student thinking about graphing. Actor-oriented transfer focuses on what students see as similar across situations, even if their perceptions of similarity are not normatively correct (Lobato, 2003). From this perspective, students' graphing activity in R^3, even if incorrect, *makes sense to them* for some particular reasons, and the goal is to uncover those reasons. In the two examples given above, students' reasons for drawing $f(x,y) = x^2$ as a parabola and $f(\mathrm{x,y}) = x^2 + y^2$ as a cylinder or sphere might indicate that they are attending to the way similar equations, $f(x) = x^2$ and $x^2 + y^2 = r^2$, look in R^2. My use of an actor-oriented transfer perspective affords identifying more of these sorts of connections that students see and use as they think about what graphs of multivariable functions look like.

Methods of Data Collection and Analysis

I asked 12 differential calculus students about multivariable functions so that I could observe the initial sense making of students who had not yet received instruction regarding these functions. I hypothesised that this would allow me to observe students' abstractions in real time. This poster focuses on data from three tasks: students' graphs of $y = 2$ in R^2, $y = 3$ in R^3, and $f(x,y) = x^2 + 6$ in R^3. I asked follow-up questions such as "why did you draw a [line, plane, curved surface] here?" I analysed my data by first identifying instances of generalisation, defined as "the influence of a learner's prior activities on his or her activity in novel situations" (Ellis, 2007, p. 225). A colleague and I then reviewed and discussed those episodes, with the goal of characterising the nature of those generalisations. Specifically, we looked for (a) any references on the students' part to graphing or functions in R^2, which we coded using Ellis' (2007) generalisation taxonomy, and (b) what mathematical concepts or ideas students leveraged as they generalised.

Results

Due to space limitations, I focus on a particular student, Alex, and then give brief details about ways other students answered these tasks. Alex drew a correct graph of $y = 3$ in R^3 despite having seen R^3 coordinate axes for the first time in the interview. Alex's work is compelling because he gave two incorrect answers, then reasoned to a correct answer by *connecting back* (c.f. Ellis, 2007) to the graph of $y = 2$ in R^2 and attending to two mathematical properties: equidistance and parallelism. He generalised these from the univariate case to describe the graph of $y = 3$ in R^3 as a plane "that is 3 away from the plane that x and z creates":

Alex: Actually, $y = 3$ … would be an entire plane….It has to be parallel to x, and this has to be parallel to z, so it would be this plane right here that is 3 away from the plane that x and z creates… like for the last question when y is equal to 2, that is every value that is 2 away from $y = 0$, right? So I'm thinking that like $y = 0$ would be the same as this [shades xz plane]. So it's 3, it's 3 in the positive [y] direction, because it's a positive 3, it's y equals that… *Interviewer:* Tell me about this parallel, like you said it's going to be parallel to x and z? *Alex:* It's going to be parallel to x in the same way that this line right here [$y = 2$ in R^2] is parallel to the x, to the x-axis. So it's kind of the same thing except it's like, it would be like that if it was a plane."

The sketching activity, and connecting back to the graph of $y = 2$ in R^2, allowed Alex to generalize that $y = b$ is a line in R^2 and a plane in R^3. He drew two incorrect graphs before drawing the correct one ("actually, $y = 3$ would be an entire plane"), and it was in the process of creating and reviewing these graphs that he appeared to focus on using the equidistance and parallelism to arrive at the correct answer. Alex's thinking about these two ideas is representative of other students. Another, asked to graph $y = 3$ in R^3, said "so on an xy [R^2] graph at 3, would be going this way. So on the y, following the x. So [on R^3 axes] this would be on the y, this is the 3 point on the y, and it's following the x axis." This student *created a new situation* (c.f. Ellis, 2007) that he viewed as similar to the current situation, and generalised by thinking about parallelism, which he stated as "following." Other ways students thought about this question were in terms of plotting points; for instance, "$y = 3$ at all points on the graph, any point you evaluate, so if you say $z = 2$ and $x = 2$, it's going to be 3." Hence the initial data analysis suggests that as students generalise, some of the things they attend to equidistance, parallelism, and coordinate points.

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