Opportunity to learn solving context-based tasks provided by business calculus textbooks: An exploratory study

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The purpose of this study was to investigate the opportunities to learn how to solve realistic context-based problems that undergraduate business calculus textbooks in the United States offer to business and/or economics students. To do this, we selected and analyzed examples and practice problems from six different textbooks that are widely used in the teaching of business calculus nationwide. There are three major findings from this study: (1) a majority of the tasks in all the textbooks uses a camouflage context, (2) all the tasks in all the textbooks have matching information, and (3) only three textbooks had reflection tasks. The findings of this study suggest that business calculus textbooks do not offer students rich and sufficient opportunities to learn how to solve realistic problems in a business and/or economic context.

Key words: opportunity to learn, textbook research, textbook analysis, business calculus, context-based tasks

Introduction

The concept of opportunity to learn (OTL) as it relates to mathematics instruction originated in the early 1960s. Carroll (1963) defined OTL as the time allowed for learning a particular topic. This study uses Husen's (1967) definition of OTL which, according to Floden (2002), is also the most common definition of OTL used in the mathematics education research literature. According to Husen, OTL refers to "whether or not … students have had the opportunity to study a particular topic or learn how to solve a particular type of problem" (pp. 162-163). Mathematics textbooks are one such opportunity from which students can learn how to solve certain types of problems. The role of mathematics textbooks as an opportunity to learn mathematics is well documented in the research literature on the learning of pre-university mathematics. Referring to mathematics textbooks, Reys, Reys, and Chavez (2004) argued "that the choice of textbooks often determines what teachers will teach, how they will teach it, and how their students will learn" (p. 61).

The existing research on pre-university mathematics focuses on students' opportunities to learn mathematical topics such as linear functions and trigonometry (Wijaya, van den Heuvel-Panhuzen, & Doorman, 2015), addition and subtraction of fractions (Alajmi, 2012; Charalambous et al., 2010), probability (Jones & Tarr, 2007), statistics (Pickle, 2012), reasoning and proof (Stylianides, 2009; Thompson et al., 2012), proportional reasoning (Dole & Shield, 2008), and deductive reasoning (Stacey & Vincent, 2009). Research on students' opportunities offered by mathematics textbooks to learn other mathematics topics, such as optimization, especially at the upper secondary and undergraduate level is lacking.

To our knowledge, only one study (Mesa, Suh, Blake, & Whittemore, 2012) examined students' opportunity to learn about the family of exponential functions, the family of logarithmic functions, and transformations of graphs provided by mathematics textbooks at the post-secondary level. The scarcity of research that examines the opportunity to learn how to solve context problems provided by college mathematics textbooks, is a motivation for this study. In particular, this study sought to examine the opportunity to learn how to solve realistic context problems (optimization problems) that occur in business and/or economics provided by undergraduate business calculus textbooks that are used nationwide. Our study sought to answer the following research questions:

1. *What type(s) of context (relevant and essential, camouflage, no context) is characteristic of tasks that are found in undergraduate business calculus textbooks?*
2. *What type(s) of information (matching, missing, superfluous) is characteristic of tasks that are found in undergraduate business calculus textbooks?*
3. *What type(s) of cognitive demand (reproduction, connection, reflection) is characteristic of tasks that are found in undergraduate business calculus textbooks?*

In this study, we viewed tasks as optimization examples and practice problems given at the end of each optimization section in each of the textbooks that we selected for the study. A description of the types of contexts, types of information, and types of cognitive demands is given in the methods section. Having rich opportunities to learn how to solve realistic optimization problems that are situated in a business or economic context is essential for students in several fields of study such as marketing, supply chain management, finance, and economics. According to Gordon (2008), over 300,000 students enroll in business calculus each year in the Unites States. We are not aware of any existing work where this particular context (business or economic) and type of problem (optimization) has been studied, which is the motivation for our study.

