

Reasoning about changes: a frame of reference approach

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In a RUME 18 Theoretical Report my co-authors and I presented our cognitive description of a conceptualized frame of reference, consisting of mental commitments to units, reference points, and directionality of comparison when thinking about measures. Here I present a pilot study on how a focus on conceptualizing a frame of reference impacts students' ability to reason quantitatively about changes. The two-part empirical study consisted of clinical interviews with several students followed by teaching interviews with three students chosen because of their varying abilities to conceptualize a frame of reference. My initial evidence shows that the ability to conceptualize a frame of reference greatly benefits students as they attempt to reason with changes.

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In a RUME 18 Theoretical Report (Joshua, Musgrave et al. 2015) my co-authors and I presented our cognitive description of a conceptualized frame of reference, consisting of mental commitments to units, reference points, and directionality of comparison when thinking about measures. At the same time, my experiences working with reform curricula for pre-calculus (Carlson, Oehrtman et al. 2013) and Calculus 1 (Thompson, Byerley et al. 2013) led me to be surprised at how much students struggle with thinking about and reasoning about changes. In mathematics, rate of change is known to be a main idea in calculus (Carlson, Jacobs et al. 2002), and important to introduce as early as Algebra 1 with the idea of slope, but in order to reason about rate of change, a student must be able to conceptualize and reason about changes themselves.

I hypothesized that student struggles were due at least in part to the fact that they were taught that measures of changes in quantities had reference points and directionality yet did not conceptualize measures of quantities themselves with reference points and directionality. Therefore, they did not have parallel attributes with which to compare and contrast the ideas of quantities versus changes, and to distinguish the two in their minds.

The pilot study I propose to share via a poster presentation is an empirical study that I conducted on the connections between a student's ability to conceptualize a frame of reference, and his or her ability to reason about changes. There are several issues surrounding changes that I explored. Among them are:

- a) Changes in Quantity vs. Values of Quantity
 - How do students conceptualize a change in a quantity versus the value of a quantity?
 - Does a focus on frames of reference affect students' ability to reason about changes in quantity and values of quantity, by drawing explicit attention to reference points?
- b) Changes in Changes

- How do students think about changes in changes, in tasks such as being asked to identify whether a function is increasing/decreasing at an increasing/decreasing rate?
 - Does a focus on frames of reference affect students' ability to reason about changes in changes, by drawing explicit attention to a directionality of comparison?
- c) Changes in the Context of Velocity & Accelerations
- How do students conceptualize velocity as it relates to both displacement and acceleration?
 - Does a focus on frames of reference affect the common misconception that "positive acceleration mean the object is speeding up?"
 - How could inconsistent use of a frame of reference (as described in the anecdote) affect student's thinking and consistency, and/or cause future problems, if at all?

The interview processes were carried out with students who have taken at least one algebra class and one physics class. The first part of the study consisted of clinical interviews on eight tasks with seven students to gather data to help me form models of each student's ability to conceptualize a frame of reference. I then picked three students that I found demonstrated varying abilities to conceptualize a frame of reference (roughly described as high, medium, and low) and conducted teaching interviews on eight new tasks with them. All interviews were videotaped and analyzed to form models of how the student thought about measures and measure comparisons before, during, and at the end of the teaching experiment, as well as hypotheses about how these ways of thinking about measures (within a frame of reference or not) affected the student's ability to reason about changes.

Through this pilot study, I found strong initial evidence that a student's ability to conceptualize a frame of reference and reason about measures within a frame of reference had a large positive effect on their ability to reason about changes. The students' abilities to reason through tasks about changes in the teaching interviews frequently reflected the initial positions of 'high' 'medium' and 'low' that I had placed them simply on their abilities to reason about a frame of reference in the clinical interviews. More significantly, the language that the students used to explain their reasoning about tasks involving changes was often about aspects of a frame of reference (units, reference points and directionality of comparison) when the students were successful, and almost never about aspects of a frame of reference when the students gave up or were unsuccessful. Finally, there were many fascinating details in my teaching interviews about how students might begin to conceptualize a frame of reference and apply such an ability to dealing with changes, that have provided starting places for my next expanded project on how a focus on conceptualizing a frame of reference impacts students' ability to reason quantitatively about changes. I believe that not only will the results of my pilot study be of interest to the RUME community, but that discussions with and advice from colleagues during and after my poster presentation would be greatly beneficial to me as I continue to design the next stage of this project.

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