An overview of research on the arithmetic mean in university introductory statistics courses.

Samuel A. Cook
Wheelock College

There is a dearth of research on the arithmetic mean at the university level. This poster will cover overlap of several studies (some unpublished) on university students' understanding of the mean and university statistics instructors' beliefs about their students' understandings of the mean.

Key words: Mean, Average, Arithmetic Mean, Introductory Statistics, Statistics

Children through grade 8 have shown the ability to calculate a mean, but have trouble thinking of the mean beyond the procedure of calculating it (Mokros & Russell, 1995). It is theorized that the early introduction of the procedure for calculating the mean may interfere with the ability to understand the mean conceptually as an object, as opposed to the result of a set of procedures, (Mokros & Russell, 1995) and that introducing the procedure early on would require a more difficult route towards a conceptual understanding (Cook & Fukawa-Connelly, 2012). This poster examines overlapping findings of university students' understanding of the mean at the, and how it relates to what is being taught in introductory statistics courses at the university level.

What Do University Students Understand About the Mean?

In one study on incoming mathematics majors' statistical knowledge, all participants believed they understood what the mean was and were confident in their responses when asked to describe the mean. However, each participant only thought of the mean as the result of a procedure, often described as "add up all the numbers and divide by the amount of numbers". In contrast, when asked about the standard deviation, each participant who had knowledge of the standard deviation was unable to describe the procedure; however, some of these participants did go on to describe it as a measure of spread. In the discussion of this paper, it was suggested that not knowing the procedure benefited the students as they had to think about standard deviation more conceptually, something they did not need to do with the mean (Cook & Fukawa-Connelly, 2015).

In a different study that examined student understanding of mean, median and standard deviation at the conclusion of a first university course on statistics, some students described the mean more conceptually in comparison to the study of incoming knowledge. In a survey, 28% of the respondents used the notion of center or representativeness in their descriptions of the mean, and nearly all respondents cited the calculating formula either alone or in conjunction with more conceptual descriptions. In contrast, very few respondents cited the calculating formula for the standard deviation and used phrases to describe smaller standard deviations as data sets with "more data in the middle" (Cook & Fukawa-Connelly, 2014), a finding consistent with a study exclusively on student understanding of the standard deviation (delMas & Liu, 2005).

This limited amount of research indicates that most students leave their first statistics course with a similar understanding of the mean as when they entered. This understanding are similar to the understandings held by the children in Mokros and Russell's study (1995).

What Do University Statistics Instructors Believe About their Students and the Mean?

The following data comes from a small online survey of current introductory statistics instructors. In the survey, 16% of respondents reported that they have no assumptions about their students' understanding of the mean and 63% assume their students can calculate the mean of a set of data. Additionally, 32% expect that their students will enter class with an understanding of what will happen to a mean if particular pieces of data are removed from the data, or new data points are included (ie get bigger, smaller or stay the same). In the study referenced above, less than half of students were able to answer a question of this type correct at the end of a statistics course, with many citing that they needed to know what all the data points were.

This survey also asked what aspects of the mean that they explicitly cover in class, with the most common responses being:

- Mean affected by outliers (100%)
- Mean is a measure of central tendency (95%)
- How to calculate a mean (89%)
- How a mean will change if data is added or removed (84%)

However, when asked how many minutes of class time they spend over then entire course covering the mean as it's own concept, the average amount of time was 15 minutes (CI: 9, 21), with 16% of respondents reporting they spend no time and 21% report spending under 5 minutes. Thus, despite 37% of respondents reporting they spend between 0 and 5 minutes teaching explicitly about the mean, over 80% report that the do explicitly teach the four items above. This seems to imply that instructors see teaching the mean explicitly as a brief review, or that students have a robust enough understanding coming in that they will easily be able to pick up important concepts related to data fluency and inference. This poster will also explore how university statistics text books introduce the mean and what conceptual aspect of the mean are explicitly covered.

Implications.

I believe that instructors and students have both miss-assessed student understanding of the mean as a trivial concept, and these assumptions (of both students and instructors) potentially hinder learning of more advanced statistical concepts. More research is required to defend this belief; however, these studies support that students rely on a procedural understanding of the mean and instructors believe they have (or can quickly develop) a conceptual understanding of the mean. The actual implications of this misalignment are unknown.

References

Cook, S. A., & Fukawa-Connelly, T. (2015). The incoming statistical knowledge of undergraduate majors in a department of mathematics and statistics. *International Journal of Mathematical Education in Science and Technology*, (ahead-of-print), 1-18.

Cook, S**.,** & Fukawa-Connelly, T. (2014); *Student Understanding of Mean and Median after a First University Course.* SIGMAA RUME: Conference on Research in Undergraduate Mathematics Education. Denver, CO

Cook, S., & Fukawa-Connelly, T. (2012). Toward a theory of symbol sense in undergraduate statistics. In *Proceedings of the 2012 SIGMAA RUME: Conference on research in undergraduate mathematics education*. Portland, OR. Retrieved from http://sigmaa.maa.org/rume/crume2012

delMas, R. & Liu, Y. (2005). Exploring students' conceptions of the standard deviation. *Statistics Education Research Journal, 4*(1), 55-82.

Mokros, J. & Russell, S. (1995). Children's concepts of average and representativeness. *Journal for Research in Mathematics Education, 26*(1), 20-39.