Relevant literature

The term, context, has been defined in several ways by researchers in mathematics education. Our view on the meaning of context is consistent with that given by White and Mitchelmore (1996). These researchers posited that "in calculus, the context of an application problem may be a realistic or artificial "real-world" situation, or it may be an abstract, mathematical context at a lower level of abstraction than the calculus concept that is to be applied" (p. 81). White and Mitchelmore's understanding of the term context is consistent with that of other researchers (e.g., Gravemeijer & Doorman, 1999; van den Heuvel-Panhuizen, 2005). According to Wijaya et al. (2015), mathematical tasks could have a realistic context, a camouflage context, or they could be bare (only mathematical symbols). Alajmi (2012) refers to bare mathematical tasks as tasks that are situated in a "purely mathematics context" (p. 243). Tasks with a camouflage context "are merely dressed up bare problems, which do not require modeling because the mathematical operations needed to solve the task are obvious" (Wijaya et al., 2015, p. 45). A realistic context is also referred to as a relevant and essential context in the research literature (e.g., de Lange, 1995; van den Heuvel-Panhuizen, 2005).

Several researchers (e.g., Maass, 2007; Maass, 2010; Wijaya et al., 2015) have identified three types of information that could be in a mathematical task: matching, missing, and superfluous. A mathematical problem with matching information is one in which all the information required to solve the problem is given in the problem statement. A mathematical problem has missing information if some of the information needed to solve the problem is not immediately available to the solver, that is, the solver has to deduce this information from the problem statement. A mathematical problem with superfluous information is one in which the problem statement not only contains the necessary information needed to solve the task but it also contains other extraneous or irrelevant information that may not be helpful in solving the given problem. Wijaya et al. (2015) argued that:

> Providing more or less information than needed for solving a context-based task is a way to encourage students to consider the context used in the task and not just take numbers out of the context and process them mathematically in an automatic way. (p. 45)

Maass (2010) recommended that students should be given opportunities to deal with these three different types of information.

A related line of research (e.g., Charalambos et al., 2010; Kolovou et al., 2009; Mesa et al., 2012; Wijaya et al., 2015) has investigated the types of cognitive demands in tasks that are presented in mathematics textbooks. The types of cognitive demands are: reproduction, connection, and reflection. These types of cognitive demands are similar to the levels of cognitive demands discussed by Stein, Grover, & Henningsen (1996). Reproduction tasks are routine problems that require the lowest level of cognitive demand to solve. These problems can be easily solved using memorized mathematical algorithms. Connection tasks are non-routine in nature and may require the solver to represent concepts in multiple representations: algebraically, numerically, graphically, and verbally. Mesa et al. (2012) analyzed, among other things, the cognitive demands of examples as well as the representations of these examples given in 10 college algebra textbooks. Five of these textbooks are used at community colleges, three textbooks are used at four-year institutions, and the other two textbooks are used at both community colleges and at four-year institutions. Mesa and colleagues found that "textbooks, independent of the type of institution in which they are used, present examples that have low cognitive demands, expect single numeric answers, emphasize symbolic and numerical representations, and give very few strategies for verifying correctness of the solutions" (p. 76). Reflection tasks require the highest level of cognitive demand to solve. These tasks "include complex problem situations in which it is not obvious in advance what mathematical procedures have to be carried out" (Wijaya et al., 2015, p. 46).

Theoretical framework

With a focus on the role of context in mathematical tasks found in business calculus textbooks, this study draws on the theory of realistic mathematics education (RME) which is both a theory of teaching and learning in mathematics education that originated in the Netherlands in the early 1970s. As a theory of learning, RME emphasizes that students should be asked to solve realistic contextual problems that are not only realistic in the sense of being connected to a real-world context but also that the context of these problems should be experientially real to the students. That is, students should be asked to solve "problem situations which they can imagine" (van den Heuvel-Panhuizen, 2000, p. 4). The economic or business context as it relates to optimization tasks may be experientially real for some students taking business calculus. This is especially true for students who take business calculus after having taken high school or college economics classes. In addition to the role of context, optimization tasks in business calculus textbooks may vary in terms of types of information and types of cognitive demands. These various types of information and cognitive demands are explained in the analytical framework presented in the next section.

Methods

The study followed a qualitative research design. Data for the study consisted of optimization examples and practice problems from undergraduate business calculus textbooks that are widely used in the teaching of business calculus in the Unites States.

Data collection and setting

To answer the research questions, we analyzed a total of 195 optimization examples and practice problems selected from the latest editions of six undergraduate mathematics textbooks that are widely used in the instruction of business calculus at large universities in the United States. Table 1 shows a list of the textbooks that were selected for the study. The textbooks were selected through google search using key words such as “business calculus textbook,” “business mathematics textbook,” “applied calculus textbooks,” etc.

