Preliminary genetic decomposition for implicit differentiation and its connections to multivariable calculus

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Derivatives are an important concept in undergraduate mathematics and across the STEM fields. There have been many studies on student understanding of derivatives, from graphing derivatives to applying them in different scientific areas. However, there is little research on how students construct an understanding of multivariable calculus from their understanding of single variable calculus. This poster uses APOS theory to hypothesize the mental reflections and constructions students need to make in order to solve and interpret an implicit differentiation problem and examine the connections to multivariable calculus. Implicit differentiation is often the first time students are introduced to the notion of a function defined by two dependent variables, a concept vital in multivariable calculus. Investigating how students initially reconcile this new idea of two variable functions can provide knowledge of how students think about multivariable calculus.

Key Words: Implicit differentiation, student understanding, genetic decomposition, multivariable, APOS theory

Introduction and Relation to Literature

Student understanding of single variable calculus has been well researched (e.g. Bardini, Pierce, & Stacey, 2004; Habre & Abboud, 2006; Lauten, Graham, & Ferrini-Mundy, 1994; Simonsen, 1995; Tall, 1985; Thompson & Silverman, 2008; White & Mitchelmore, 1996; Williams, 1991). Comparatively, there are relatively few investigations of student understanding of multivariable calculus (e.g. Dorko & Weber, 2014; Fisher, 2008; Kerrigan, 2015; Martínez-Planell & Gaisman, 2012; McGee & Moore-Russo, 2014). In particular, while there is an abundance of research on the topic of single variable differentiation (García, Llinares, & Sánchez-Matamoros, 2011; Habre & Abboud, 2006; Haciomeroglu, Aspinwall, & Presmeg, 2010; Orhun, 2012; Santos & Thomas, 2001), there is very little knowledge of student understanding of multivariable differentiation (Martínez-Planell, Gaisman & McGee, 2015; McGee & Moore-Russo, 2014; Tall, 1992). Similarly, many researchers have explicitly investigated the mental generalizations and reflections students need to construct the concept of differentiation (Asiala, Cottrill, Dubinsky, & Schwingendorf, 1997; Clark et al., 1997; García et al., 2011). However, there has been little work done on what reflections and mental constructions students need to make in order to understand implicit differentiation. This poster is theoretical in nature and focuses, through APOS theory, on the connections between the mental reflections and constructions need for implicit differentiation and those needed for multivariable calculus.

APOS Theory

APOS theory emerged from Piaget's notion reflection abstration and is a theoretical framework for investigating mental construction of mathematical objects (Dubinsky & McDonald, 2001). There have been several addititions the the original *action*, *process*, *object*, *schema* stages in the theory, including *procept* and *procedural process*. The term *procept* refers to a duality of having both a process and object understanding and *procedural process* refers to when a student can mentally run through an action and has interiorized it but may not yet have a deeper conceptual understanding of the process. In APOS theory, a genetic decomposition is a hypothetical model of mental constructions needed to learn a specific mathematical concept (Arnon, 2014). This poster will exhibit of a genetic decomposition for implicit differentiation and examine the connections between the mental actions/reflections students make in implicit differentiation which may be useful later in multivariable calculus. This linking broadens the current ways researchers have been looking at the connections between single- and multivariable calculus.

Discussion

There are several key reflections and connections that students must make in solving an implicit differentiation problem that similarities to those made in multivariable settings. Due to space limitations, I provide two examples; the poster will contain a complete genetic decomposition..

The first reflection students must make in solving implicit differentiation problems is to identify an implicitly defined function. To identify that a function cannot be explicitly expressed in terms of a single variable requires students to reorganize their notion for function. Students often think of functions as a variable set equal to an expression, such as $y=2x+7$ (Thompson, 2013). However, an implicitly defined function it is dependent on both variables as opposed to one. This is an essential concept in the multivariable setting because the functions are explicitly defined in terms of two variables. A student who has already seen implicit differentiation should have at least a process level of understanding of function dependent on two variables. Thus when introduced more formally to two variable functions in multivariable calculus, students already have a process to reflect on in order to build the concept of multivariable functions.

Another key construction students must make in implicit differentiation is taking the derivative of y with respect to x, rather than taking the derivative of x with respect to x. The methodology for finding the derivative with respect to a single variable when multiple are present is different between implicit differentiation and multivariable differentiation, however, constructing the concept of looking at the change in one dependent variable with respect to another dependent variable is common to both. For instance, to find the derivative with implicit differentiation, students must take the derivative of each variable with respect to a single variable. This requires several new mental constructions including an encapsulate the process of the chain rule to be able to apply as an object in implicit differentiation. However, finding a derivative of a multivariable function requires the student to reflection on what variable the derivative is being taken with respect to and to treat the other variables as fixed. This does not require the same mental structure as implicit differentiation but the main reflection of taking a derivative with respect to a single variable when more that one is present is vital to both concepts.

These are just two examples of mental constructions that students make first in implicit differentiation that are vital to those in multivariable calculus. Understanding how student think about implicit differentiation and the underlying mental actions needed to construct the concept, can not only help implement better instructional methods but also lend insight into how students think about multivariable calculus.

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