A national investigation of Precalculus through Calculus 2

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We present findings from a recently completed census survey of all mathematics departments that offer a graduate degree (Master's and/or PhD) in mathematics. The census survey is part of a larger project investigating department-level factors that influence student success over the entire progression of the introductory mathematics courses that are required of most STEM majors, beginning with Precalculus and continuing through the full year of single variable calculus. The findings paint a portrait of students' curricular experiences with Precalculus and single variable calculus, as well as the viewpoints held by departments of mathematics about that experience. We see that departments are not unaware of the value of particular features characteristic of more successful calculus programs, but that they are not always successful at implementation. However, our data also suggest hope for the future. Our work not only reveals what is currently happening, but also what is changing, how, and why.

Keywords: Census Survey, Precalculus, Calculus, Program success

There is a growing body of research pointing to why students are leaving STEM fields in general and first-year mathematics courses in particular. Contrary to common belief, introductory mathematics courses are not serving as a filter for students who are academically unprepared (Steen, 1988). Students who leave STEM majors are consistently shown to be as academically prepared as their persisting counterparts (Berrett, 2011; Rasmussen & Ellis, 2013; Reich, 2011; Taylor, 2011). Instead, students leaving STEM fields often cite poor instructional experiences in introductory level courses as the primary reason for their departure. These results are consistent with Tinto's integration framework, which emphasizes the effects of student engagement and integration on retention, especially in the first year of college (Kuh et al., 2008; Tinto, 1975, 2004). Integration occurs through a negotiation between the students' incoming social and academic norms and the norms of the department and broader institution. From this perspective, student persistence is viewed as a function of the dynamic relationship between the student and other actors within the institutional environment, including the classroom environment.

Literature focusing on student success in the pre-calculus to calculus sequence provides further insights into why students are leaving first-year mathematics courses (and therefore STEM fields). This research consistently indicates that: students are not learning what we want them to in these courses (Breidenbach et al., 1992; Carlson, 1998; Tallman et al., 2015; Thompson, 1994); these courses are not adequately preparing students for subsequent courses (Carlson, 1995, 1998; Selden & Selden, 1994; Thompson, 1994); students lose interest in STEM after taking these courses (Bressoud, Mesa, & Rasmussen, 2015; Seymour & Hewitt, 1997). These findings point to significant shortcomings in students' experiences. Unfortunately, many of these studies are focused on a limited number of institutions, a small number of students, or a single course.

What is currently missing is a national portrait of students' Precalculus through calculus curricular experiences and how these experiences relate to what is known about effective programs that support student success. In this paper we present initial findings from the *Progress though Calculus* (PtC) project, which builds on the insights from a recently completed five-year project, *Characteristics of Successful Programs in College Calculus* (CSPCC) (Bressoud, Mesa, & Rasmussen, 2015). The overall goal of the PtC project is to investigate, at a national level, department-level factors that influence student success over the entire progression of the introductory mathematics courses that are required of most STEM majors, beginning with Precalculus and continuing through the full year of single variable calculus. We refer to this sequence as Precalculus to Calculus 2 (P2C2).

As reported in Bressoud & Rasmussen (2014), the CSPCC study found that institutions with more successful Calculus I programs shared many of the following characteristics: (1) Calculus I was coordinated across sections and individual instructors contributed significantly to communal course decisions; (2) faculty used local data to check on the effectiveness of their program and make improvements; (3) for programs that made use of graduate, there was an extensive training of the Graduate-student Teaching Assistants; (4) faculty supported and encouraged active learning strategies; (5) the department had rigorous courses and high expectations for students; (6) the university offered many student supports, such as all day free tutoring centers and Supplemental Instruction; and (7) had adaptive placement systems that sought to place students in the highest course for which they could succeed.

In this report we address the following research questions:

1. How do mathematics departments prioritize the importance of the seven characteristics found in the CSPCC study?
2. How do mathematics departments characterize their implementation of the practices of successful programs identified in CSPCC study, what changes are being considered, and why?
3. What instructional format and structures (e.g., bridge courses, stretched out calculus) are currently in place in the P2C2 sequence and how common are they nationally?