Table 1

Analyzed Textbooks

Textbook Name	Author (s)	Textbook Abbrev-iation	Section(s) analyzed	Textbook Publisher
Applied Calculus for the Managerial, Life, and Social Sciences (9th ed)	Tan, S. T. (2013)	TBK 1	Optimization I Optimization II (chapter 4)	Brooks/Cole (Cengage Learning)
Introductory Mathematical Analysis for Business, Economics, and the Life and Social Sciences (13th ed)	Haeussler, E. F., Paul, R. S., & Wood, R. J. (2011)	TBK 2	Applied Maxima and Minima (chapter 13)	Pearson Education
Applied Calculus (6th ed)	Waner, S. & Costenoble, W. (2014)	TBK 3	Applications of Maxima and Minima (chapter 5)	Brooks/Cole (Cengage Learning)
Applied Calculus (5th ed)	Hughes-Hallet et al. (2013)	TBK 4	Profit, Cost, and Revenue Average Cost (chapter 4)	Wiley
Calculus and its Applications (11th ed)	Bittinger, M. L., Ellenbogen, D. J., & Surgent, S. J. (2015)	TBK 5	Maximum-Minimum Problems; Business, Economics, and General Applications (chapter 2)	Pearson Education
Calculus for Business, Economics, Life Sciences, and Social Sciences (13th ed)	Barnett, R. A., Ziegler, M. R., & Byleen, K. E. (2015)	TBK 6	Optimization (chapter 4)	Pearson Education

Table 2

Analytical Framework

Task Characteristic	Sub-category	Explanation
Type of context	No context	-Refers to only mathematical objects, symbols, or structures.
	Camouflage context	-Experiences from everyday life or common sense reasoning are not needed. -The mathematical operations needed to solve the problems are already obvious. -The solution can be found by combining all numbers given in the text.
	Relevant and essential context	-Common sense reasoning within the context is needed to understand and solve the problem. -The mathematical operation is not explicitly given. -Mathematical modeling is needed.
Type of information	Matching	-The task contains exactly the information needed to find the solution.
	Missing	-The task contains less information than needed so students need to find the missing information.
	Superfluous	-The task contains more information than needed so students need to select information.
Type of cognitive demand	Reproduction	-Reproducing representations, definitions, or facts. -Interpreting simple and familiar representations. -Memorization or performing explicit routine computations/procedures.
	Connection	-Integrating and connecting across content, situations, or representations. -Non-routine problem solving. -Interpretation of problem situations and mathematical statements. -Engaging in simple mathematical reasoning.
	Reflection	-Reflecting on, and gaining insight into, mathematics. -Constructing original mathematical approaches. -Communicating complex arguments and complex reasoning.

Table 2: Analytical Framework reproduced from (Wijaya et al., 2015, p. 52)

Data analysis

The data (optimization examples and practice problems) were coded using the mathematics textbook analysis framework developed by Wijaya et al. (2015) shown in Table 2. In particular, there were three dimensions of analysis, namely, type of context, type of information, and type of cognitive demand. We coded a total of 195 tasks. We illustrate our coding with the following examples:

Acrosonic's total profit (in dollars) from manufacturing and selling x units of their model F loudspeaker systems is given by $P(x) = -0.02x^2 + 300x - 200{,}000 \quad (0 \leq x \leq 20{,}000)$. How many units of the loudspeaker system must Acrosonic produce to maximize its profit? (TBK 1, p. 301).

We coded this example as: (1) having a camouflage context because the operations needed to solve the problem are already obvious and the context can be ignored when solving this problem, (2) having matching information because it contains the exact amount of information needed to solve it, and (3) a reproduction task because the strategy required to solve it requires performing explicit routine procedures. In the same textbook, this example is also given:

Dixie Import-Export is the sole agent for the Excalibur 250-cc motorcycle. Management estimates that the demand for these motorcycles is 10,000 per year and that they will sell at a uniform rate throughout the year. The cost incurred in ordering each shipment of motorcycles is $10,000, and the cost per year of storing each motorcycle is $200. Dixie's management faces the following problem: Ordering too many motorcycles at one time ties up valuable storage space and increases the storage cost. On the other hand, placing orders too frequently increases the ordering costs. How large should each order be, and how often should orders be placed, to minimize ordering and storage cost? (TBK 1, p. 317).

