Investigating college students difficulties with algebra

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Algebra is frequently referred to as the "gateway" course for high school mathematics. Even among those who complete high school Algebra courses, many struggle with more advanced mathematics and are frequently underprepared for college level mathematics. For many years, college instructors have viewed the final problem solving steps in their respective disciplines as "just Algebra", but in reality, a weak foundation in Algebra maybe the cause of failure for many college students. The purpose of this project is to identify common algebraic errors students make in college level mathematics courses that plague their ability to succeed in higher level mathematics courses. The identification of these common errors will aid in the creation of a model for intervention.

Keywords: Algebra, Common Errors, Symbolic World

Algebra is often referred to as a gateway course because it is foundational and fundamental to STEM subjects and it is clearly here to stay. At college level, algebra content is considered as assumed knowledge and the professors are not expected to re-teach it. Calculus curricula are demanding and fast moving leaving no extra time to resolve basic algebra issues. However, it seems that many college level instructors are only concerned with one side of the coin and are somewhat disconnected from students' prior experiences, let alone the psychological effects and possible negative experiences that originated many years ago. Although, Stacey, Chick and Kendal (2004) in their edited book titled: *The Future of the Teaching and Learning of Algebra*, discussed the main problems on Algebra in school Algebra, very little mentioned in the way of consequences in college level.

A survey published by the National Center for Education Statistics reported that nationwide, in 2000, 28% of incoming freshmen took a remedial class (U.S. Department of Education, National Center for Education Statistics (NCES), 2004). Beyond those who find themselves underprepared for college level mathematics coursework, the majority of students struggle due to incomplete or insecure understandings of many important Algebraic topics. The impact of weak or incomplete mathematical understanding at the middle school and high school level, and Algebra in particular, has a profound impact on the future mathematical success of students and their educational possibilities.

This research will employ Tall's (2008, 2010, 2013) framework of embodied, symbolic and formal mathematical thinking in an effort to construct a model of mathematical thinking for investigating students' understanding of algebra concepts. Tall (2010) defines the worlds as follows: The *embodied world* is based on "our operation as biological creatures, with gestures that convey meaning, perception of objects that recognise properties and patterns...and other forms of figures and diagrams" (p. 22). Embodiment can also be perceived as giving body to an abstract idea. The *symbolic world* is based on practicing sequences of actions which can be achieved effortlessly and accurately as operations that can be expressed as manipulable symbols. The *formal world* is based on "lists of axioms expressed formally through sequences of theorems proved deductively with the intention of building a coherent formal knowledge structure" (p. 22). Through in-depth qualitative research we anticipate ascertaining more about each of the three worlds as well as the blending of relationships *between* the worlds, and ultimately proposing a model that is applicable to interventions for the algebra skills needed for success in calculus.

In an effort to identify common student errors and gain insight about how best to develop appropriate interventions, this project is focused on the following research questions: 1) In what areas of algebra do the major difficulties occur? 2) How would algebra interventions affect a student's understanding of calculus and help them with more symbolic ways of interpreting mathematics? 3) As students advance into various STEM disciplines and encounter more formal mathematics, how does the lack of understanding algebra affect them and what interventions would help them? 4) What are some of the pedagogical challenges related to the symbolic world of mathematics?

For the purpose of this poster presentation we will only answer the first research question.

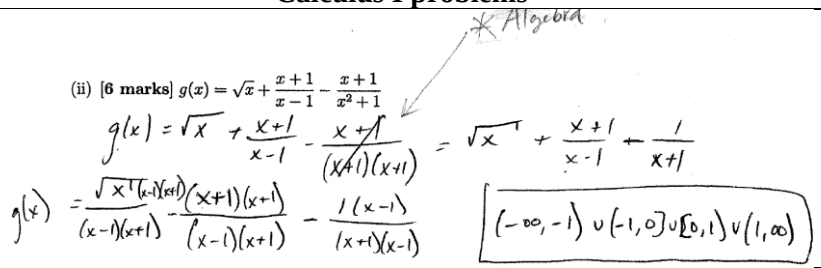
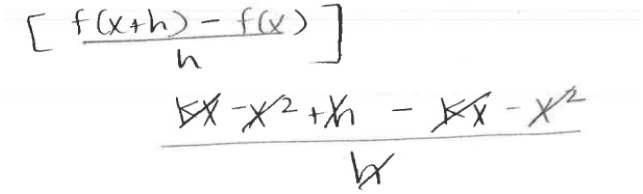
Method

Our interdisciplinary, multi-institutional team includes two mathematics educators, two mathematicians, a cognitive psychologist who specializes in children's algebra thinking process, an elementary school algebra teacher and two graduate students. To achieve the project goals and answer the research questions, a pilot study was conducted at a mathematics department at a large research university in South-West of the United States. Data were gathered from approximately 2500 students' final exams from the following five different math courses: College Algebra; Pre-Calculus and Trigonometry; Pre-Calculus for Business, Life and Social Sciences; Calculus I for Business, Life and Social Sciences; and Calculus and Analytical Geometry. A small sample of the pilot data were analyzed in order to provide preliminary results. We plan to use the results of this study to develop a model for intervention. Data will be collected before and after the intervention to further our understanding of both the common errors and how best to help students overcome them.

Preliminary Results

The initial analysis of the pilot study data show several themes emerging among the types of common errors made by students while encountering *fractions* and *exponents* and dealing with *variables* and *mathematical properties*. The frequent occurrence of these categories of errors amplify significantly in calculus courses and will have a negative impact on students' overall performance. For the purpose of this proposal, we have provided two examples from the data that reflect the common errors while *encountering fractions* (see Table 1).

Table 1. Sample students' common errors causing significant disruption in solving problems.

Calculus I problems	*Errors*
	*Simplifying fractions causing difficulties in finding the domain*
	*Simplifying fractions causing difficulties in finding the derivative*

More than a century ago, De Morgan (1910) wrote about the difficulties students face in learning mathematics noting common errors related to arithmetic and rational number computation. Since that time, other researchers have catalogued common errors in computation and algebra (Ashlock, 2010; Benander & Clement, 1985; Booth, Barbieri, Eyer & Pare-Blagoev, 2014). Connecting with the existing body of knowledge on students' persistent problems related to algebra our preliminary findings seem to parallel some of these categorizations and document that these errors persist in college level. In many cases, students are working to learn concepts that are new to them in college calculus courses and the results on assessments, formative or summative, are often more reflective of student difficulties with algebra than the newer concepts. The frequency of such errors creates frustration for both students and their instructors and may create barriers to student advancement in college level mathematics. We believe that the model generated by this project will be generalizable and can be used to examine the effect of students' understanding of Algebra in other sciences e.g. Physics and Chemistry. Moreover, the study will have an enormous impact on our understanding of the symbolic world and its pedagogical complexities.

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