Separating issues in the learning of algebra from mathematical problem solving

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Students' difficulty in learning school algebra has motivated a plethora of research on knowledge and skills needed for success in algebra and subsequent undergraduate mathematics courses. However, in gateway mathematics courses for science, technology, engineering, and mathematics majors, student success rates remain low. One reason for this may be to the lack of understanding of thresholds in student mathematical problem solving (MPS) practices necessary for success in later courses. Building from our synthesis of the literature in MPS, we developed Likert scale items to assess undergraduate students' MPS. We used this emerging assessment and individual, task-based interviews to better understand students' MPS. Preliminary results suggest that students' issues in algebra do not prohibit them from using their typical problem solving methods. Thus, the assessment items reflect students' MPS, regardless of possible misconceptions in algebra, and provide a mechanism for examining MPS capacity separate from procedural and conceptual issues in algebra.

Keywords: college algebra, problem solving, algebra learning

Issues in Algebra

Research shows several sources of difficulty in learning algebra. For example, students struggle in understanding the meaning of variables. In algebra instruction, x is frequently called a *variable*, accompanied with statements such as "x can be anything." But this conception is particularly misleading for equations such as $2x + 7 = 13$, in which x is actually an unknown quantity (Kieran, 2007). Additionally, in functions, variables stand for inputs and outputs. Many students use x and y to write equations and functions but do not actually attend to the meaning of those symbols; Students use these letters solely as a placeholder in equations and functions to replicate examples they have seen (Chazan, 2000). This confusion increases student difficulty converting word problems into equations (Kieran, 1992).

Algebra students also struggle with the meaning of the equal sign. Although the equal sign is often used to indicate a relationship between two quantities, for a function, the equal sign represents a naming of an object. Further, many students view the equal sign as a connector or operation, with little meaning beyond indicating the direction of the solution path (Schoenfeld & Arcavi, 1988). This connector usage leads to student concatenating operations using an equals sign as if they are using a calculator (i.e. 4 + 7 = 11 + 3 = 14). In elementary school, students use a "guess and check" method of solving equations. However, formal procedures taught in algebra can be difficult for students to internalize, as they have not previously needed to maintain symmetry across the equal sign (Kieran 1992). Further, in solving $2 + \_\_ = 5$, elementary students place 3 in the blank, which appears to add 3 to only one side of the equation.

Although Kieran (2007) asserts that the use of technology in the algebra classroom improves student understanding of functions, technology can also lead to some misunderstandings about equations. Though the equations $y = 2x + 8$ and $-2x + y = 8$ are equivalent, if such an equations are part of a system of equations, x and y correspond to specific values in a solution set rather than inputs and outputs (Chazan & Yerushalmy, 2003).

Studying mathematical problem solving

Nationwide, more than 40% of undergraduates pursuing science, technology, engineering, and mathematics (STEM) majors failed to complete their degrees (President's Council of

Advisors on Science and Technology, 2012), and for many students, progress is blocked by their lack of success in foundational mathematics courses. For example, as few as 10% of calculus-bound STEM intended College Algebra students reach calculus (Dunbar, 2005). Though important skills and procedures needed for success in calculus are identified in the research (e.g., Carlson, Oehrtman, & Engelke, 2010), the specific knowledge and skills emphasized in gateway mathematics courses seems insufficient for students' progression in STEM majors. Students appear to lack the necessary mathematical problem solving (MPS) skills and reasoning to be successful. MPS is at the forefront of instructional goals in mathematics education (e.g., National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010; National Council of Teachers of Mathematics, 2000); however, little is understood the thresholds students must meet at various levels to ensure success in subsequent courses.

Campbell (2014) synthesized the research literature in MPS and separated MPS into five components. These components are sense-making, representing/connecting, reviewing, justification, and challenge/difficulty. Building from this work, we developed items to assess student's MPS in these areas. The MPS components or domains attend to three of the six problem solving components identified by Jonassen (1997) and the other components are controlled for in the design of the items. The roles of different components of MPS are largely unstudied due to the time and effort that must be invested to use existing tools (e.g. Oregon Department of Education, 2000; Dawkins & Epperson, 2014). By contrast, the goal of the emerging tool is to create a set of problem solving items that can be machine scored to quickly learn about students' problem solving techniques and practices. In the instrument, students complete a series of problems and then items targeting specific components of MPS. The items and instrument development are explained fully in Epperson, Rhoads, and Campbell (in press).

Student understandings in MPS and Algebra

This research takes place at a large, public university in the Southwest. We administered the MPS instrument to 70 (calculus-bound) College Algebra students and selected 11 for individual, one-hour problem-solving video-recorded interviews. In an interview, the researcher asks the student to explain his or her usual problem solving approaches and the specific MPS used on the problems and items from the assessment. The interview participants also complete a new problem and associated items. The recorded interviews were transcribed for analysis. The research adopts a mixed grounded theory approach to characterize the MPS used by the participants (Corbin & Strauss, 2008; Charmaz, 2006).

Interviews show interesting trends in students' MPS. First, participants only used diagrams and representing/connecting practices at the beginning of the problem solving process and did not incorporate them later. In addition, participants fixated on the problem statement, spending extra time rereading or rewriting the problem statement before attempting to solve the problem. These activities aligned with MPS capacity identified by their work on the MPS items. Participants' difficulties in algebra arose in the interviews. Students used confusing language pertaining to variable or unknown, such as "running through variables" to mean checking multiple values. A student also suggested that the needed *function* was an *inequality*. Despite these difficulties with algebra, students showed reluctance to use less analytical approaches. The students desired elegant functions or equations even if they proposed adequate solution paths using other logical means. However, many eventually used their less-preferred approach. These results indicate that students' limitations in algebra do not necessarily halt their problem solving practices. The implications of equal sign confusion are also under investigation. Separating the challenges of algebra learning from problem solving can provide a window into aspects of MPS necessary for student success in gateway mathematics courses for STEM.

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