Using the chain rule to develop secondary school teachers' Mathematical Knowledge for Teaching, focused on the rate of change: Secondary mathematics teachers' knowledge of the chain rule and its' impact on their teaching of the rate of change.

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The unit described in this study was designed to connect secondary and advanced mathematical topics. It focused on how the knowledge of chain rule impacts secondary teachers' understanding and teaching of rate of change so that they can address students' misconceptions. This project is informed by the idea of Mathematical Knowledge for Teaching, which encompasses both subject-matter knowledge and pedagogical content knowledge of teachers. The goal was to enhance secondary school teachers' teaching of the rate of change and the unit featured tasks connecting rate of change problems as seen in high school algebra to the concept of chain rule. The unit was designed to engage mathematics teachers in discourse about the content learned at the college level to content that is taught at the secondary school level.

Key words: [Chain rule, secondary school teaching, rate of change, Mathematical Knowledge for Teaching]

Introduction

According to the CCSSM standards, the concept of nonlinear models should be introduced in eighth grade but this introduction is limited to analyzing graphs to understand the functional relationship between two quantities (CCSS, 2013). This leads to several gaps in students' understanding of the rate of change of non-linear models. Furthermore since students learn the concept of linearity early in their primary and secondary school careers and this concept is reinforced they feel comfortable using it and tend to apply it without discretion (De Bock, 2002). Consequently, when students are introduced to higher level concepts, specifically in differential calculus, they struggle to develop conceptual understanding because of their assumptions of linearity (Brabham, 2014). It is important therefore to introduce rate of change as it relates to both linear and non-linear models so students get a deeper conceptual understanding of the rate of change. For this purpose we created an instructional unit as part of our class project.

Our unit was created with the goal to make connections between secondary and advanced mathematical topics. Specifically, our research question is: How does the knowledge of chain rule impact secondary school teachers' understanding and teaching of the rate of change?

Conceptual Framework

Our project is informed by the idea of Mathematical knowledge for teaching (MKT) (Ball, Thames & Phelps, 2008), which encompasses both subject-matter knowledge and pedagogical content knowledge. For example teachers need the knowledge to check correct answers, definitions and concepts but also specialized content knowledge to meet the demands of teaching mathematics. These include skills needed to pose questions, interpret students' responses, use multiple representations to provide explanations and most importantly make connections. Some MKT is a result of blending mathematics with other knowledge, like knowing the students, the curriculum, pedagogy etc. A recently added type

of MKT is “horizon knowledge” which gives teachers a mathematical “peripheral vision” that is so important for effective teaching. It is “an awareness of how mathematical topics are related over the span of mathematics included in the curriculum” (Ball et al., 2008, p. 403). It provides a larger view of mathematics that gives the teachers a sense of where to place the content that they are teaching and how it is connected to higher level mathematics (Ball and Bass, 2009).

Research suggests that strong MKT is linked to certain habits of mind like careful attention to mathematical detail, reasoning skills, dexterity with various forms of mathematics curricula, working with students etc. Sometimes teachers develop this knowledge on their own by engaging in mathematics focused professional development but other times they need support (Hill & Ball, 2009). Based on the nature of MKT, there is a need to develop instructional guidance for teachers (Hill & Ball, 2009). For this reason, a unit was developed with the understanding that it will enhance secondary school teachers’ teaching of the rate of change. The unit featured tasks connecting rate of change problems as seen in high school algebra to the concept of chain rule. Our goal in creating this unit was to engage mathematics teachers in discourse about the content learned at the college level to content that is taught at the secondary school level. Our unit employed tasks designed by Hill and Ball (2009) which focused on analyzing student errors, experiencing alternative solutions, choosing examples etc.

Research Methodology

Participants

The two participants in this study are fulltime students in a doctoral level mathematics education course. Of the two, one is currently a high school mathematics teacher in the United States and the other participant has several years of teaching experience in India.

Data Collection

The class was audio taped and observation notes of the lesson were taken focusing on participants’ mathematical conversations and comments in regards to the lesson. Participants’ work was also collected

Instructional Unit

The unit was designed to engage the teachers in discourse about the content learned and to make connections between mathematical content at the secondary and tertiary education levels. It posed a series of thought provoking questions which led the participants to examine the teaching of rate of change at the secondary level; specifically, the relationship between the instantaneous rates of change, the constant rate of change, and the chain rule. At the end of the lesson, the participants shared their reflection on the topic, both verbally and in writing.

Data Analysis

Our group analyzed participants’ work to find recommendations for content and pedagogy to improve teaching of the concept of rate of change in secondary education. We also recorded any connections between secondary and tertiary mathematics as well as student misconceptions on this topic.

Implications

This pilot study focused on a unit developed as part of a class project. In the future, we plan to revise our unit and conduct this study with pre-service mathematics teachers.

References

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching what makes it special? *Journal of teacher education*, 59(5), 389-407.

Ball, D. L., & Bass, H. (2009). With an eye on the mathematical horizon: Knowing mathematics for teaching to learners' mathematical futures. Paper prepared based on keynote address at the 43rd Jahrestagung für Didaktik der Mathematik held in Oldenburg, Germany.

Brabham, K. P. (2014). Does exploring non-linear models address high school students' misconceptions of linearity and rate of change? (Unpublished Thesis), Louisiana State University, LA.

De Bock, D., Van Dooren, W., Janssens, D., & Verschaffel, L. (2002). Improper use of linear reasoning: An in-depth study of the nature and the irresistibility of secondary school students' errors. *Educational Studies in Mathematics*, 50(3), 311-334.

Hill, H., & Ball, D. L. (2009). The curious—and crucial—case of mathematical knowledge for teaching. *Phi Delta Kappan*, 91(2), 68-71.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards*. Washington, DC: Authors.www.corestandards.org (accessed May 29, 2013).