Methods

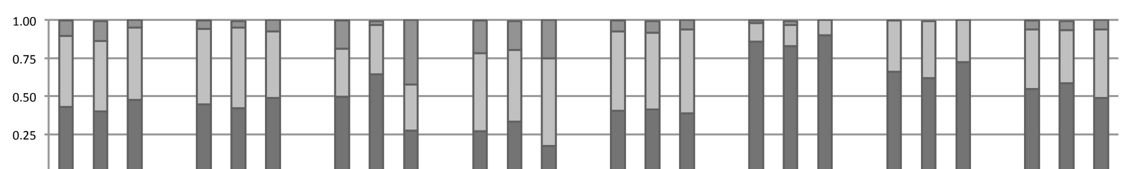
The five-year PtC project, which began in early 2015, is being conducted in two phases. Phase 1 is a census survey of all mathematics departments that offer a graduate degree in mathematics. These institutions were selected because they produce the bulk of STEM graduates while often struggling to find a balance between the demands of research and teaching. Phase 2 will consist of in depth longitudinal case studies. In this report we focus on initial findings from the census survey. In the United States there are a total of 341 institutions that offer either a Masters or PhD in mathematics. All 341 institutions, which included 181 PhD granting institutions and 160 Master's degree granting institutions, were surveyed. The overall response rate was 59.5%, with a response rate of 68% from the PhD institutions and 50% from the Master's institutions.

We designed the census survey to gather information on the implementation of the seven features of successful programs identified by the CSPCC project as well as to gain an understanding of the variety of P2C2 programs currently being implemented across the country, the prevalence of such programs, and what institutions are doing to improve their programs. The survey consisted of three main parts. Part I asked for a list of all courses in the mathematics department mainstream Precalculus/calculus sequence. "Mainstream" refers to any course in this sequence that would be part of student preparation for higher-level mathematics courses such as a sophomore- or junior-level course in differential equations or linear algebra. Part II asked about the departmental practices in support of the Precalculus/calculus sequence. Part III asked for detailed information about each course in the mainstream P2C2 sequence, including enrollment data and details about course delivery. The survey was closed mid August 2015.

Given the fact that the survey has only recently been closed, we begin analysis of the cleaned data set with descriptive statistics (counts, frequencies, means, standard deviations) which will then be followed by additional descriptive methods (e.g., Multiple Correspondence Analysis; clustering; Principal Components Analysis) to reveal patterns in the census data. Our aim is to identify models of existing P2C2 programs in their entirety rather than simply identifying patterns within individual components.

Sample Results

For research question 1, our data allow us to see how departments of mathematics view the practices identified in CSPCC as characteristic of successful institutions. Participants were asked to consider eight characteristics and group them by their importance to a successful P2C2 sequence. The results from this question are summarized in Figure 1. Note that in general, PhD- and MA-granting institutions agree on the importance of individual features, with the exception of GTA teaching preparation programs, in which case MA-granting institutions report the feature as less important than PhD-granting institutions. Of course many MA-granting institutions do not make extensive use of GTAs and so this difference is expected.