We coded this task as: (1) having a relevant and essential context because reasoning within the context of the task is needed to understand and solve the problem, (2) having matching information because it contains the exact amount of information needed to solve it, and (3) a reflection task because the solver must construct original mathematical approaches e.g. the average inventory level of $x/2$ if x is the lot size. The results of the coding of all 195 tasks from the six textbooks are summarized in Table 3.

Results

There are three major findings from this study. First, a majority of the optimization tasks given in the business calculus textbooks reviewed in this study use a camouflage context. All of the textbooks except TBK 4 rarely had tasks with no context. Only TBK 5 has a significant number of tasks with a realistic (relevant and essential) context relative to the number of economic problems given in each of the textbooks we analyzed.

Table 3

Textbook Analysis Results

TA	NEP	Type of Context	Type of Information	Type of Cognitive Demand
TBK 1	24	No context: 0	Matching: 24	Reproduction: 19 (79%)
		Camouflage context: 19 (79%)	Missing: 0	Connection: 2 (8%)
		Relevant & essential context: 5 (21%)	Superfluous: 0	Reflection: 3 (13%)
TBK 2	29	No context: 0	Matching: 29	Reproduction: 28 (97%)
		Camouflage context: 26 (90%)	Missing: 0	Connection: 1 (3%)
		Relevant & essential context: 3 (10%)	Superfluous: 0	Reflection: 0
TBK 3	29	No context: 2 (7%)	Matching: 29	Reproduction: 25 (86%)
		Camouflage context: 26 (90%)	Missing: 0	Connection: 3 (10%)
		Relevant & essential	Superfluous: 0	Reflection: 1 (3%)

		context: 1 (3%)		
TBK 4	56	No context: 23 (41%)	Matching: 56	Reproduction: 43 (77%)
		Camouflage context: 31 (55%)	Missing: 0	Connection: 7 (13%)
		Relevant & essential context: 2 (4%)	Superfluous: 0	Reflection: 6 (11%)
TBK 5	32	No context: 8 (25%)	Matching: 32	Reproduction: 13 (41%)
		Camouflage context: 12 (38%)	Missing: 0	Connection: 19 (59%)
		Relevant & essential context: 12 (38%)	Superfluous: 0	Reflection: 0
TBK 6	25	No context: 0	Matching: 25	Reproduction: 12 (48%)
		Camouflage context: 21 (84%)	Missing: 0	Connection: 13 (52%)
		Relevant & essential context: 4 (16%)	Superfluous: 0	Reflection: 0

TA stands for textbook abbreviation. NEP stands for number of economic examples and practice problems in the section(s) of each textbook that we analyzed.

Second, all six textbooks have tasks which contain the exact amount of information students need to solve the tasks. As a result, students do not have to make sense of the context (if any) of the tasks in order to either deduce missing information or identify important information (in the case of superfluous information) that is necessary to solve the tasks from the problem statements. Third, only three textbooks (TBK 1, TBK 3, and TBK 4) had reflection tasks, that is, tasks with a higher cognitive demand. However, the number of such tasks was extraordinarily low, with only 13% (n=3) of the tasks in TBK 1, 3% (n=1) of the tasks in TBK 3, and 11% (n=6) of the tasks in TBK 4. Hence, the opportunity to learn from such tasks via textbooks is minimal. Reproduction tasks were common in all six textbooks.

Discussion and conclusion

The results of this study has some implications for different stakeholders, namely textbook authors, textbook selection committees, and instructors. Textbook authors need to include a much broader range of economic-based optimization examples and practice problems in terms of types of context, types of information, and types of cognitive demands to maximize the learning opportunities provided by their textbooks. Textbook selection committees need to select textbooks that contain a balance of optimization tasks in terms of types of context, types of information, and types of cognitive demands to avoid limiting students' opportunity to learn about optimization problems in an economic context to tasks with matching information, camouflage context, and tasks of low cognitive demand as the findings of this study suggest. Research (e.g., Reys et al., 2004) suggest "that the choice of textbooks often determines what teachers will teach, how they will teach it, and how their students will learn" (p. 61). Business calculus instructors may have to supplement the examples and practice problems given in business calculus textbooks to include tasks with superfluous (or missing) information and/or tasks of higher cognitive demands in order to maximize students' opportunity to learn from such tasks which are rare in the textbooks we analyzed.

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