On the Variety of the Multiplication Principle's Presentation in College Texts

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The Multiplication Principle is one of the most foundational principles of counting. Unlike foundational concepts in other fields, where there is uniformity in presentation across text and instruction, we have found that there is much variety in the presentation of the Multiplication Principle. This poster highlights the multiple aspects of this variety, specifically those with implications for the combinatorial research and education community. Such topics include the statement types, language and representation of statements, and mathematical implications.

Key words: Combinatorics, Multiplication Principle, Student Thinking, Textbook Analysis

Introduction and Research Questions

Combinatorics problems embody a duality of accessibility and difficulty for students at various levels. Because of the growing need for discrete mathematics in scientific fields, it is important for the mathematics education community to understand student conceptions of foundational counting principles and techniques. The multiplication principle (MP) is widely accepted as an important and fundamental principle in combinatorics, and serves as the basis for many basic counting formulas (Gersting, 1999; Mazur, 2009; Richmond & Richmond, 2009). We have experienced a variety in the presentation of the multiplication principle. This variety, and the importance of the principle motivated a formal analysis of a large sample of textbooks in combinatorics, finite, and discrete mathematics textbooks. This poster presents on the results of that study which sought to answer the following two research questions: 1) What is the nature and extent of the variation of statements of the multiplication principle presented in combinatorics, discrete mathematics, and finite mathematics textbooks?
2) What mathematical issues arise in comparing and contrasting different statements of the multiplication principle?

Relation to Literature

There have been recent studies that investigated student thinking in combinatorial contexts in which correct application of multiplication was a vital component of the learning process (eg. Lockwood & Coughman, 2015; Kavousian, 2008; Tillema, 2011; Tillema, 2013). There are also a number of researchers (e.g., Dubois, 1984; Fischbein & Gazit, 1988; Piaget, 1975) who have studied student discovery and application of counting formulas which rely heavily on the multiplication principle. While the above studies relate multiplication to counting, there is a lack of studies directly involving student thinking on the MP. This textbook analysis offers a glance at the pedagogical issues surround the MP that students are exposed to in their learning process.

Theoretical Perspectives

Researchers have examined textbooks to better understand how ideas are presented to students in the fields of linear algebra (Cook & Stewart, 2014; Harel, 1987), trigonometry (Mesa

& Goldstein, 2014), and abstract algebra (Capaldi, 2012). We adopt this examination to combinatorics texts.

We also utilize Lockwod's (2013) model for student combinatorial thinking in terms of sets of outcomes and counting processes. We combine Lockwood's model with Sfard's (1991) dual nature of mathematical conceptions. Her language reflects that students can think of mathematical concepts as objects (reflecting a structural conception) and process (reflecting an operational conception). This dualistic language proves vital to statement analysis.

Methodology

We selected textbooks for our analysis from a list of 76 colleges nation-wide. The list was made to include colleges from each state, as well as colleges of differing size and ranking. 6 colleges were excluded from the study. In total, we analyzed 32 textbooks that served as the assigned reading for 92 different courses from the 70 universities. We also then added textbooks from our personal libraries to make a total of 64 textbooks analyzed.

Our analysis followed Strauss and Corbin's (1998) constant comparative method of qualitative analysis. The data collected was scanned textbook sections introducing the MP and the surrounding narratives (Thompson, et al., 2012). In our initial glances at the data we noted emergent observed phenomenon, and built a coding scheme inductively. In each section we were specifically interested in statements of the MP, and so with each statement given we characterized the different statement types, the language used, and the representations given to accompany the statement. We were also interested in the mathematics of the statement types, specifically noting if each statement discussed independence of events, distinctness of composite outcomes, and subtleties involving the Cartesian product.

Results and Implications

This poster will demonstrate the variety across the different statements and textbooks. We categorized three inherently distinct statement types: structural, operational, and bridge. The former two statement types are in accordance with Sfard's dualistic concept notions and the latter merges the characterizations. We note, and will display, that there were differences in the combinations of these statement types in the textbooks. For instance, 6 discrete mathematics and 4 combinatorics books gave only structural statements. These different statement types further research in student thinking on the MP by providing researchers different conceptions of the MP to leverage when investigating combinatorial thought.

We will also display the extent of the diversity of the other considerations we accounted for in our analysis. We found that the languages and representations of the MP varied greatly. This variety is noteworthy to educators in that they may now be made aware of the kinds of presentations of the MP that exist in textbooks. Educators with this awareness can make more informed decisions when choosing the textbooks for their classes.

Finally, we found that statements accounted for differing combinations of the three mathematical considerations listed above. It is pedagogically important to note that these considerations can affect the accuracy of students' applications of the MP. For instance, not accounting for the distinctness of the composite outcomes may lead a student to misapply the MP and over-count when solving a particular counting problem. This discussion will be useful for the combinatorial instruction.

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Lockwood & Coughman Primus "set partitions"