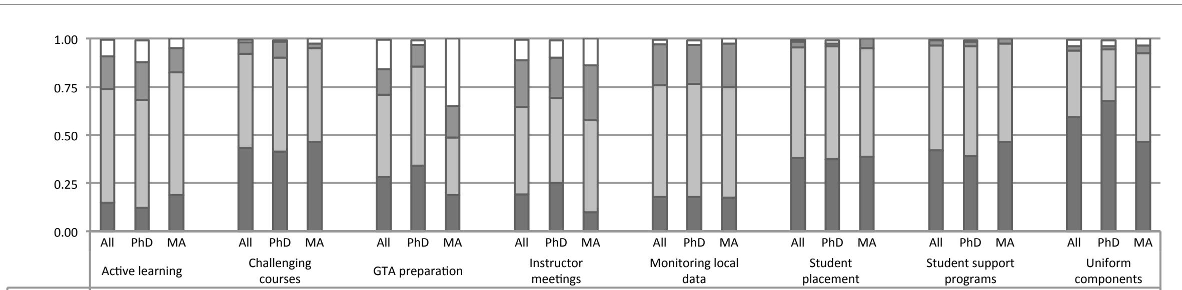


	Active learning			Challenging courses			GTA preparation			Instructor meetings			Monitoring local data			Student placement			Student support programs			Uniform components		
	All	PhD	MA	All	PhD	MA	All	PhD	MA	All	PhD	MA	All	PhD	MA	All	PhD	MA	All	PhD	MA	All	PhD	MA
Not	0.10	0.13	0.05	0.05	0.04	0.08	0.18	0.02	0.43	0.21	0.19	0.25	0.07	0.07	0.06	0.01	0.02	0	0	0	0	0.06	0.06	0.06
Somewhat	0.47	0.46	0.48	0.49	0.53	0.44	0.32	0.33	0.30	0.51	0.47	0.58	0.52	0.50	0.55	0.12	0.14	0.10	0.33	0.37	0.28	0.39	0.35	0.45
Very	0.43	0.40	0.48	0.45	0.42	0.49	0.50	0.64	0.28	0.27	0.33	0.18	0.40	0.41	0.39	0.86	0.83	0.90	0.66	0.62	0.73	0.55	0.59	0.49

Figure 1. Reported importance of CSPCC features for successful P2C2 sequence. $N_{all} = 203$, $N_{PhD}=123$, $N_{MA}=80$.

Further, participants were asked how successful their program is with each of these features. The results from this question are summarized in Figure 2. Again we see general agreement between the institution types as to their relative success at implementation, with the exception of GTA teaching preparation, where MA-granting institutions reported a much higher rate of "NA."

Student placement and student support programs are the two CSPCC features where the widest gap was observed between perceived importance and perceived success. Both were reported as very important to the success of a P2C2 sequence, but most participants reported that they were only somewhat successful at implementation of these features.

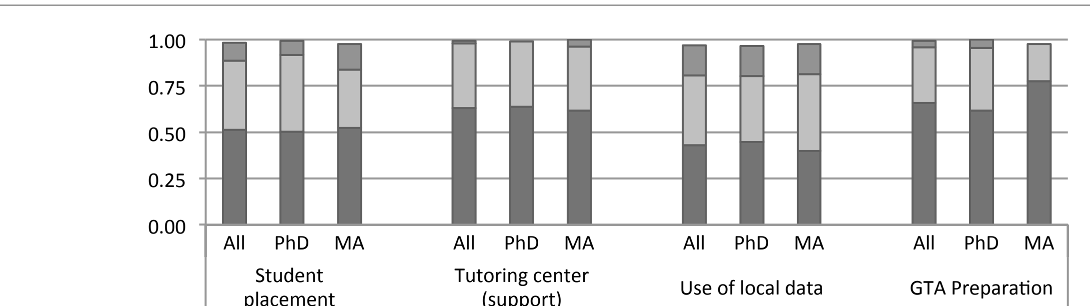


	Active learning All	Active learning PhD	Active learning MA	Challenging courses All	Challenging courses PhD	Challenging courses MA	GTA preparation All	GTA preparation PhD	GTA preparation MA	Instructor meetings All	Instructor meetings PhD	Instructor meetings MA
NA	0.09	0.11	0.05	0.01	0.01	0.03	0.15	0.02	0.35	0.11	0.09	0.14
Not	0.17	0.20	0.13	0.06	0.08	0.03	0.13	0.11	0.16	0.24	0.21	0.29
Somewhat	0.59	0.56	0.64	0.49	0.49	0.49	0.43	0.51	0.30	0.45	0.44	0.48
Very	0.15	0.12	0.19	0.43	0.41	0.46	0.28	0.34	0.19	0.19	0.25	0.10

