Calculus Students' Understanding of the Vertex of the Quadratic Function in Relation to the Concept of Derivative

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Abstract

The purpose of this study was to gain insight into thirty Calculus I students' understanding of the relationship between the concept of vertex of a quadratic function and the concept of the derivative. APOS (action-process-object-schema) theory (Asiala et al., 1996) was used in analysis on student written work, think-aloud, and follow up group interviews. Students' personal meanings of the vertex, including misconceptions, were explored, and how students relate the vertex to the understanding of the derivative. Results give evidence of students' lack of connection between different problem types which use the derivative to find the vertex. Implications and suggestions for teaching are made based on the results. Future research is suggested as a continuation to improve student understanding of the vertex of quadratic functions and the derivative.

Keywords: Quadratic function, Vertex, Derivative, APOS

It is well documented that students have trouble with quadratic functions (Afamasaga-Fuata'i, 1992; Eraslan 2008; Metcalf, 2007; Zaslavsky, 1997). It is also well documented that students have trouble with the concept of the derivative. As the derivative can be used to find the maximum or minimum of a quadratic function, also known as the vertex, this study aims to explore thirty *Calculus of Variable I* students' understanding of the relationship between the two concepts, vertex of a quadratic function and its' derivative. Understanding of a vertex by calculus I students is closely tied to students' understanding of a quadratic function for which the vertex is a particular point; a point with respect to which many algebraic and graphical properties of a quadratic function could be described (such as, the extreme value, axis of symmetry, increasing/decreasing values of a function). For this study, understanding the relationship between the vertex of a quadratic function and the derivative function includes recognizing that at the vertex of a quadratic function, the slope of the tangent line is zero, as well as being able to relate other properties between a quadratic function f and its derivative f', such as where the function is increasing or decreasing in relation to the values of the slope of the tangent line. This presents the following research question: How do Calculus I students perceive and relate the concept of the vertex of a quadratic function to the derivative in different problem situations?

There are some studies that specifically include a component on the vertex of a quadratic function (Borgen and Manu 2002; Ellis and Grinstead, 2008) Most of the studies that focus on the vertex of quadratic functions were done with students in classes that usually precede Calculus I. Studies with Calculus I students often involve concepts such as functions and variables (Vinner & Dreyfus, 1989), limits and continuity (Ferrini-Mundy & Graham, 1991), derivative (Maharaj, 2013; Orton, 1983a), and integrals (Orton, 1983b) without a specific focus on the vertex of a quadratic function. Asiala, Cottrill, Dubinsky, & Schwingendorf (1997) explored calculus students' graphical understanding of a function and its derivative and suggested that students who had an instructional treatment based on theoretical analysis may have more success

in fostering an understanding of the graph of a function and its derivative versus those in traditional courses. Other studies, such as White and Mitchelmore (1996) found that calculus students had an "underdeveloped concept of a variable" (White & Mitchelmore, 1996, p. 88). However, most studies do not necessarily make an explicit connection between the derivative and the vertex of a quadratic function. This study aims to look at Calculus I students' understanding of the relationship between these two concepts.

Analysis of student written work, think-aloud sessions, and follow up group interviews were done using APOS theory as an assessment tool to classify and make distinctions in students' answers and reasoning on problems relating the vertex of a quadratic function and the derivative (Asiala, Brown, DeVries, Dubinsky, Mathews, & Thomas, 1996). APOS framework was most appropriate to analyze student perception and understanding of the concept of vertex of the quadratic function in relation to the derivative because of the theories ability to describe and analyze possible mental constructions representing different levels of students' understanding. This theory has been proven useful for constructing a genetic decomposition of a function (Breidenbach et al., 1992), as well as a good model for studying about learning and teaching of other important mathematical concepts (Asiala et al., 1997; Clark et al., 1997; Cottrill, Dubinsky, Nichols, Schwingendorf, Thomas, & Vidakovic, 1996). By attempting to characterize student understanding of the vertex of a quadratic function in relation to the derivative based on the action, process, and object levels, this framework proved to be a useful tool in interpreting students' performance and understanding.

As part of a larger study, this poster presentation offers results from two questions from the think aloud interviews, an algorithmic problem and a real world application problem, used to determine if students could recognize the relationship between the vertex of a quadratic function and the derivative in two different problem contexts. Misconceptions of the vertex of a quadratic function, including misconceptions of the vertex as always being an intercept, misconceptions of the vertex as the origin, and misconceptions of the vertex as a point of inflection all contributed to student difficulty with answering and describing questions pertaining to the relationship of the vertex of a quadratic function and its' derivative function. Many of the students work appeared to be consistent with action conception of understanding the vertex and its' relationship to the derivative according to APOS. On the other hand, the more a student was able to talk and describe about the vertex accordingly, the more conceptually the student was able to talk about the concepts and the relationships between the vertex of the quadratic function and the derivative function. According to APOS, those students who could speak with meaning possibly exhibited at least a process level of understanding, as they could reflect and describe the reasons behind the steps.

Several implications for teaching are suggested based on the results and discussion. First, since many students appear to be performing at an action level of understanding, it is important to reassess how students are taught. It is understood that teaching goals should be to help students to develop their understanding of concepts beyond the action level. Pedagogical methods might include students talking out loud either in groups or as a class to explain reasons behind their procedures. A combination of individual and group activities on various APOS levels, class discussion, and individual exercises could help foster conceptual growth in students.

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