Using Adjacency Matrices to Analyze a Proposed Linear Algebra Assessment

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An assessment of student learning of major topics in linear algebra is currently being created as part of a larger study on inquiry-oriented linear algebra. This includes both the assessment instrument and a way to understand the results. The assessment instrument is modeled off of the Colorado Upper-division Electrostatics (CUE) diagnostic (Wilcox & Pollock, 2013). There are two parts to each question: a multiple-choice part and an explanation part. In the explanation part, the student is given a list of possible explanations and is asked to select all that could justify their original choice. This type of assessment provides information on the connections made by students. However, analyzing the results is not straightforward. We propose the use of adjacency matrices, as developed by Selinski, Rasmussen, Wawro, & Zandieh (2014), to analyze the connections that students demonstrate.

Key words: Linear Algebra, Assessment, Adjacency Matrices

As part of a larger study on developing materials for inquiry-oriented approach to Linear Algebra we created an assessment instrument to measure student understandings of major topics in linear algebra, including span, linear independence/dependence, invertibility, solutions to systems of linear equations, and transformations. In an attempt to gain a deeper understanding of student thinking on these topics without the use of a free-response assessment, we modeled our assessment off of the Colorado Upper-division Electrostatics (CUE) diagnostic (Wilcox & Pollock, 2013). In this type of an assessment, students are asked a standard multiple choice-question and then they are prompted to select all of the "because" choices that could justify their choice. An example is given in Figure 1.

1) The set of vectors $\left\{\begin{bmatrix}4\\5\end{bmatrix}, \begin{bmatrix}2\\3\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}\right\}$ is:

A) linearly independent
B) linearly dependent

Because … (select ALL that could justify your choice)

i) the set includes the vector $\begin{bmatrix}0\\0\end{bmatrix}$.
ii) the vector $\begin{bmatrix}0\\0\end{bmatrix}$ can be excluded from the set.
iii) the set has 3 vectors in $\mathbb{R}^2$.
iv) the **only** solution to $c_1\begin{bmatrix}4\\5\end{bmatrix} + c_2\begin{bmatrix}2\\3\end{bmatrix} + c_3\begin{bmatrix}0\\0\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$ is the trivial solution (i.e., $c_1 = c_2 = c_3 = 0$).
v) the vectors span all of $\mathbb{R}^2$.
vi) $\begin{bmatrix}4 & 2\\5 & 3\end{bmatrix}$ is row equivalent to the identity matrix.

Figure 1: Assessment format

Interpreting the results of such questions, however, is complicated and prior work gives little insight into how to compile and make sense of the results. Moreover, in light of the fact that linear algebra is rich in connections, we were interested in measuring the nature of connections students made between topics in linear algebra. One possibility is to analyze student responses through the use of adjacency matrices.

Adjacency Matrices

Recent studies in linear algebra have used adjacency matrices as a way to analyze the connections students make between topics (e.g., Selinski et al., 2014). The use of adjacency matrixes starts by creating a list of codes that are topics and sub-topics in linear algebra. These codes make up the sides of the matrix and whenever a student makes a connection between two topics or subtopics, the matrix is marked in the corresponding cell of the row and column. From there, a quantitative measure can be placed on the types of connections made between topics by the students as well as preserving a connection to the qualitative data that created the matrix.

For the current study, we explored how adjacency matrices may be leveraged to analyze results from the assessment. The goal is to obtain a portrait of the connections students made between topics in linear algebra through the use of their assessment answers. However, before the assessment answers could be analyzed separate from the students, we needed to have knowledge of the ways in which the students were reading and understanding the assessment as well as how they were choosing their answers. Such information requires individual interviews.

Assessment Interviews

In the fall of 2014, 11 interviews were conducted using the current version of the linear algebra assessment at a large public university. Students were asked to explain their choices so as to gain an understanding of how they understood the problems and answers. The interviews were transcribed and coded independently by two researchers. The researchers discussed each code until consensus was reached. The codes were generated through a combination of open coding and a priori coding, with the a priori codes coming from previous studies done on student understanding in linear algebra (Selinski et al., 2014). The codes were used to create ordered pairs, much like what one may use to describe placement on a matrix, so as to create a basis for the adjacency matrix for each student. It is through these codes that an adjacency matrix was created. In the table below is an example of the coding done throughout the interviews. In this example, the student had just finished answering the multiple choice section of the problem shown in Figure 1 and had chosen the answer of linearly dependent. He is going through a couple of the explanation answers in the quotes below and chosing the ones he believes supports his answer choice.

Student	The set includes the vector [0, 0]. I think that's why.	(B1, B)
	The set has 3 vectors in R2. I didn't even think about that but that's why, right? Yeah. Pretty sure. [H: okay]	(B2, B)

The third column gives an example of how the coding was done and is read as B1 implies B2 and B2 implies B, where B1 stands for "the set includes the zero vector," B2 stands for "the set has more vectors than dimensions," and B stands for "set of vectors is linearly dependent." Much of the coding was done based on what the student had said previously as well as what was specifically stated in the utterance.

Conclusions

Adjacency matrices have provided both a quantitative and qualitative way of looking at student understanding in linear algebra. Through the use of interviews, for example, prior work provided a detailed explanation of the depth and types of connections students made between topics in linear algebra. In the current study, there is an additional level of complexity as we are attempting to use the adjacency matrix to help analyze whether or not the assessment shows the types of connections we expect. Our analysis suggests the following questions to discuss: If the adjacency matrix shows different or additional connections made by the students than what the assessment captures, how might the adjacency matrix inform future versions of the assessment? The number of explanations we can provide students is limited so how do we determine the optimal number that captures most of the possible explanations? What other possible frameworks could be useful in analyzing the assessment?

References

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