	Monitoring local data All	Monitoring local data PhD	Monitoring local data MA	Student placement All	Student placement PhD	Student placement MA	Student support programs All	Student support programs PhD	Student support programs MA	Uniform components All	Uniform components PhD	Uniform components MA
NA	0.02	0.02	0.03	0.01	0.02	0.00	0.00	0.01	0.00	0.03	0.03	0.04
Not	0.21	0.20	0.23	0.03	0.02	0.05	0.02	0.02	0.03	0.02	0.02	0.04
Somewhat	0.58	0.59	0.58	0.58	0.59	0.56	0.55	0.57	0.51	0.34	0.27	0.46
Very	0.18	0.18	0.18	0.38	0.37	0.39	0.42	0.39	0.46	0.59	0.67	0.46

Figure 2. Reported success at CSPCC features for successful P2C2 sequence. N_{all} = 203, N_{PhD}=123, N_{MA}=80.

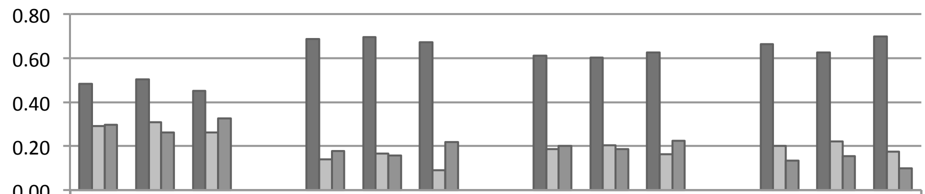
Regarding research question 2, our data also captures detailed aspects of how CSPCC practices are being implemented by departments of mathematics as well as what changes are being planned and why. In the full paper we will report on how departments initially place students into the P2C2 sequence, how they gather and use local data to monitor and modify the sequence, and what supports (in particular tutoring centers) are in place for P2C2 students. A separate proposal has been submitted that details how GTAs are prepared for their teaching roles. We also have information about satisfaction levels and the status of these features (i.e., if changes have recently occurred or are being planned). These results are summarized in Figures 3 and 4. Satisfaction ratings with the tutoring center and GTA preparation programs were collected only from departments reporting that they have these programs in place, while queries about status were asked of all participants.



	Student placement All	Student placement PhD	Student placement MA	Tutoring center (support) All	Tutoring center (support) PhD	Tutoring center (support) MA	Use of local data All	Use of local data PhD	Use of local data MA	GTA Preparation All	GTA Preparation PhD	GTA Preparation MA
Not adequate	0.10	0.07	0.14	0.01	0.00	0.04	0.16	0.16	0.16	0.03	0.05	0.00
Could be improved	0.37	0.42	0.31	0.35	0.35	0.35	0.38	0.36	0.41	0.30	0.34	0.20
Adequate	0.51	0.50	0.53	0.63	0.64	0.62	0.43	0.45	0.40	0.66	0.62	0.78

Figure 3. Satisfaction levels with selected CSPCC features. Sample sizes for student placement and use of local data: N_{All}=203, N_{PhD}=123, N_{MA}=80. For the tutoring center: N_{All}=157, N_{PhD}=102, N_{MA}=55. For GTA preparation: N_{All}=150, N_{PhD}=110, N_{MA}=40.

Again we see that PhD- and MA-granting institutions report similar levels of satisfaction for each of these program features. However, the reports of being satisfied (program is adequate) are higher than reports of being "very successful" with these same programs. This appears to indicate that many departments are satisfied with being somewhat successful in their management of the P2C2 sequence.



	Student placement			Tutoring Center (Support)			Use of local data			GTA Preparation		
	All	PhD	MA	All	PhD	MA	All	PhD	MA	All	PhD	MA
No changes planned	0.48	0.50	0.45	0.69	0.70	0.67	0.61	0.60	0.63	0.67	0.63	0.70
Recent changes	0.29	0.31	0.26	0.14	0.17	0.09	0.19	0.20	0.16	0.20	0.22	0.18
Under discussion	0.30	0.26	0.33	0.18	0.16	0.22	0.20	0.19	0.23	0.13	0.15	0.10

Figure 4. Status of selected CSPCC features. N_{all} = 203, N_{PhD}=123, N_{MA}=80. Options were not mutually exclusive.

