Exploring student understanding of the negative sign in introductory physics contexts

Suzanne White Brahmia, Rutgers University
Andrew Boudreaux, Western Washington University

Recent studies in physics education research demonstrate that although physics students are generally successful executing mathematical procedures, they struggle with the use of mathematical concepts for sense making. In this poster we investigate student reasoning about negative numbers in contexts commonly encountered in calculus-based introductory physics. We describe a large-scale study (N > 900) involving two introductory physics courses: calculus-based mechanics and calculus-based electricity and magnetism (E&M). We present data from six assessment items (3 in mechanics and 3 in E&M) that probe student understanding of negative numbers in physics contexts. Our results reveal that even mathematically well-prepared students struggle with the way that we symbolize in physics, and that the varied uses of the negative sign in physics can present an obstacle to understanding that persists throughout the introductory sequence.

Introduction

Signed numbers carry rich information about physics contexts. A confounding feature in physics is that the operations of addition and subtraction (represented by the symbols "+" and "−") can easily be confused with the descriptors, positive and negative, that can characterize the opposite natures of some physical quantities (position, charge, velocity, etc.)

Developing flexibility with negative numbers is a known challenge in math education. Vlassis[1] used written diagnostic questions and interviews to investigate the understanding of negative numbers by Belgian students taking algebra. She found that in order to fully understand the concept of a negative number, students had to develop a flexibility with the various ways in which negative numbers are used in context. The most challenging context is common to physics – quantifying opposites.

Sherin[2] refers to quantifying opposites in physics as the symbolic form "competing terms cluster," which includes the notion of zero to represent balance, and positive and negative quantities as competing terms in an expression. This cluster is built on a stable set of coordinated resources that includes a conceptual understanding of signed numbers and zero. He observes that flexibility with this symbolic form is a feature of expert problem solving in introductory physics.

Bajracharya, Wemyss, and Thompson[3] investigated student understanding of integration in the context of *P-V* diagrams in introductory physics. Their results suggest an incomplete understanding of the criteria that determine the sign of a definite integral. Students struggle with the concept of a negative area, and with the concept of positive and negative directions of integration. Even for students in calculus-based physics, negative quantities pose challenges.

Experimental Design

We administered a set of three questions at the end of the fall 2015 semester in the calculus-based introductory courses in Mechanics, and in E&M; a portion of each class was given a MC version of the questions while the rest were given an open ended version and asked to explain their reasoning. Each set probes the use of a signed quantity: 1) to represent a component of a vector quantity in 1-D, 2) to quantify opposites, and 3) to represent a difference of a position dependent quantity measured at two different locations (see Fig. 1).

Discussion

Our results show that engineering students really struggle to make sense of the physics use of the negative sign in almost every context except the one that is familiar from math class.

Surprisingly after a semester of calculus-based physics, one-third of the engineering students fail to recognize the context in which they learned about negative numbers - the position on a number line (see Fig 1, Mech 3)

Figure 1: Assessment items, "Mech" was administered in the Mechanics course and "EM" was administered in the E&M course.

Mech 1: An object moves along the x-axis, and the acceleration is measured to be $a_x = -8$ m/s^2. Consider the following statements about the "–" sign in "$a_x = -8$ m/s^2". Pick the statement that best describes the information this negative sign conveys about the situation.

a. The object moves in the negative direction
b. The object is slowing down
c. The object accelerates in the $-x$-direction
d. Both a and b
e. Both b and c

Mech 2: A hand exerts a force on a block as the block moves along a frictionless, horizontal surface. For a particular interval of the motion, the hand does W = – 2.7 J of work.
Consider the following statements about the "–" sign in the statement W = –2.7 J. The negative sign means:

I. the work done by the hand is in the negative direction
II. the force exerted by the hand is in the negative direction
III. the work done by the hand decreases the mechanical energy associated with the block

Which statements are true?

a. I only
b. II only
c. III only
d. I and II only
e. II and III only

Mech 3: A cart is moving along the x-axis. At a specific instant of time the cart is at a position x = –7 m. Consider the following statements about the "–" sign in "x = –7 m". Pick the statement that best describes the information this negative sign conveys about the situation.

a. The cart moves in the negative direction
b. The cart is to the negative direction from the origin
c. The cart is slowing down
d. Both a and b
e. Both a and c

EM 1: At a location along the x-axis, the electric field is measured to be $E_x = -10$ N/C. Consider the following statements about the "–" sign in "$E_x = -10$ N/C". Pick the statement that best describes the information this negative sign conveys about the situation.

a. The test charge is negative
b. The field is being created by negative charge
c. The field points in the $-x$-direction
d. Both a and b
e. Both b and c

EM 2: Valeria combs her hair in the winter and there is a transfer of charge such that $DQ_{comb}= -5$ mC. Consider the following statements about the "–" sign in the mathematical statement $DQ_{comb}= -5$ mC. The negative sign means:
I. negative charge was added to the comb. II. charge was taken away from the comb.
III. all of the electric charge in the comb is negative
Which statements could be true?

a. I only
b. II only
c. III only
d. I and III only
e. II and III only

EM 2: In physics lab, a student uses a voltmeter to measure the voltage across the terminals of a battery. The voltmeter reads –5V.
Consider the following statements about the "–" sign in the voltmeter reading "– 5V". Pick the statement that best describes the information this negative sign conveys about the situation.

a. the voltage is in the opposite direction as the current
b. there are 5V of negative charge in the battery
c. the voltage is in the negative direction
d. the voltage at one terminal is 5V less than the voltage at the other terminal
e. this battery has negative voltage

Table 1: Response rates, Mechanics(n=310) and E&M (n=402). Correct response rate is in bold type

Choice	*Mech 1*	*Mech 2*	*Mech 3*		*EM 1*	*EM 2*	*EM 3*
a	8%	17%	6%		16%	**33%**	32%
b	26%	17%	**67%**		21%	28%	14%
c	**26%**	**23%**	6%		**36%**	18%	18%
d	6%	29%	19%		12%	15%	**33%**
e	34%	14%	2%		14%	5%	3%

Explanations (from open-ended responses, Mechanics n=85, E&M, n=138)
Excerpts to be included in poster

1. Vlassis, J., “Making sense of the minus sign or becoming flexible in ‘negativity’”, Learning and instruction, 14, pp.469-484, (2004).

2. Sherin, B. L., “How students understand physics equations”, Cognition and instruction, 19, pp.479-541, (2001).

3. Bajracharya, R. R., Wemyss, T. M., and Thompson, J. R., “Student interpretation of the signs of definite integrals using graphical representations”, AIP Conference Proceedings 1413, pp.111, (2012).