The data regarding the status of individual CSPCC features indicates that most departments of mathematics are *not* planning changes to department-run tutoring centers, their use of local data, or GTA teaching preparation programs. While about half of participating schools indicate that no changes to their placement procedures are planned, it seems that this feature is the least static, tallying with our discovery that institutions across the nation feel that initial placement into the P2C2 sequence is important and that they are not entirely successful with this process. That most departments are not planning changes to their tutoring centers is more surprising, as it was widely reported that student supports are important to successful programs and departments do not report high rates of success. Note that in with all four features, the reports of "no changes planned" are higher than rates of "very successful." It appears that while many departments believe they are not entirely successful with their implementation of these CSPCC practices, they are not prepared to amend these processes.

In addition to the broad characterizations of satisfaction and status of department programs presented in this proposal, our presentation will include details of how the seven features identified in the CSPCC study are implemented across the nation with regards to the P2C2 sequence. For placement this will include initial placement procedures (e.g., AP exam results; MAA placement exam) and what (if any) procedures for revisiting and adjusting initial placement exist. Resources to support students include detailed information about the existence and format of tutoring centers for students in the P2C2 sequence, as well as supports available to students (e.g., online tutoring; arranged study groups) and any supports aimed particularly toward "at-risk" and/or underrepresented groups in mathematics (e.g., scholarships; targeted supplemental instruction). We will present also the types of local data that departments of mathematics collect and how departments have been using that data to inform decisions about their undergraduate program. In addition to reporting on the variety of implementation models, we will report on their relative frequency across institutions.

For research question 3, information about P2C2 instruction and structures in place across the nation was ascertained through Part III of the census survey. Therein, participants were queried about details regarding the implementation of each course that is part of the mainstream P2C2 sequence at their institution. Of particular interest are the data regarding primary instructional format. We collected detailed information about 1060 P2C2 courses, and found that nearly 70% of these are reportedly taught in a lecture format and 15% are taught in a format that incorporates some active learning techniques alongside lecture. Around 10% of courses did not report a single primary instructional format, and fewer than 5% fell into the categories: mainly active learning (including flipped), lecture plus computer-based instruction, or "other." These values reflect the primary instructional format across all P2C2 courses, but the general pattern is the same for each category of courses (e.g., Precalculus courses, Calculus I courses). However, we note that the proportion of classed being taught in traditional lecture format increases through the sequence (from 57% to 75%), while all other formats decreased in frequency. That fewer than 20% of all P2C2 courses report incorporating any level of active learning is remarkable, particularly in light of the fact that 43% of institutions noted that active learning strategies are "very important" to having a successful P2C2 sequence, and 74% reported being at least "somewhat successful" at implementing active learning strategies.

In the presentation we will provide more detail as to different P2C2 progressions that are in place as well as course-specific details such as enrollment data, DFW rates, instructor profiles, contact hour breakdown, prevalence and form of recitation sections, coordinated aspects of parallel sections, coordinator profiles, and the status of each course (e.g., if changes are being discussed). This will further illuminate what P2C2 sequences are experienced by undergraduates across the country.

Conclusion

This paper provides the first overview of the information we have gathered with regard to introductory undergraduate mathematics programs across the country. The findings paint a national portrait of students' curricular experiences with Precalculus and single variable calculus, as well as the viewpoints held by departments of mathematics about that experience. We see that departments are not unaware of the value of particular features, but that they are not always successful at implementation. However, our data also suggest hope for the future. Our work not only reveals what is currently happening, but also what is changing, how, and why. We note that many institutions reported in open-ended questions that they *want* to make improvements, but are not sure how. We believe that our work will not only describe what is happening in mathematics departments at the national scale, but will illuminate ways of reaching institutions interested in change – of which there are many. One institution wrote to us saying, "We should do more. This survey is giving me ideas." We suspect there are many other institutions ready for change. This report provides a first in its kind baseline of what is happening in the P2C2 sequence across the nation.

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